Quantum electricity: from Ampère to electron quantum optics

P. Degiovanni (ENS Lyon)

C. Chaubet (Univ. Montpellier)

A. San Miguel (Univ. Claude Bernard)

A. Feller & B. Souaille (Univ. Lille)

B. Roussel (Univ. Aalto)

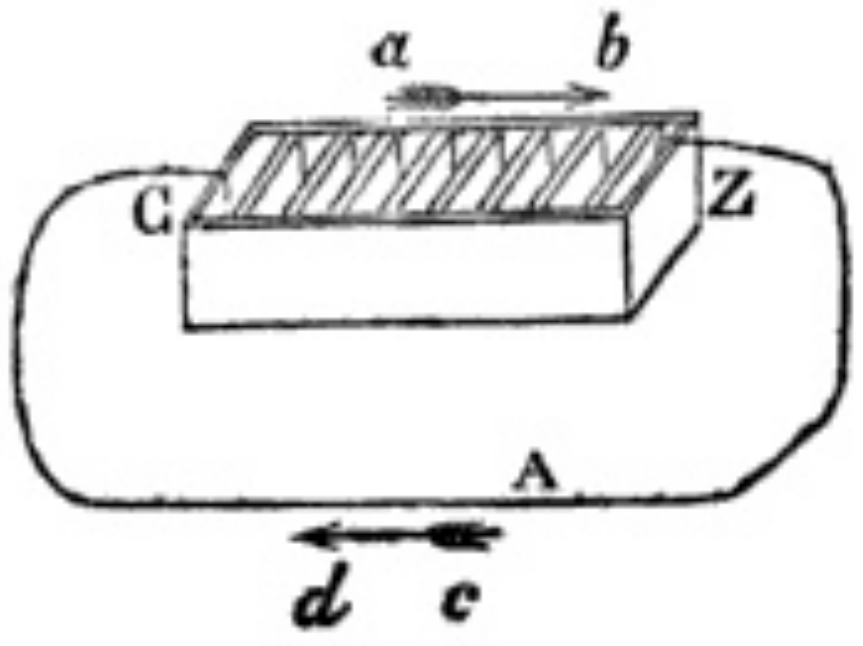
G. Rebora & M. Paulet (ENS Lyon)

Le courant électrique de 1821 à 2025

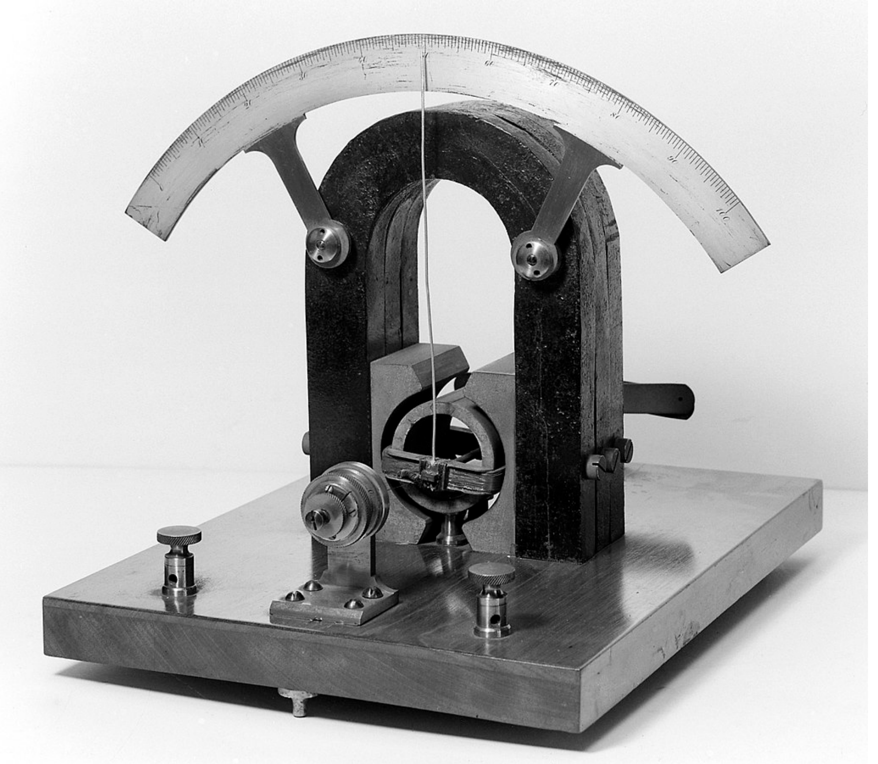
Le courant électrique de 1821 à 2025

A.M. Ampère

Fig. 1.

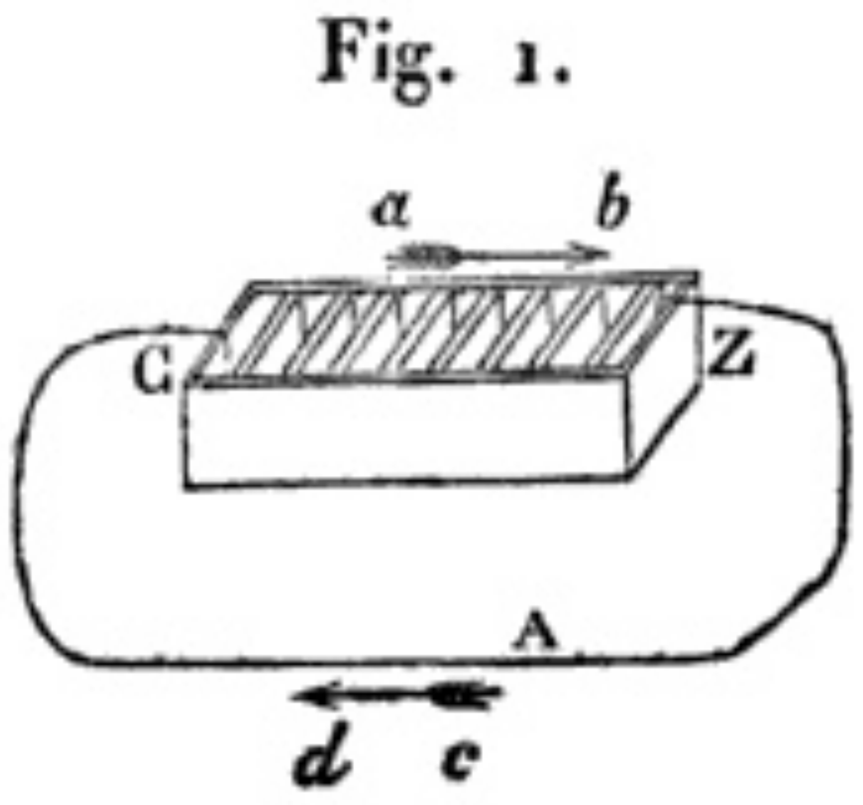


Galvanometer

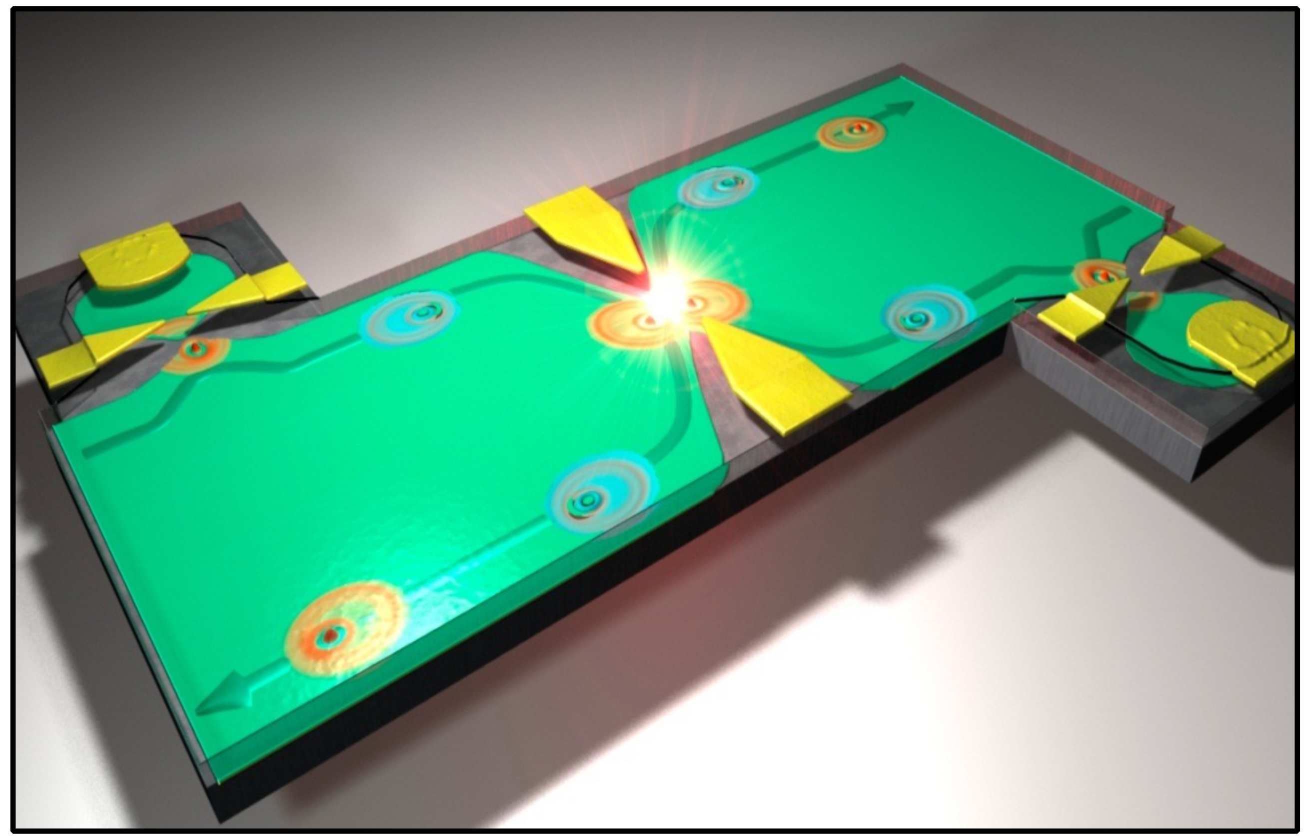


Le courant électrique de 1821 à 2025

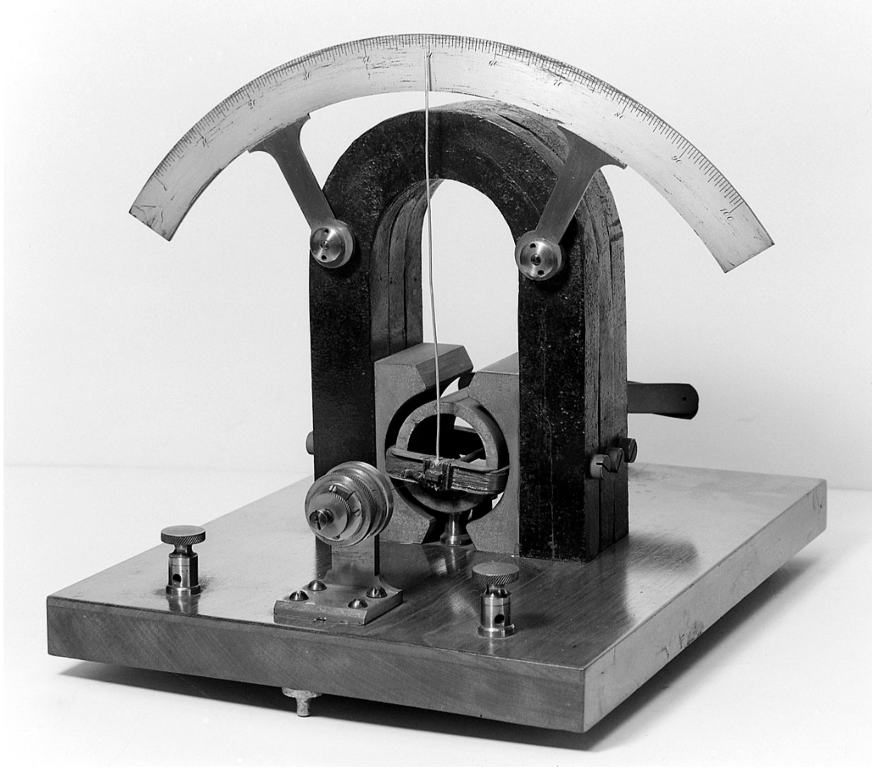
A.M. Ampère

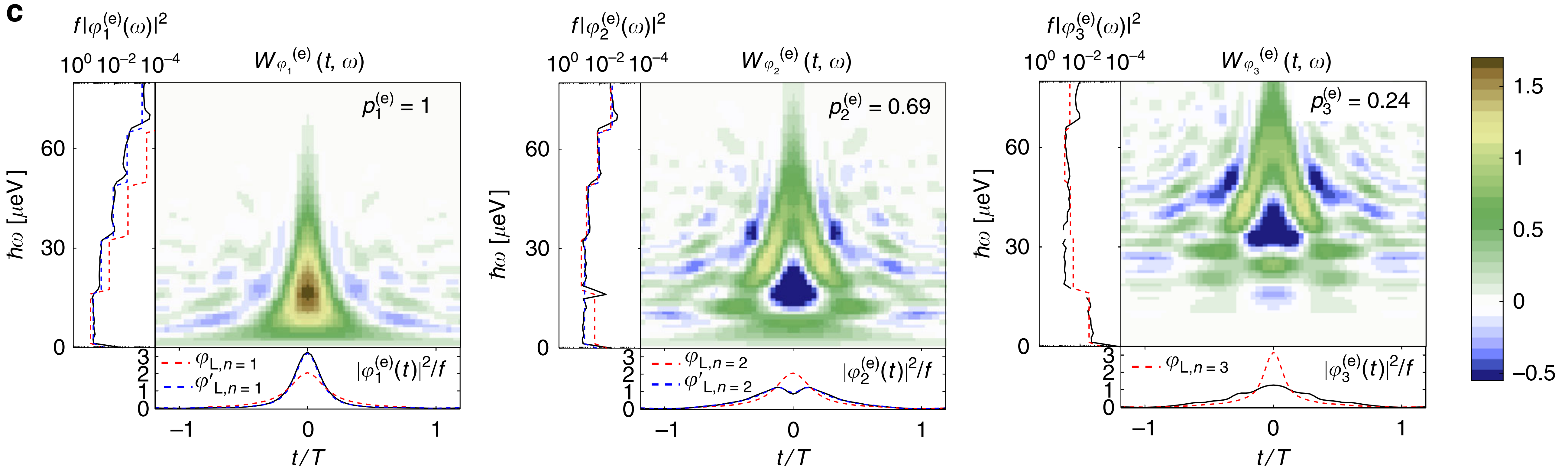


Galvanometer



Quantum current analyzer





Outline

- From Ampère to electrical circuits
- Quantum physics within electricity
- Towards quantum coherent electronics
- Presentation of the PCP 2025 cycle

Part I: From Ampère to electrical circuits

- From Ampère to Maxwell
- Classical circuit theory
- A story of fields, charges and currents
- How does energy flow ?

From Ampère to Maxwell

From Ampère to Maxwell

1775

1836

From Ampère to Maxwell

1775

1836

1800

1820

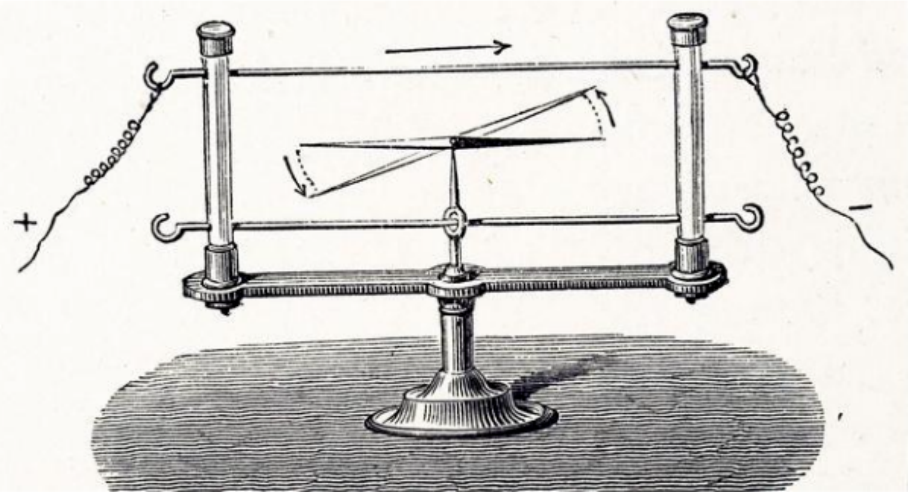
1827

1831

1745

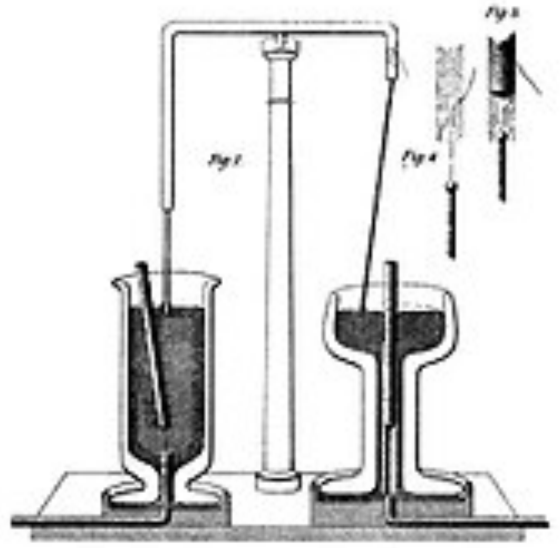
Capacitor

Battery (Volta)

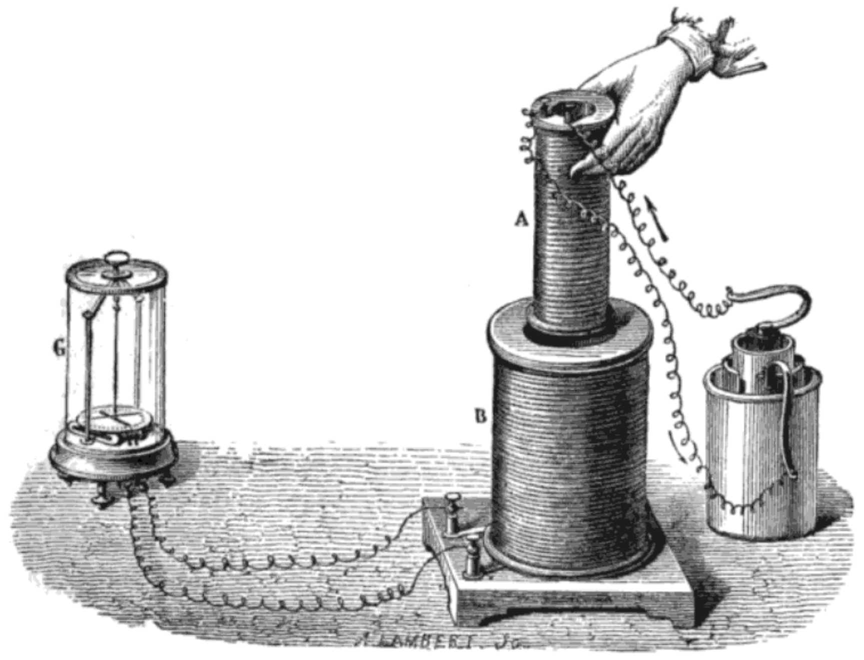


Expérience d'Oersted

Loi d'Ohm

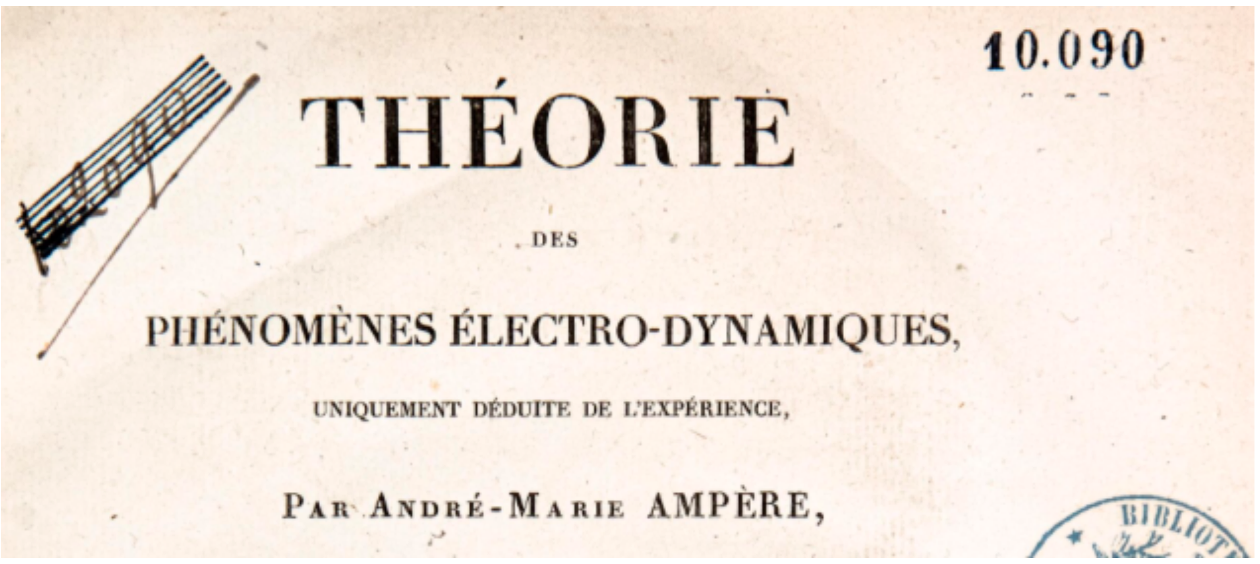


Electromagnetic rotation
(Faraday 1821)



Induction (Faraday)

From Ampère to Maxwell


10.090

THÉORIE

DES

PHÉNOMÈNES ÉLECTRO-DYNAMIQUES,

UNIQUEMENT DÉDUITE DE L'EXPÉRIENCE,

PAR ANDRÉ-MARIE AMPÈRE,

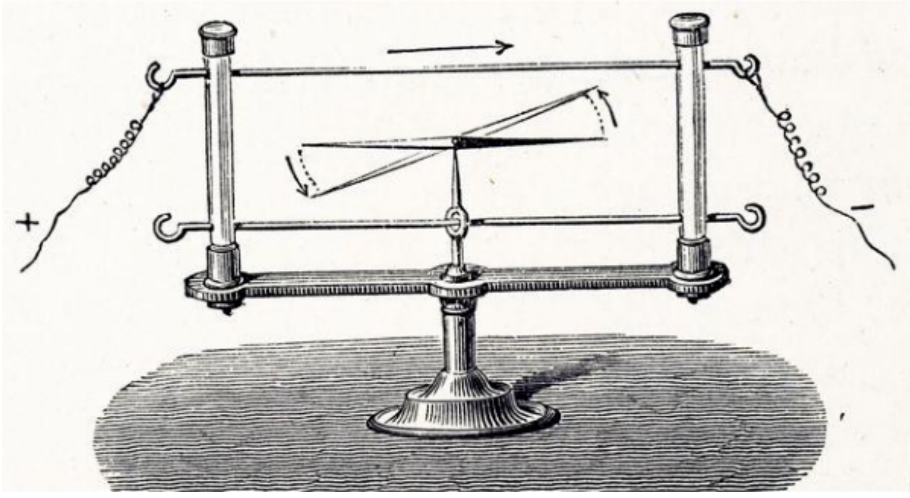
1775 — 1826 — 1836

1800 — 1820 — 1827 — 1831

1745

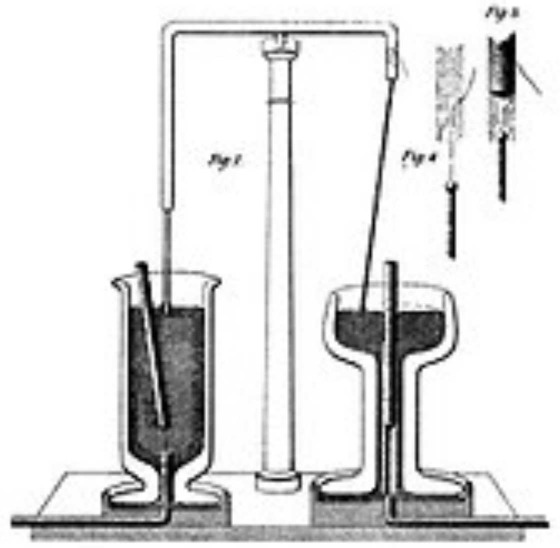
Capacitor

Battery (Volta)

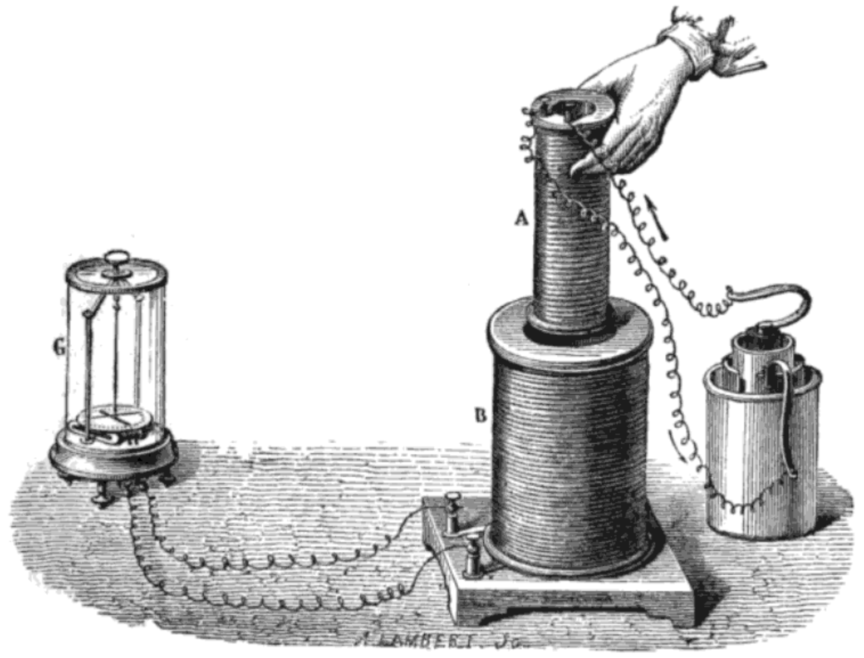


Expérience d'Oersted

Loi d'Ohm

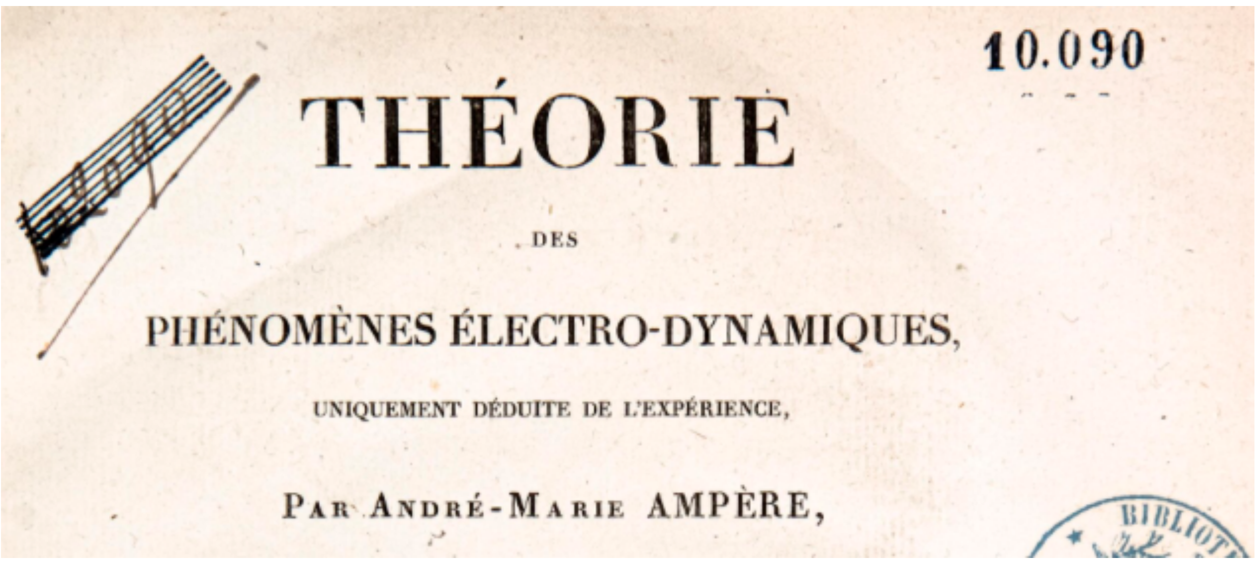


Electromagnetic rotation
(Faraday 1821)



Induction (Faraday)

From Ampère to Maxwell


10.090

THÉORIE

DES

PHÉNOMÈNES ÉLECTRO-DYNAMIQUES,

UNIQUEMENT DÉDUITE DE L'EXPÉRIENCE,

PAR ANDRÉ-MARIE AMPÈRE,

1775

1820 ⟶ 1826

1836

1800

1820

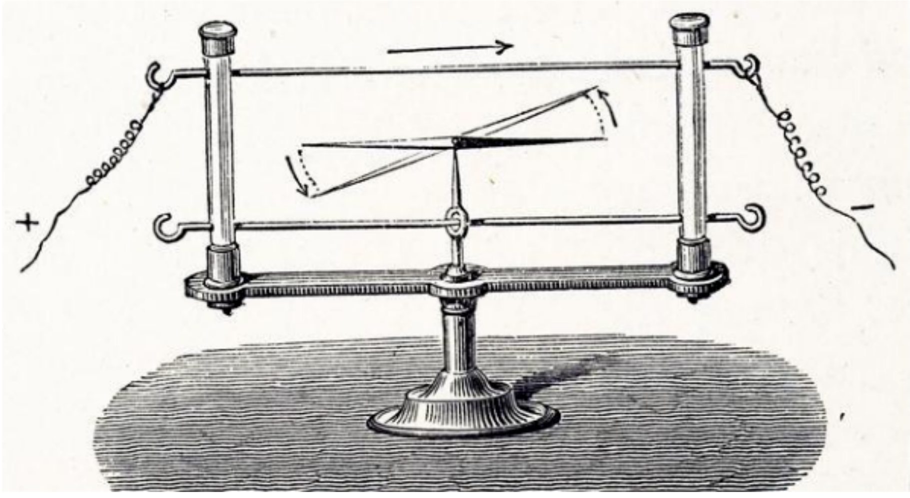
1827

1831

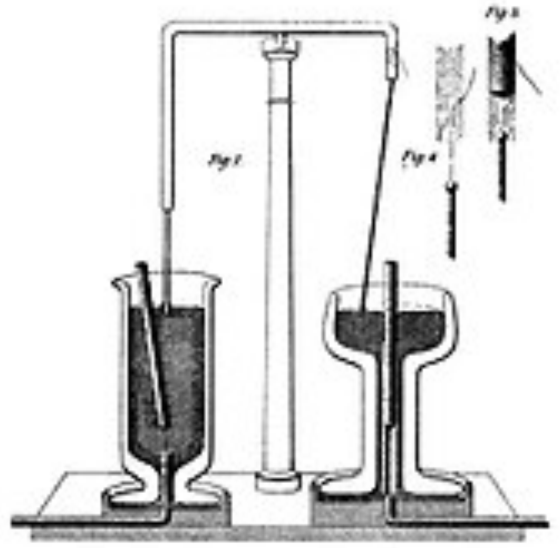
1745

Capacitor

Battery (Volta)

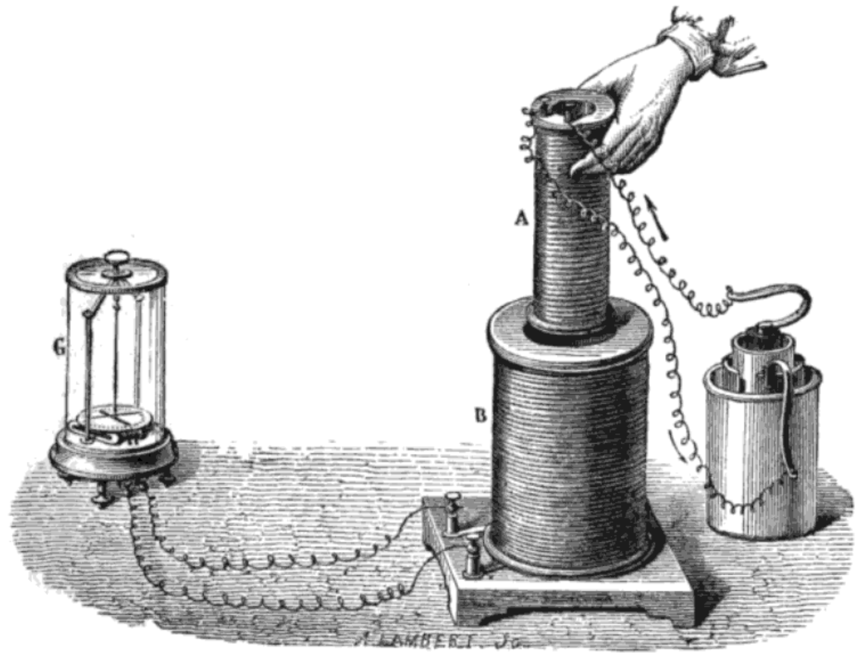


Expérience d'Oersted



Electromagnetic rotation (Faraday 1821)

Loi d'Ohm

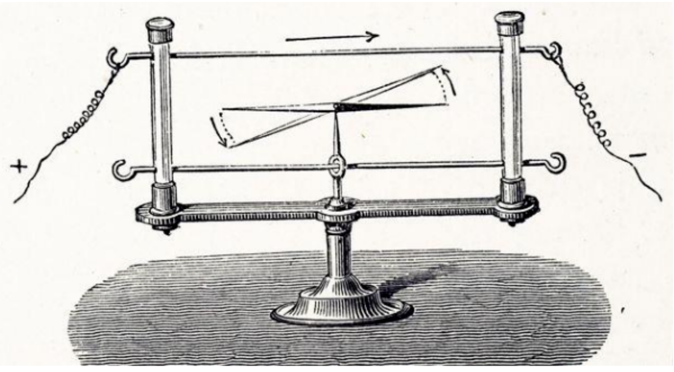


Induction (Faraday)

From Ampère to Maxwell

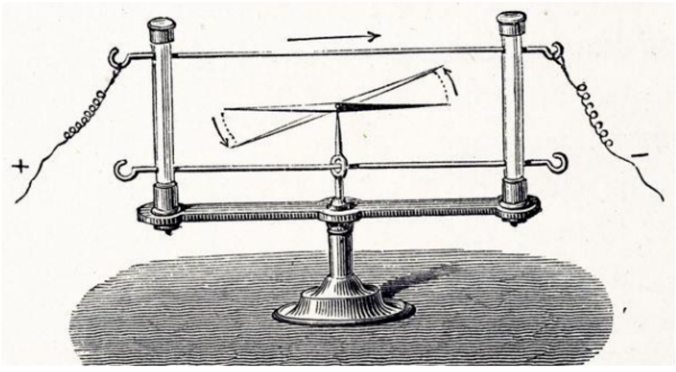
From Ampère to Maxwell

Notion de courant électrique

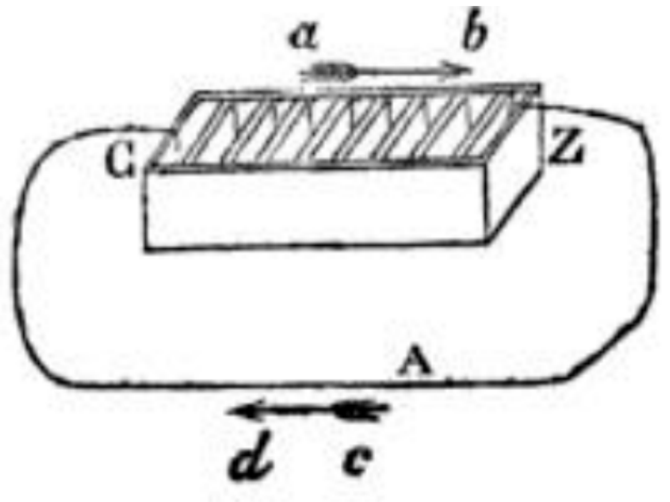


From Ampère to Maxwell

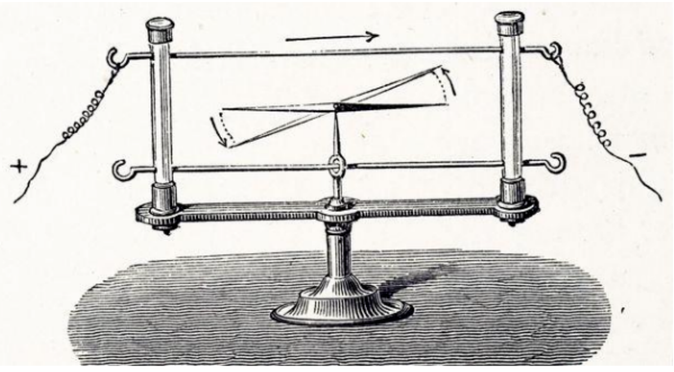
Notion de courant électrique



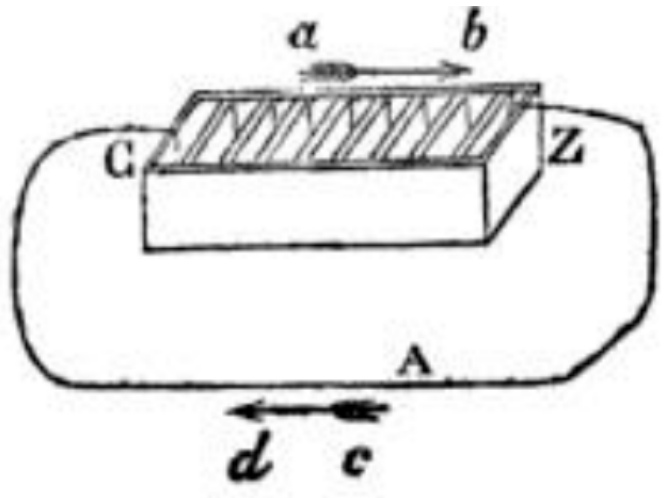
Notion de circuit électrique



Notion de courant électrique



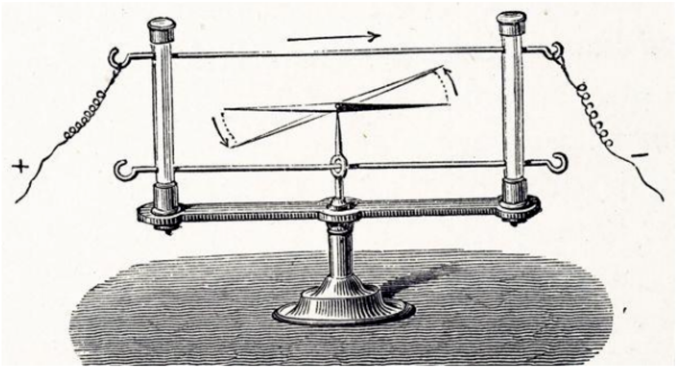
Notion de circuit électrique



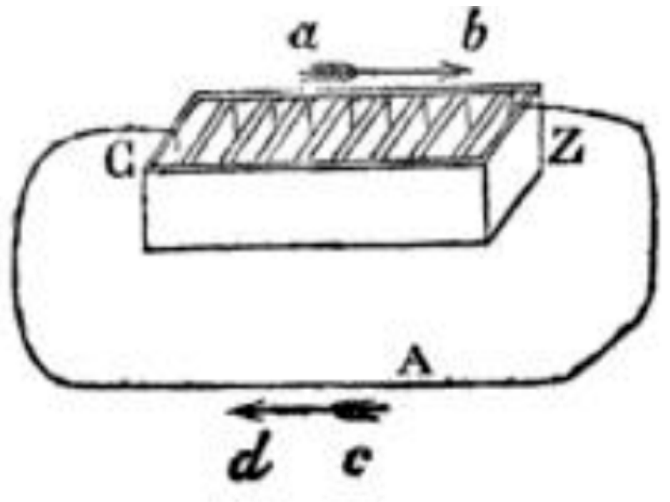
Unification électricité et magnétisme

From Ampère to Maxwell

Notion de courant électrique

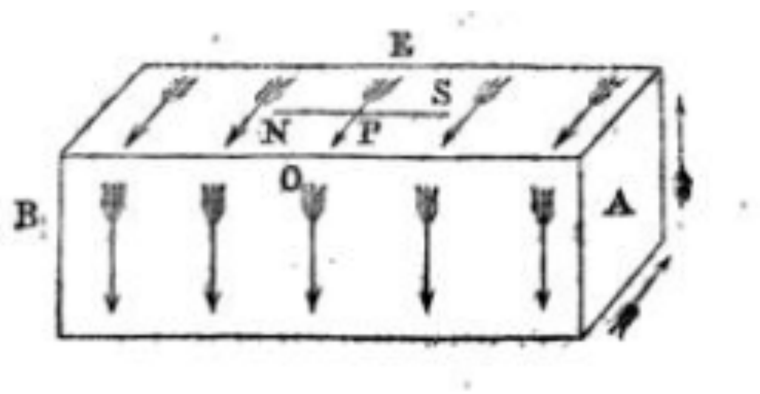


Notion de circuit électrique

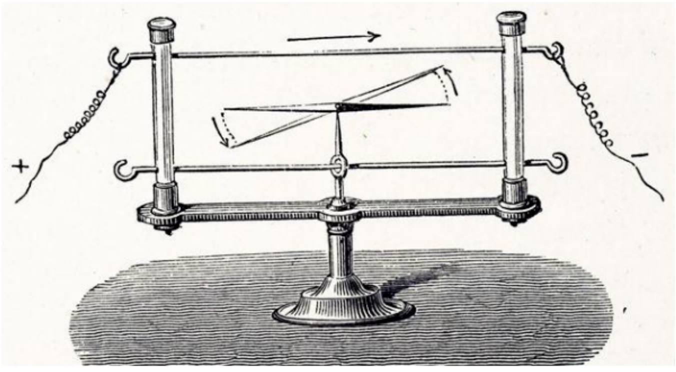


Unification électricité et magnétisme

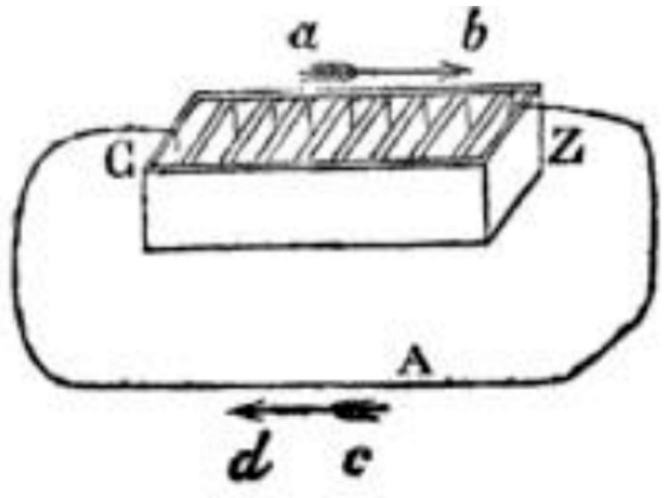
Courants moléculaires



Notion de courant électrique

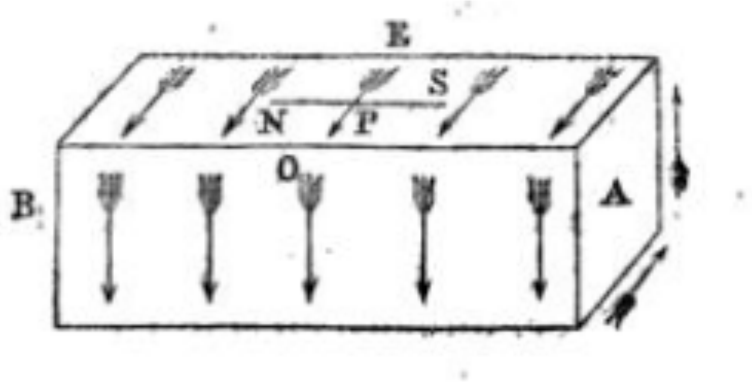


Notion de circuit électrique

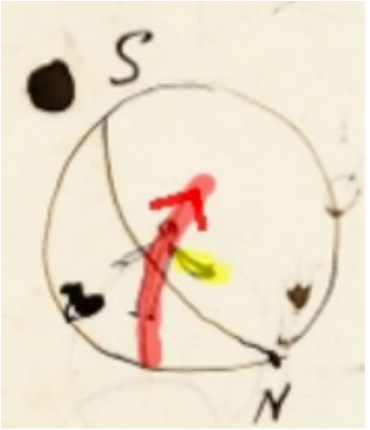


Unification électricité et magnétisme

Courants moléculaires

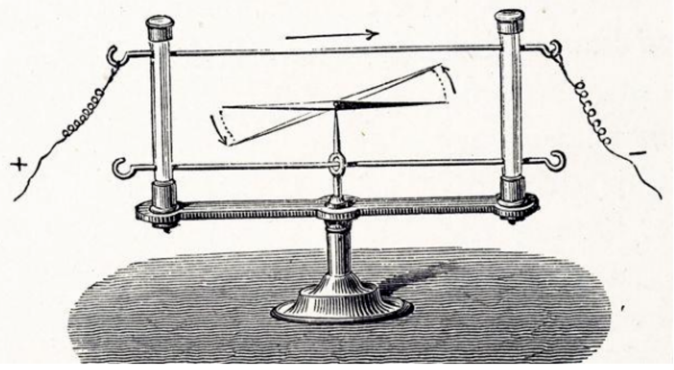


Géo-dynamo

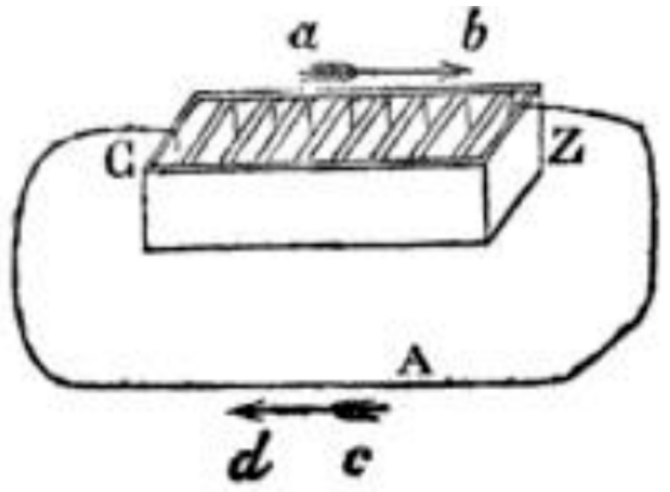


From Ampère to Maxwell

Notion de courant électrique

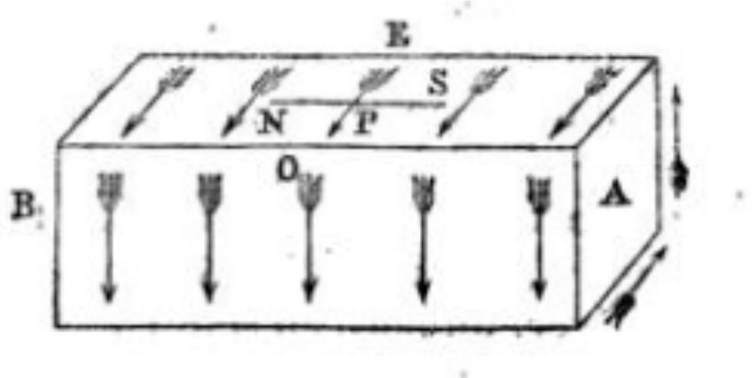


Notion de circuit électrique



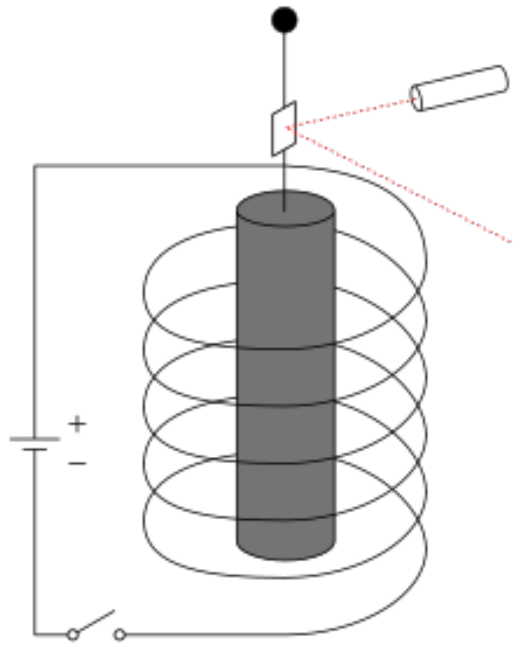
Unification électricité et magnétisme

Courants moléculaires

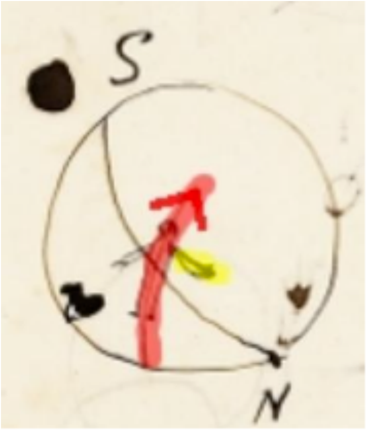


Einstein-de Haas (1915)

Experimenteller Nachweis der Ampereschen Molekularströme

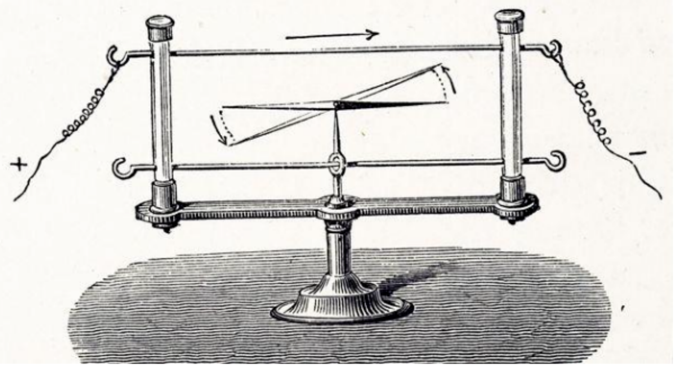


Géo-dynamo

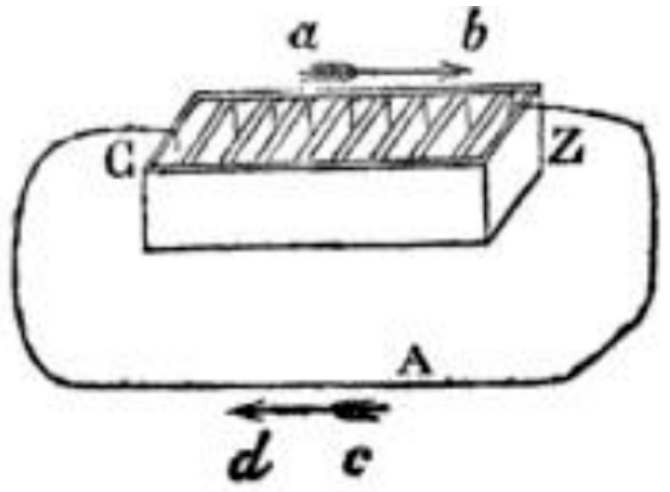


From Ampère to Maxwell

Notion de courant électrique

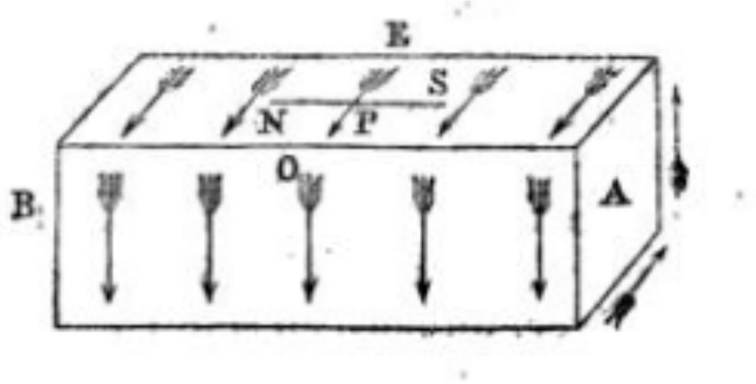


Notion de circuit électrique



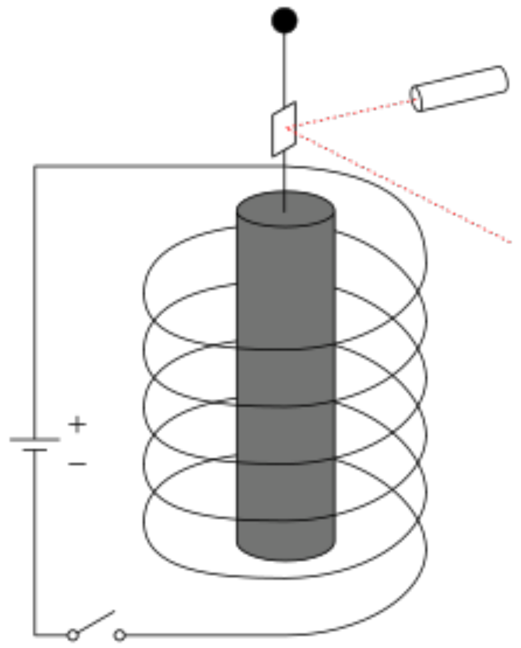
Unification électricité et magnétisme

Courants moléculaires

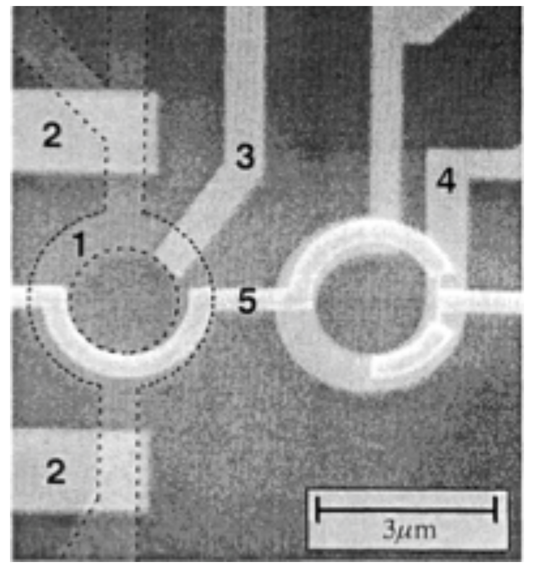


Einstein-de Haas (1915)

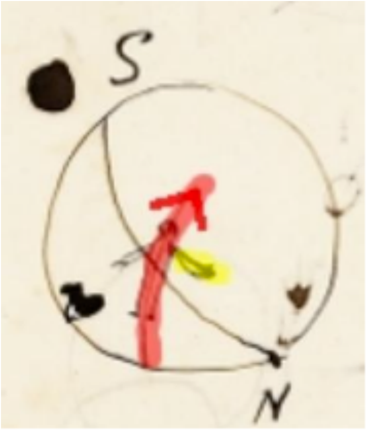
Experimenteller Nachweis der Ampereschen Molekularströme



Courants permanents (1983)



Géo-dynamo

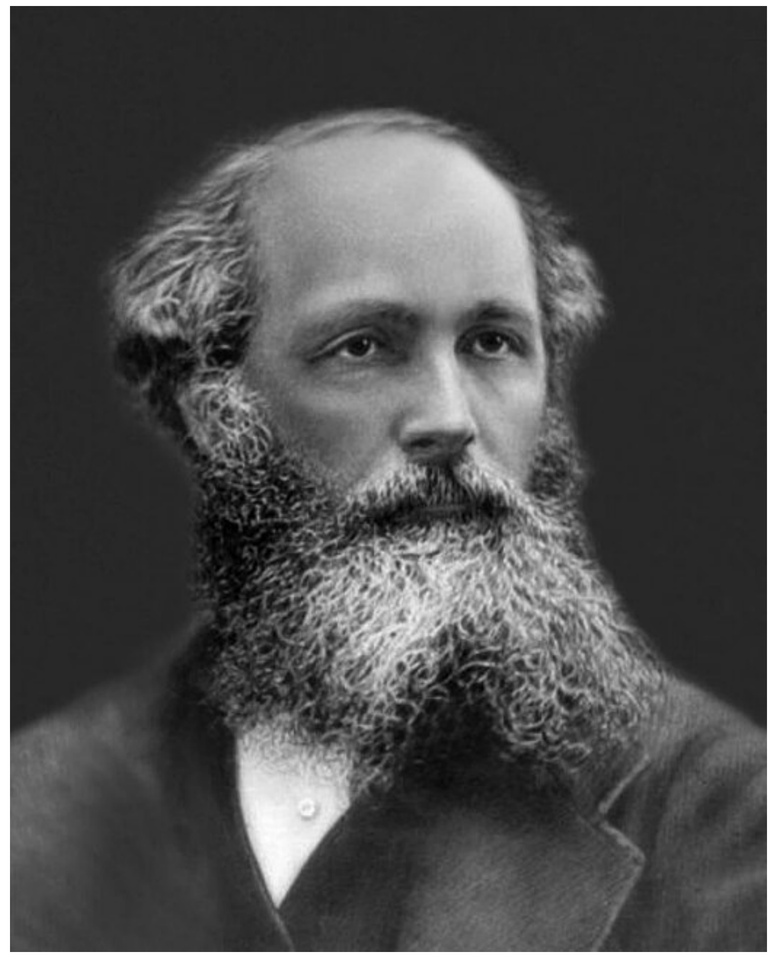


Maxwell's theory (1865)

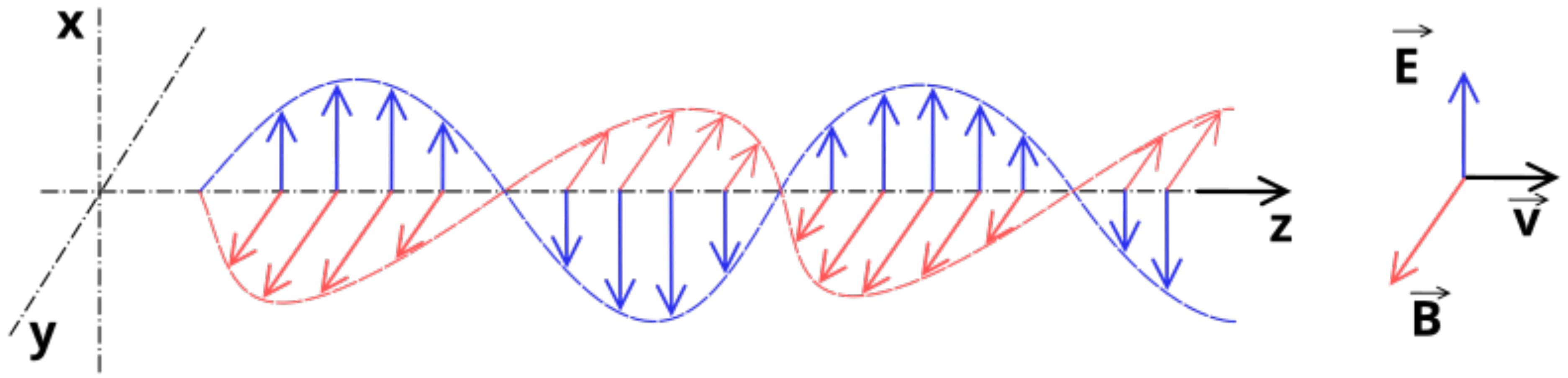
$$\mathrm{div}(\mathbf{E}) = \frac{\rho}{\varepsilon_0} \qquad \overrightarrow{\mathrm{rot}}(\mathbf{E}) = -\frac{\partial \mathbf{B}}{\partial t} \qquad \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$$

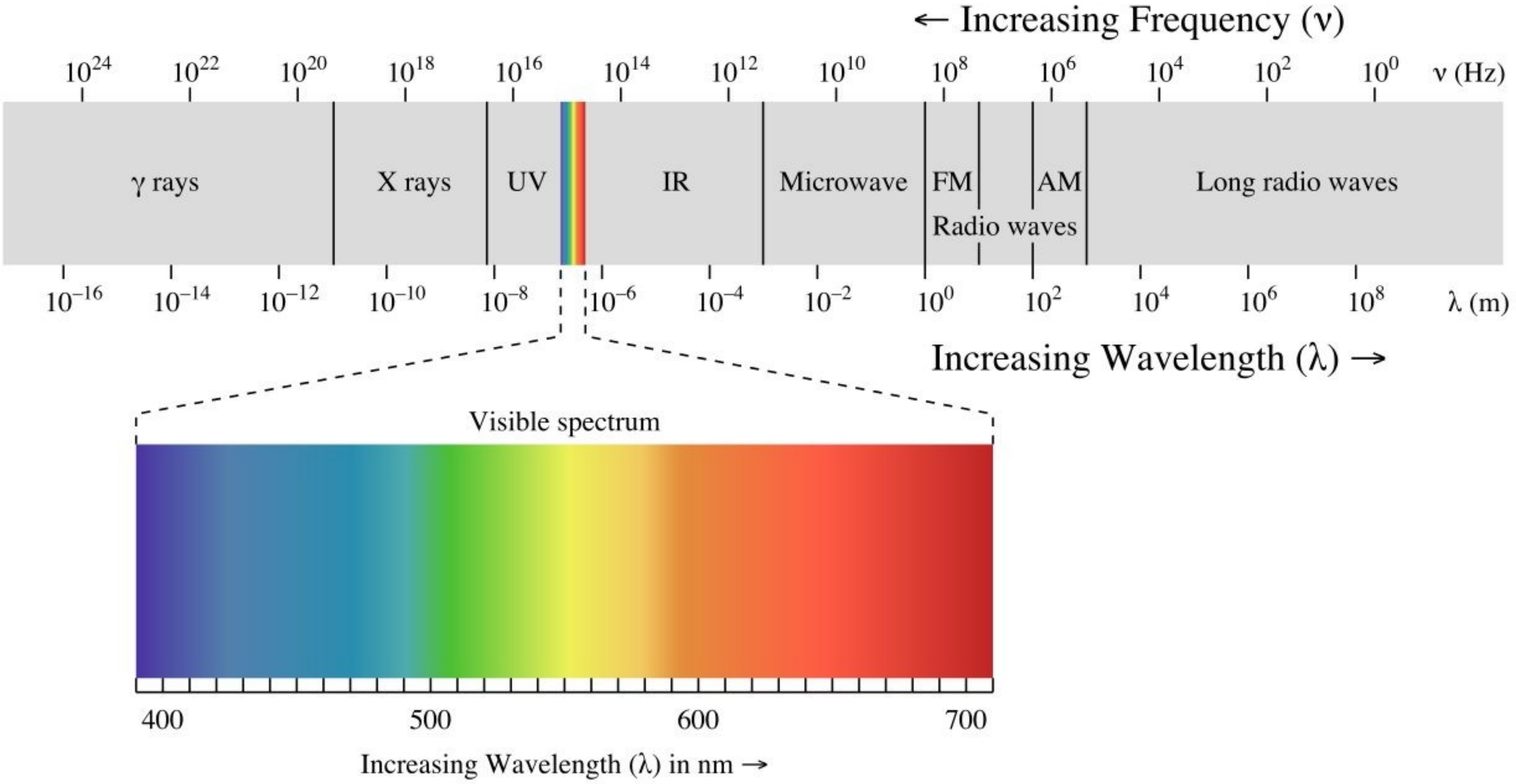
$$c^2\,\overrightarrow{\mathrm{rot}}(\mathbf{B}) = \frac{\mathbf{j}}{\varepsilon_0} + \frac{\partial \mathbf{E}}{\partial t} \qquad \mathrm{div}(\mathbf{B}) = 0 \qquad \mathbf{B} = \overrightarrow{\mathrm{rot}}(\mathbf{A})$$

$$\mathcal{L} = \frac{\varepsilon_0}{2}\left(\mathbf{E}^2 - c^2\mathbf{B}^2\right) + \mathcal{L}_{\mathrm{mat}} + \mathbf{j}\cdot\mathbf{A} - \rho V \qquad \mathbf{f} = \rho\,\mathbf{E} + \mathbf{j}\wedge\mathbf{B}$$



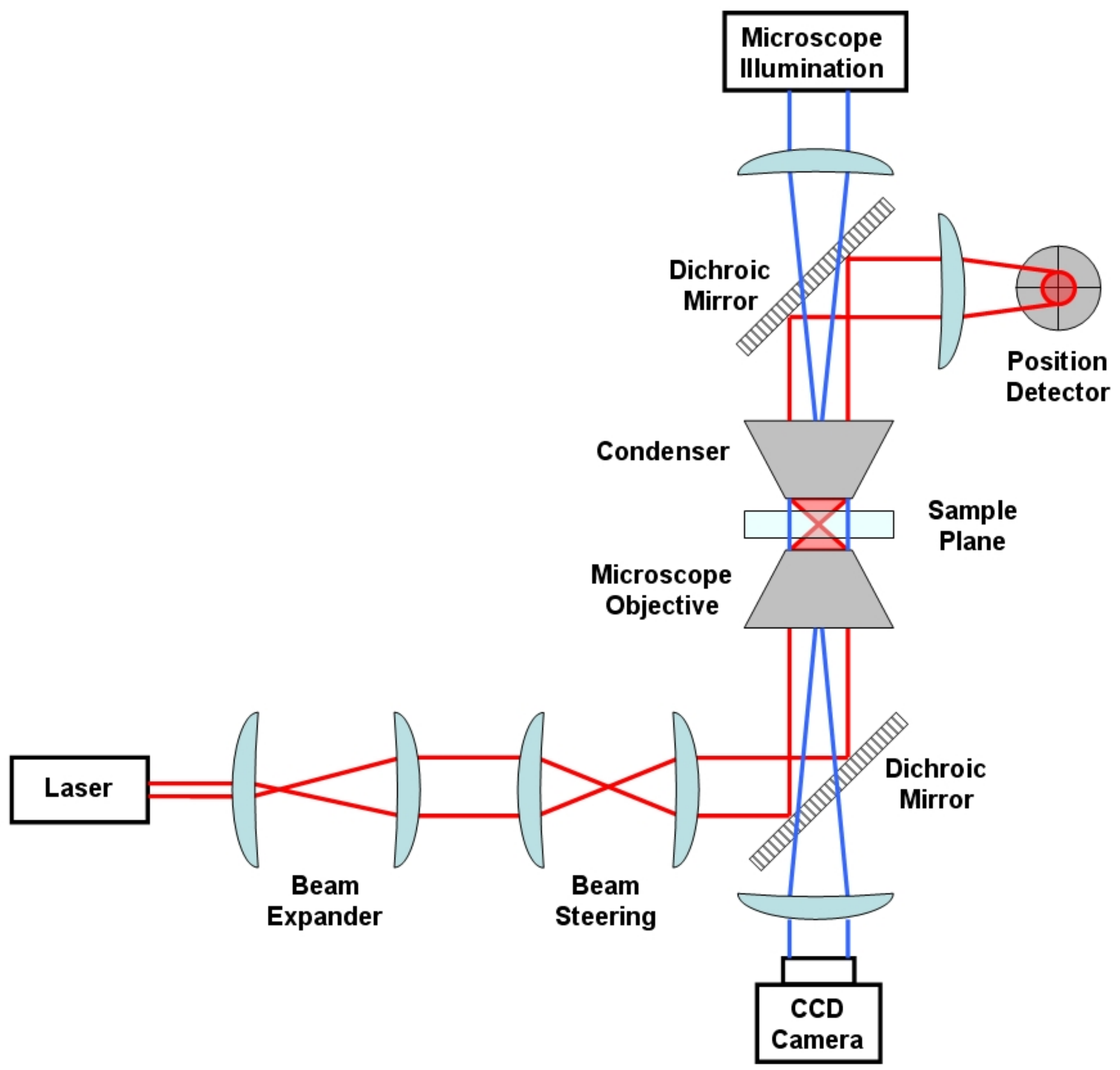
Electromagnetic waves





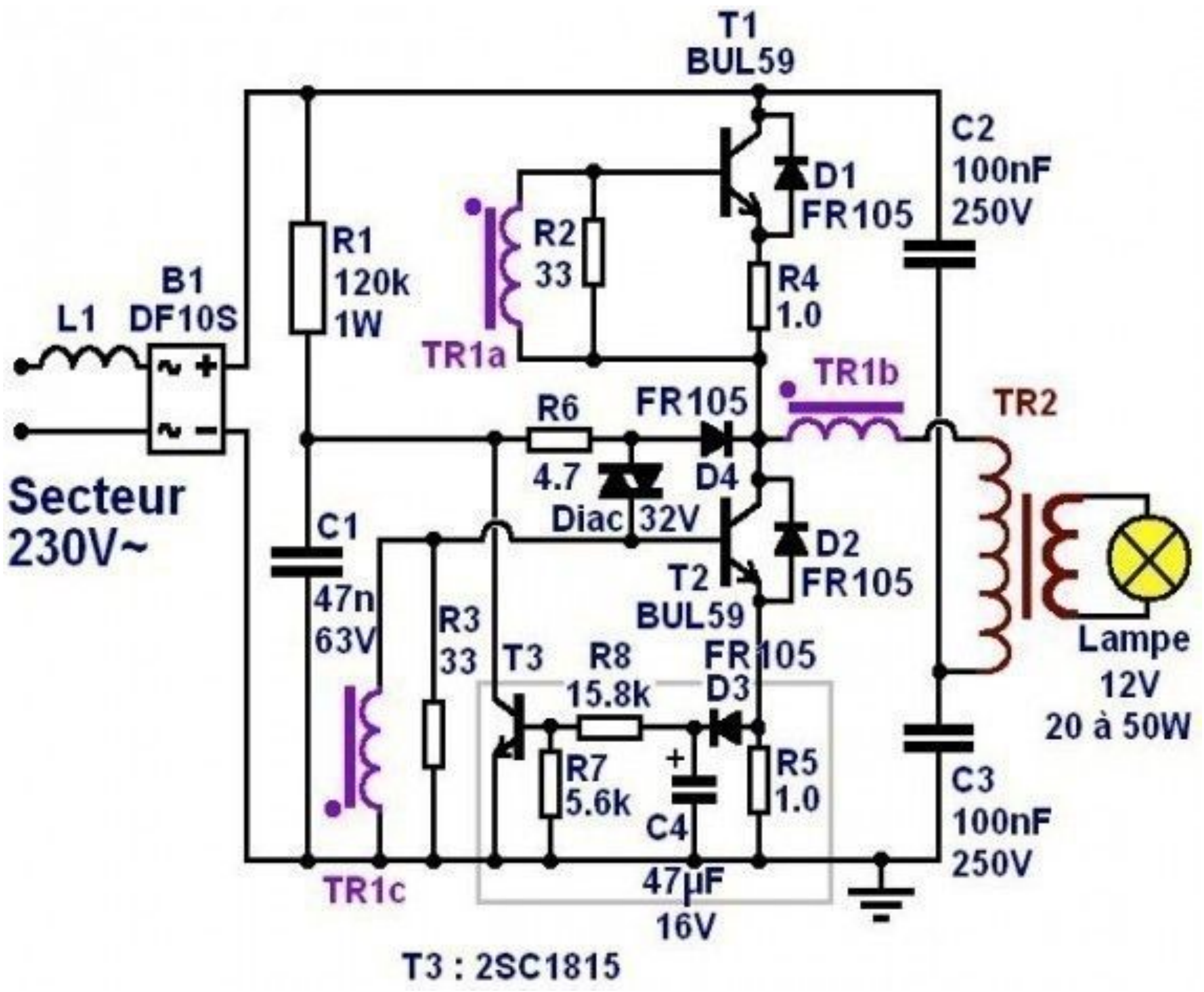
Dr. Maxwell & Mr. Kirchhoff ?

Optics



Fields, optical sources & detectors
but no voltages, no currents, no circuit

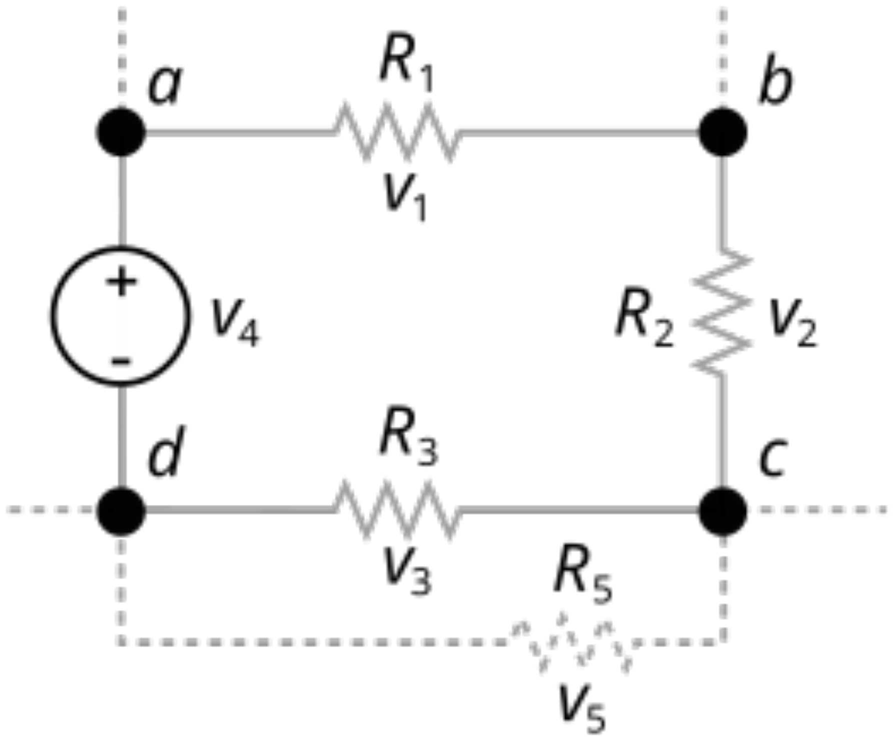
Everyday life electricity



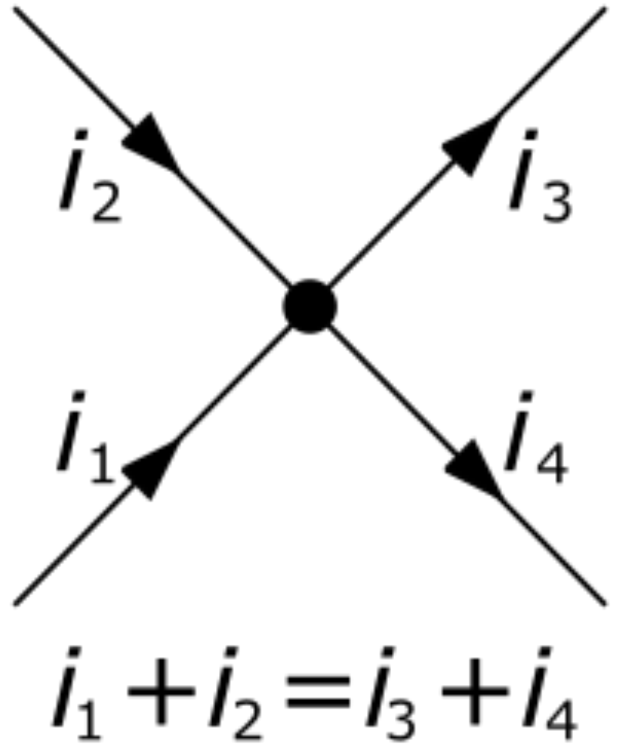
Voltages, currents & components
but no fields

Kirchhoff laws for electrical circuits

$$\sum_{b/\ b\in\partial\Sigma} V_b = 0$$



$$\sum_{b/\partial b=(\star,i)} I_b = 0$$



Original form: dc case

G. Kirchhoff, Annalen des Physik und Chemie **54**, 497-514 (1845)

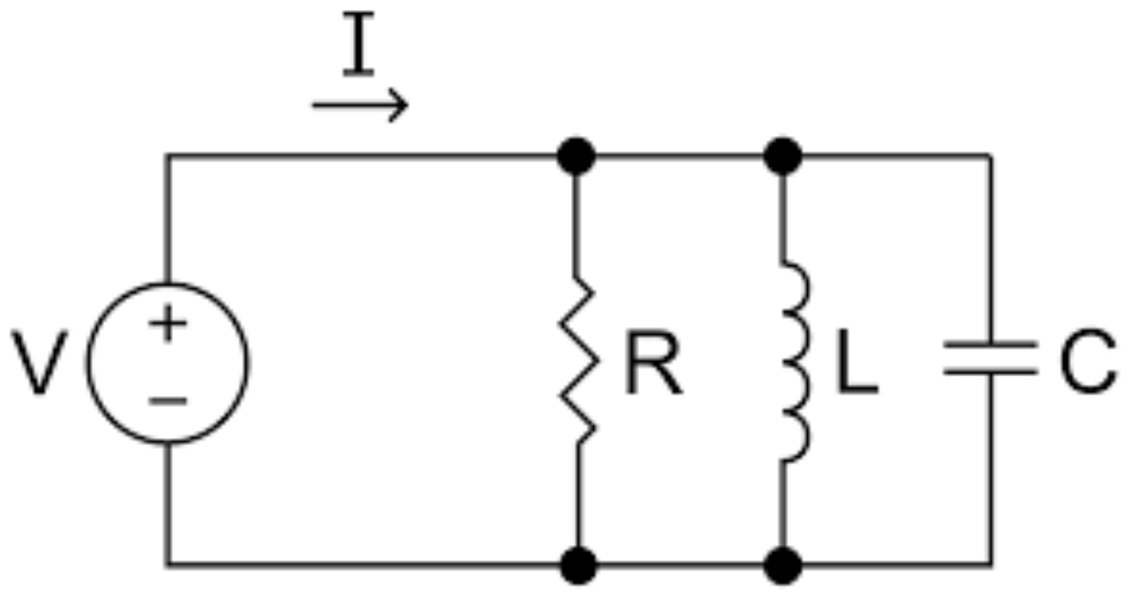
Extensions: ac case but « low frequency », with induction

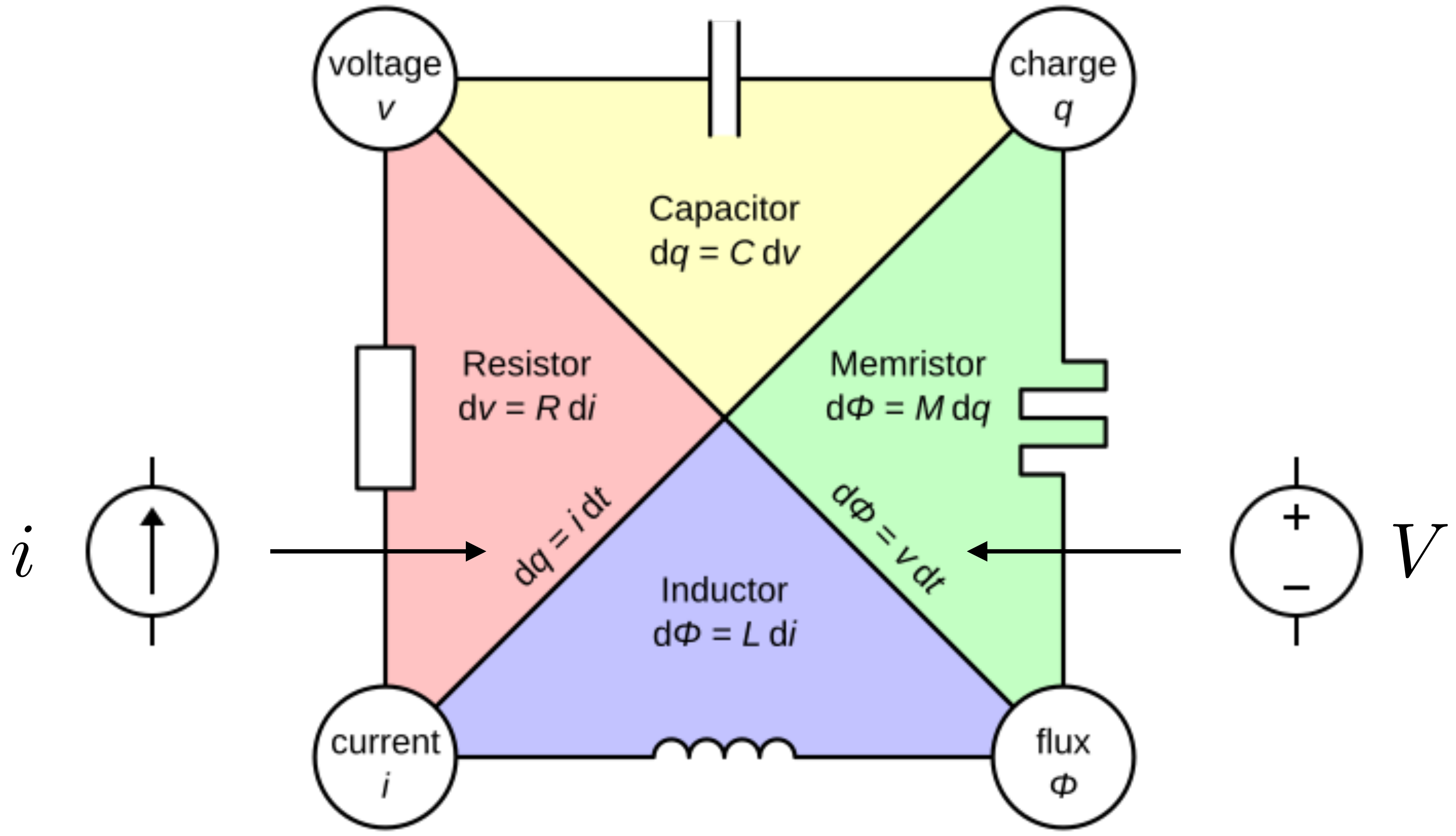
$$\oint_C \mathbf{E}\cdot \mathrm{d}\mathbf{l} = -\frac{\mathrm{d}\Phi_{\mathbf{B}}(\Sigma)}{\mathrm{d}t}$$

J. R. Carson, *Electromagnetic theory and the foundation of electrical circuit theory*, The Bell Technical Journal **6**, 1-27 (1927)

Lumped elements model of electrical circuits

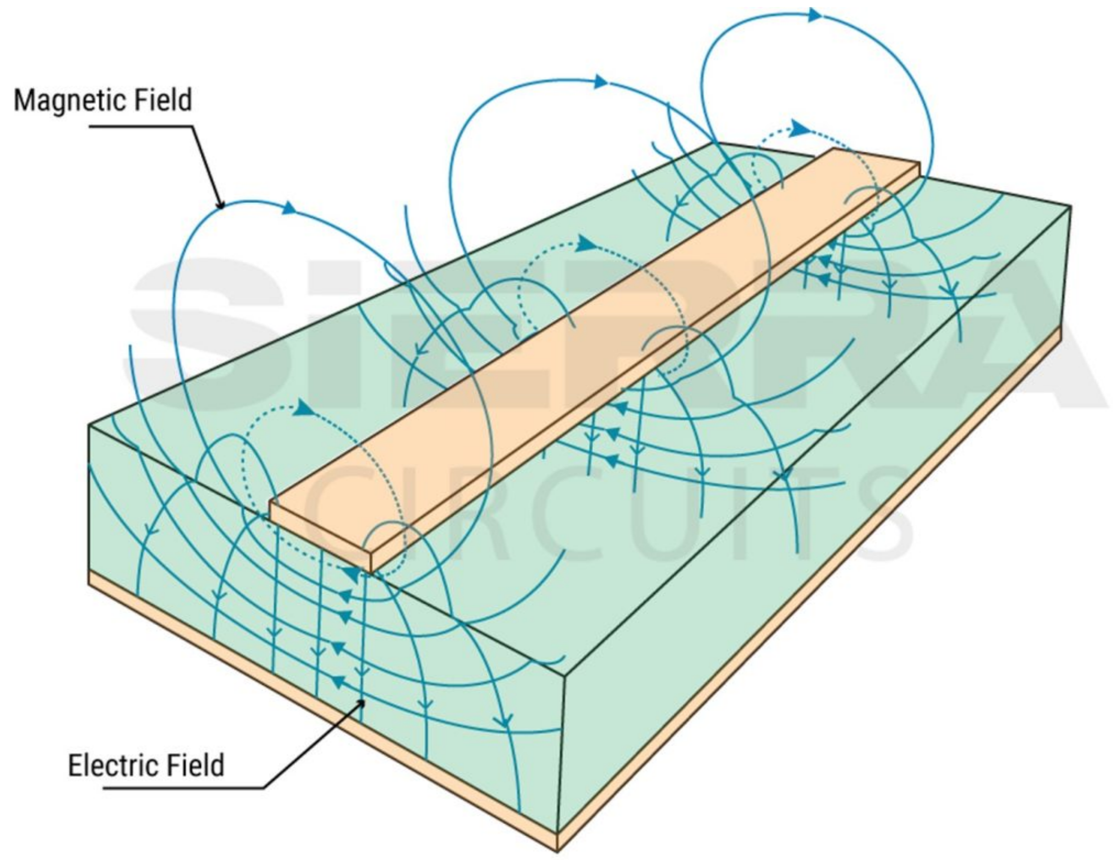
Discrete representation of electrical circuits



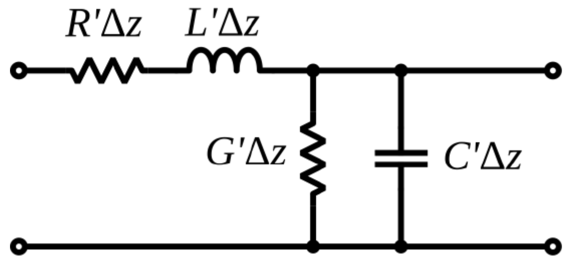


Also applicable at low frequencies to continuous circuits

Microstrip line

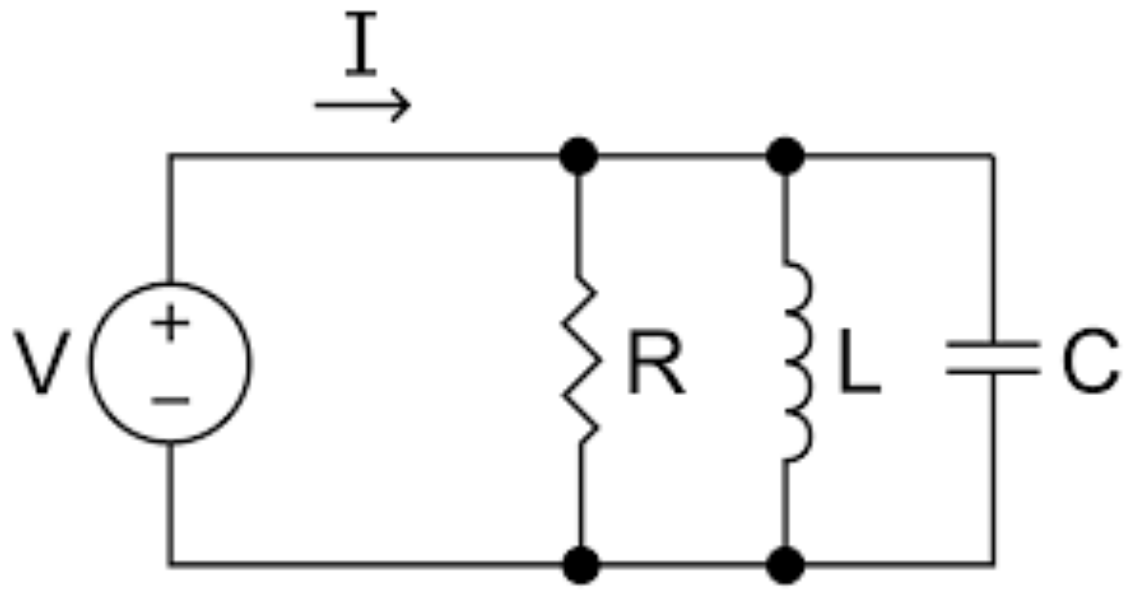


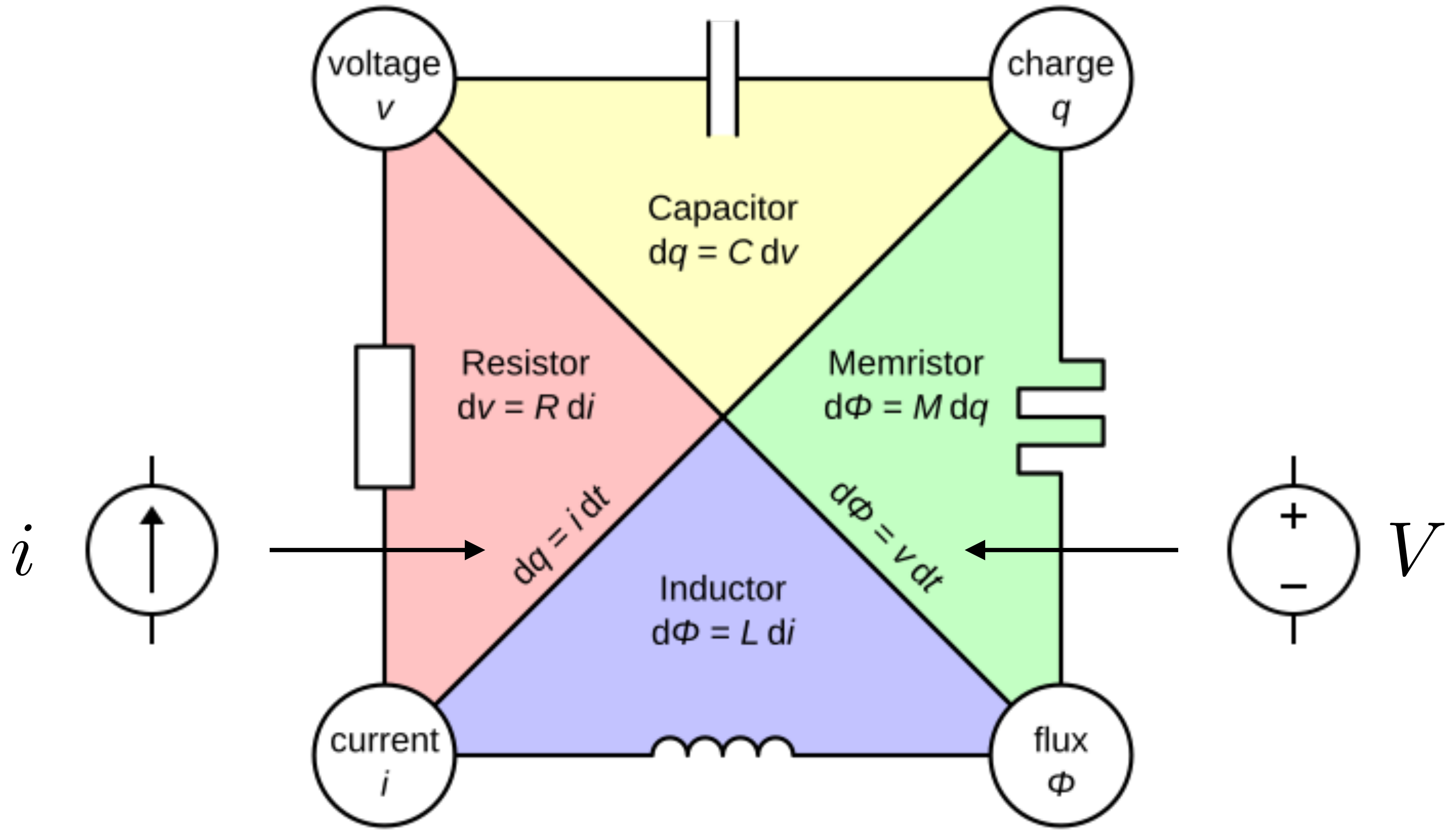
Lumped elements



Lumped elements model of electrical circuits

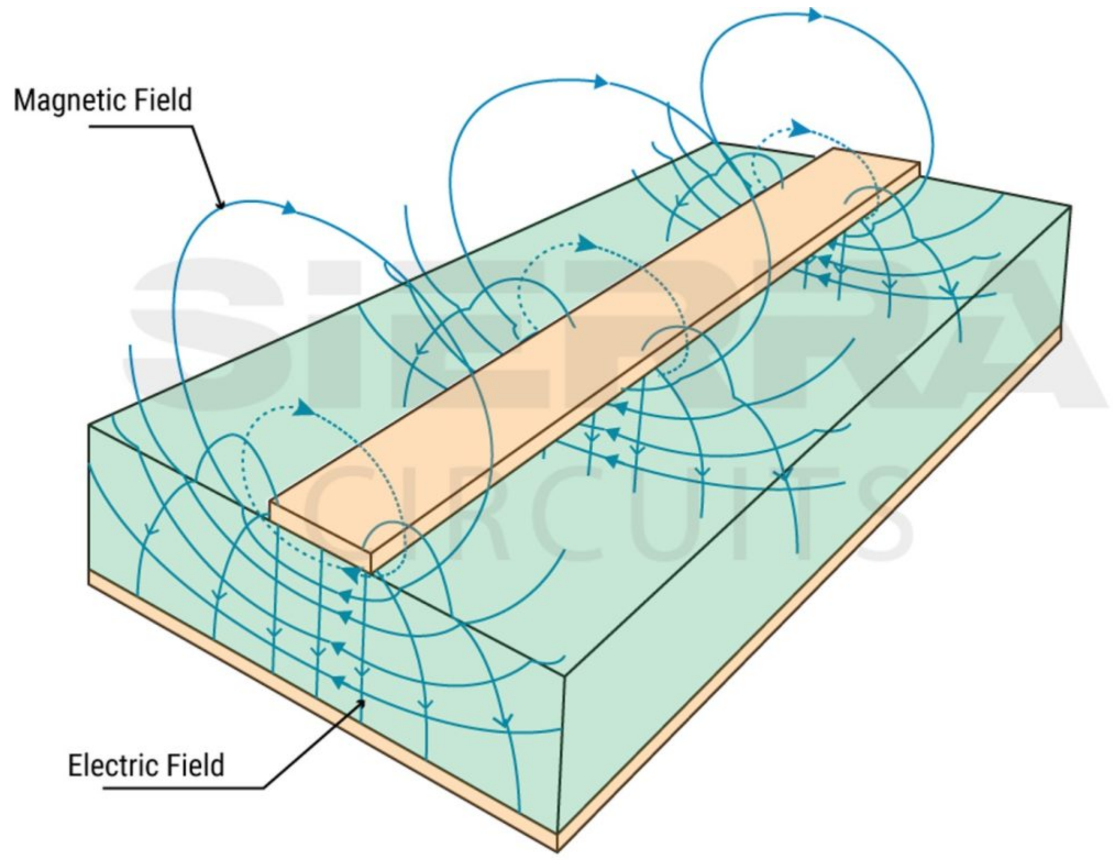
Discrete representation of electrical circuits



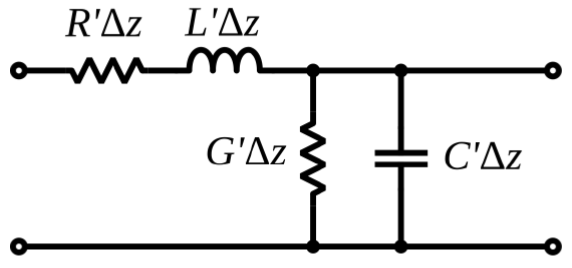


Also applicable at low frequencies to continuous circuits

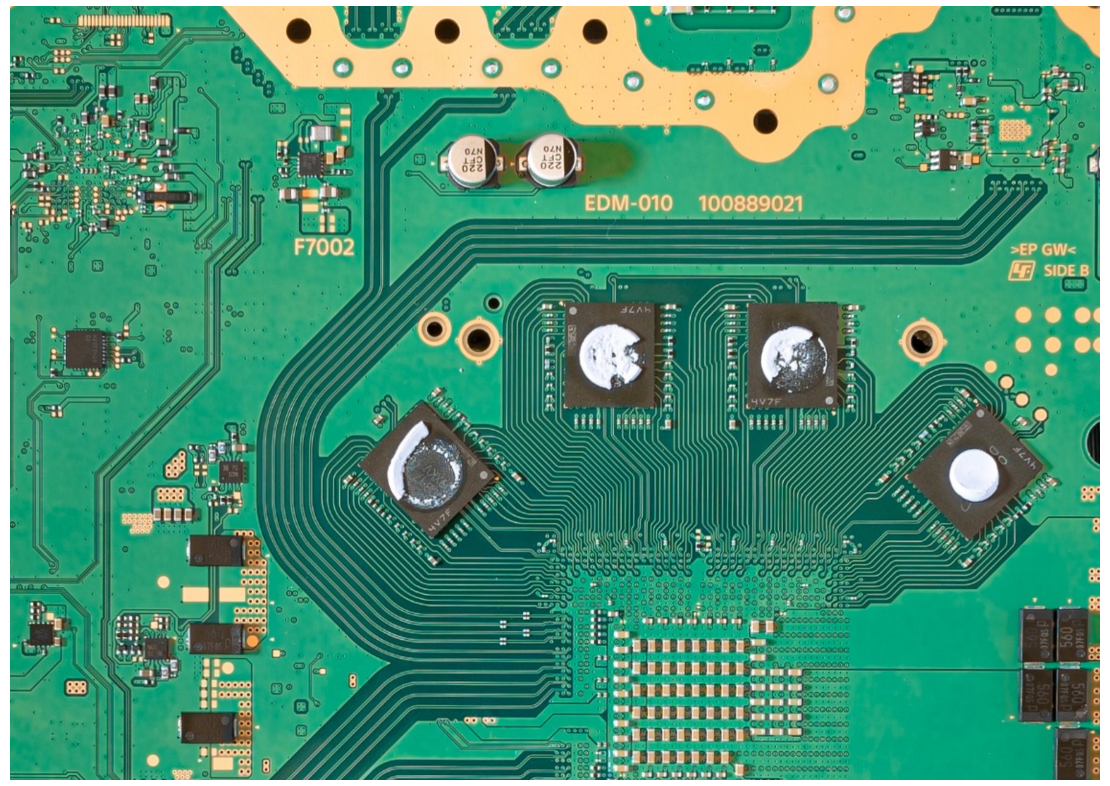
Microstrip line



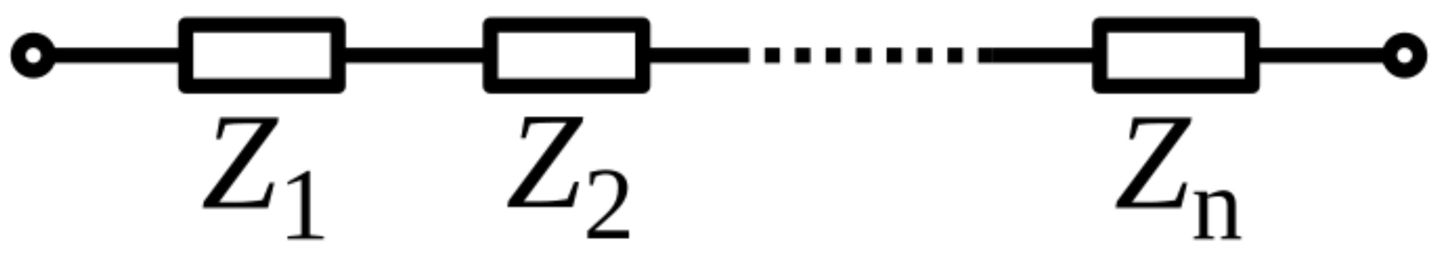
Lumped elements



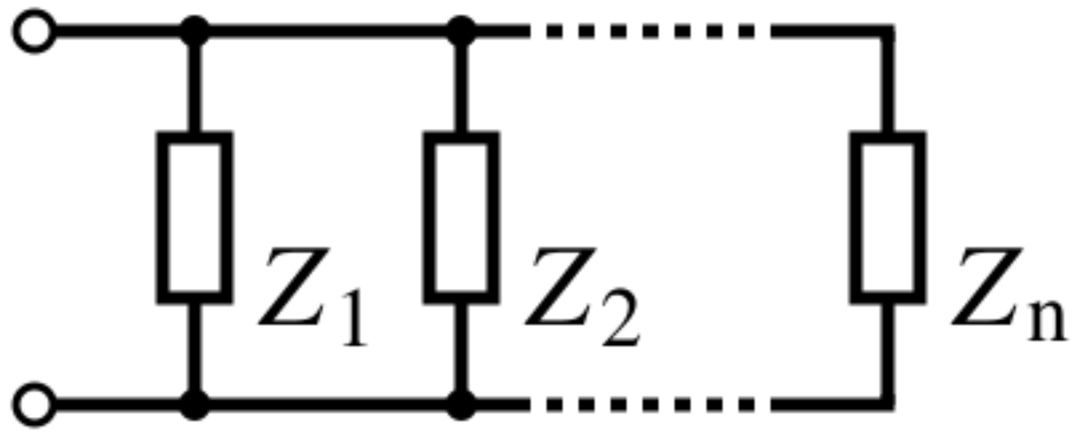
High frequency ?



Composition of impedances and admittances



$$Z_{\text{tot}} = \sum_{j=1}^{n} Z_j$$



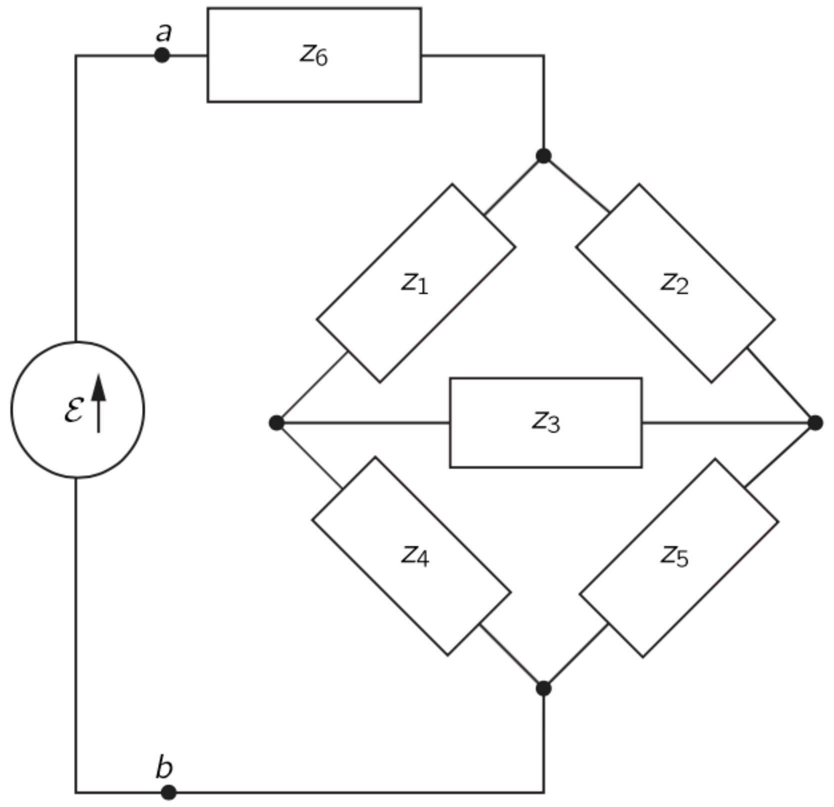
$$Z_{\text{tot}}^{-1} = \sum_{j=1}^{n} Z_j^{-1}$$

Teleggen's theorem

$$\sum_b I_b V_b = 0$$

Follows from Kirchhoff laws

Valid for any circuit (*even with active and non linear elements*)

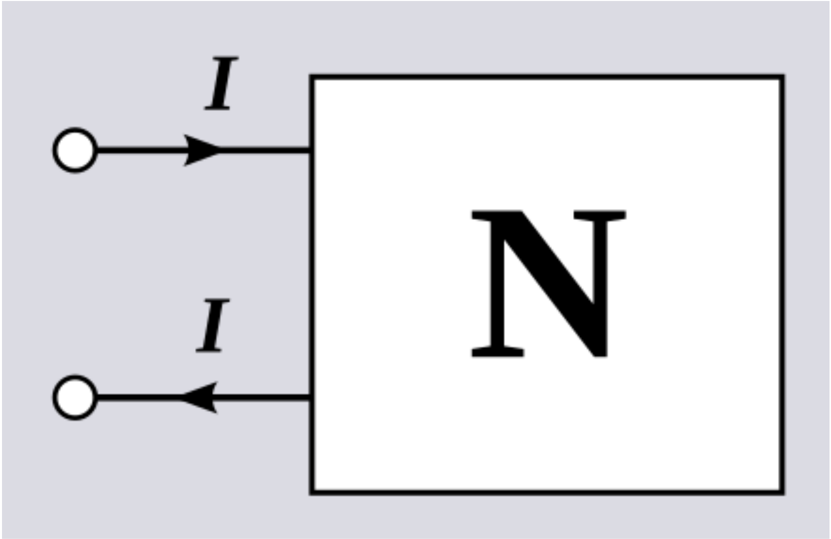


B.D.H. Tellegen, Phillips Research Reports **7**, 259 (1952)

Electrical circuit synthesis

Impedance of a passive dipole element

$$\Re(Z(s)) > 0 \text{ for } \Im(s) < 0$$
$$\Im(Z(s)) = 0 \text{ for } \Re(s) = 0$$



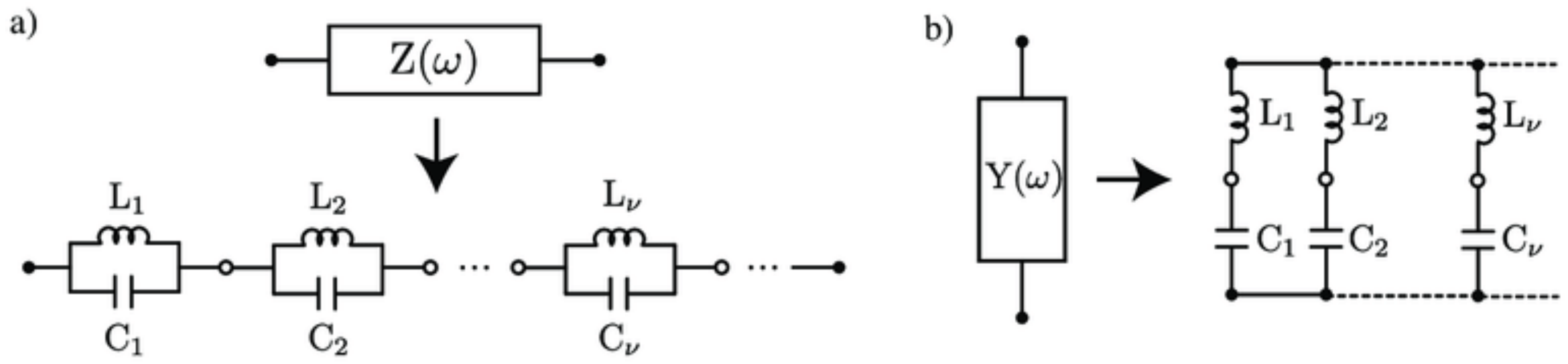
G. Cauer

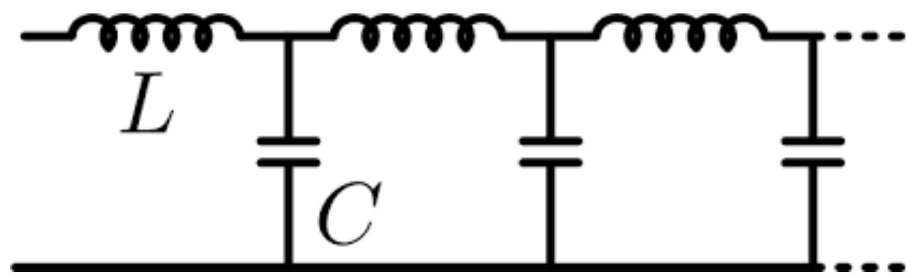
O. Brune, *Synthesis of passive networks*, PhD thesis MIT (1929)

Forster 1st form

Forster 2nd form

Cauer 1st form





Partial fraction expansion

Continuous fraction expansion

R. M. Forster, The Bell Technical Journal **3**, 259-267 (1924)

G. Cauer, PhD thesis (1926)

R. P. Feynman *et al*, *Lectures on Physics*, chap. 22, sec. 22-4, 22-6 & 22-7.

Linear electrical circuits: response coefficients

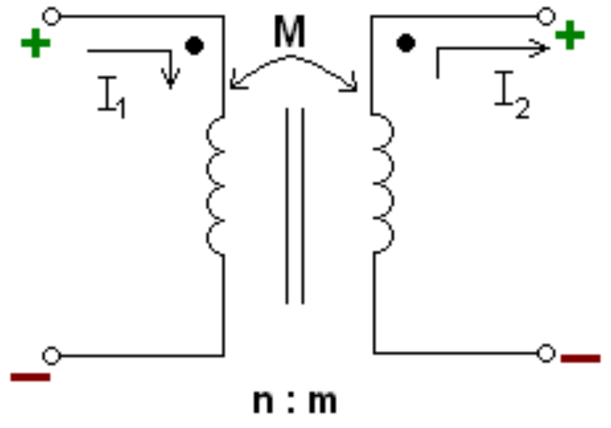
Linear electrical circuits: response coefficients

Mutual inductance matrix

$$\boldsymbol{\Phi} = \mathbf{L} \cdot \mathbf{I}$$

$${}^t\mathbf{L} = \mathbf{L}$$

$$L_{i,j} = \frac{\mu_0}{4\pi} \oint_{C_i,C_j} \frac{\mathrm{d}\mathbf{r}_i \, \mathrm{d}\mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|} \qquad i \neq j$$



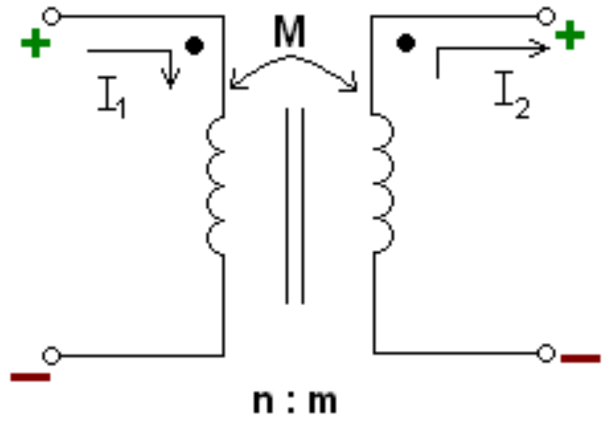
Linear electrical circuits: response coefficients

Mutual inductance matrix

$$\mathbf{\Phi} = \mathbf{L} \cdot \mathbf{I}$$

$${}^t\mathbf{L} = \mathbf{L}$$

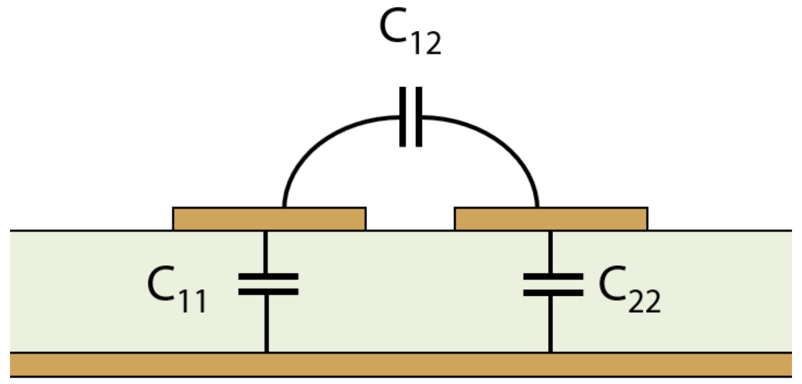
$$L_{i,j} = \frac{\mu_0}{4\pi} \oint_{C_i, C_j} \frac{\mathrm{d}\mathbf{r}_i \, \mathrm{d}\mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|} \qquad i \neq j$$



Mutual capacitance matrix

$$\mathbf{Q} = \mathbf{C} \cdot \mathbf{U}$$

$${}^t\mathbf{C} = \mathbf{C}$$



Total mutual influence

$$\mathbf{C} \cdot \mathbf{1} = \mathbf{0}$$ Gauge invariance

$${}^t\mathbf{1} \cdot \mathbf{C} = \mathbf{0}$$ Total neutrality

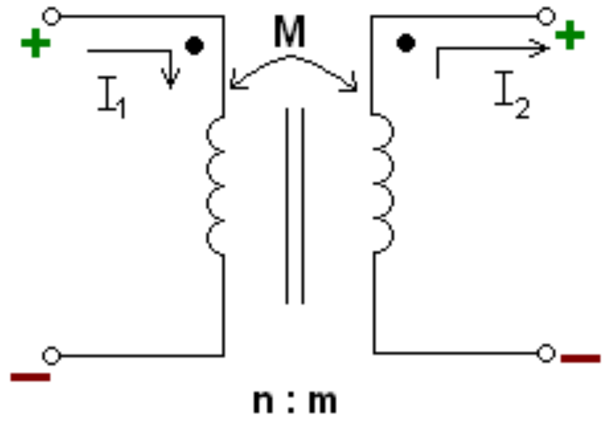
Linear electrical circuits: response coefficients

Mutual inductance matrix

$$\mathbf{\Phi} = \mathbf{L} \cdot \mathbf{I}$$

$${}^t\mathbf{L} = \mathbf{L}$$

$$L_{i,j} = \frac{\mu_0}{4\pi} \oint_{C_i, C_j} \frac{\mathrm{d}\mathbf{r}_i \, \mathrm{d}\mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|} \qquad i \neq j$$



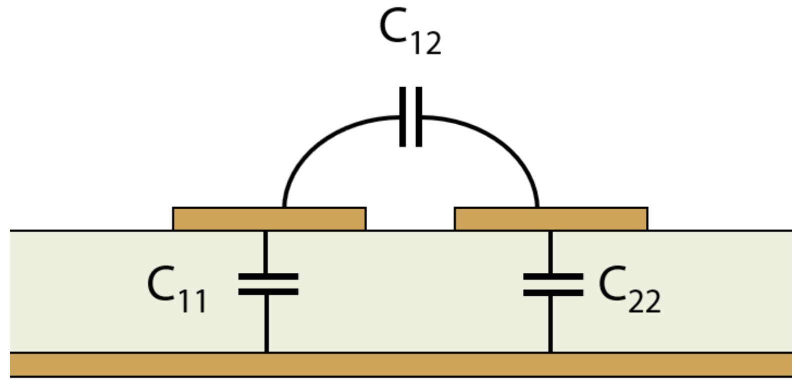
Computational methods: numerics (COMSOL)

Yu. Ya. Iossel, E.S. Kochanov, and M.G. Strunskly,
The calculation of electrical capacitance (1969)

Mutual capacitance matrix

$$\mathbf{Q} = \mathbf{C} \cdot \mathbf{U}$$

$${}^t\mathbf{C} = \mathbf{C}$$



Total mutual influence

$$\mathbf{C} \cdot \mathbf{1} = \mathbf{0}$$ Gauge invariance

$${}^t\mathbf{1} \cdot \mathbf{C} = \mathbf{0}$$ Total neutrality

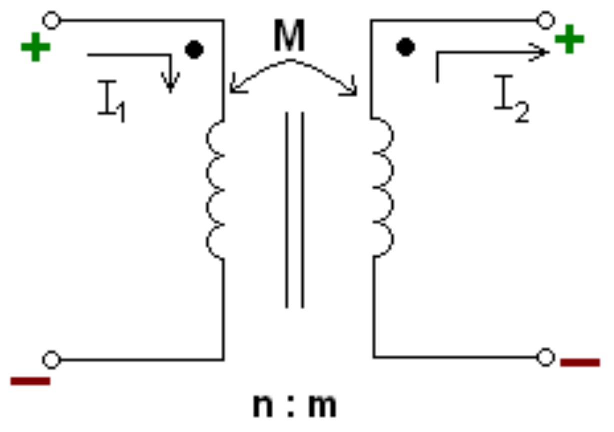
Linear electrical circuits: response coefficients

Mutual inductance matrix

$$\mathbf{\Phi} = \mathbf{L} \cdot \mathbf{I}$$

$${}^t\mathbf{L} = \mathbf{L}$$

$$L_{i,j} = \frac{\mu_0}{4\pi} \oint_{C_i, C_j} \frac{\mathrm{d}\mathbf{r}_i \, \mathrm{d}\mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|} \qquad i \neq j$$



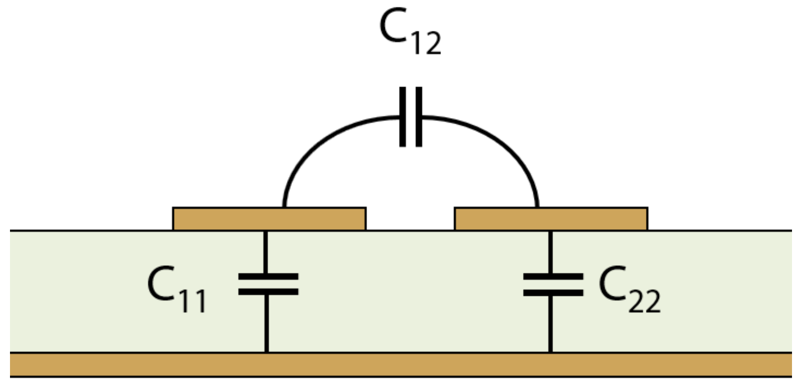
Computational methods: numerics (COMSOL)

Yu. Ya. Iossel, E.S. Kochanov, and M.G. Strunskly,
The calculation of electrical capacitance (1969)

Mutual capacitance matrix

$$\mathbf{Q} = \mathbf{C} \cdot \mathbf{U}$$

$${}^t\mathbf{C} = \mathbf{C}$$



Total mutual influence

$$\mathbf{C} \cdot \mathbf{1} = \mathbf{0}$$ Gauge invariance

$${}^t\mathbf{1} \cdot \mathbf{C} = \mathbf{0}$$ Total neutrality

Resistance

$$\mathbf{j} = \sigma \, \mathbf{E}$$ (Local form)

For a wire: $I = G\,V$ with $G = \frac{\sigma S}{L}$

Empirical law: σ is material specific (**not** of geometric / electrostatic origin)

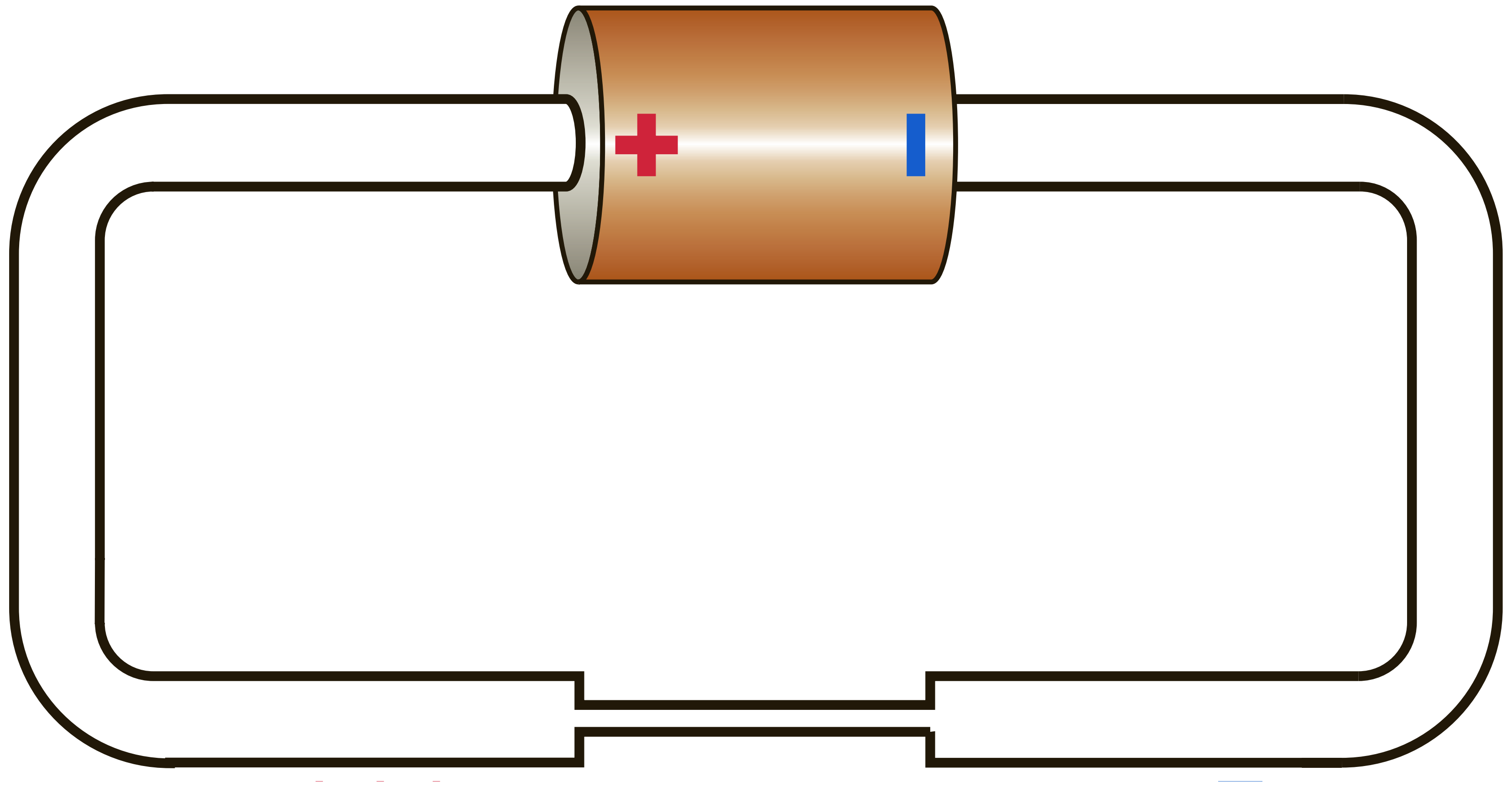
How electricity flows: a story of charges, currents and fields

Je suis sur que vous ne savez pas exactement comment marche l'électricité. Et je ne parle même pas de choses compliquées comme comme du courant alternatif ou des transistors. Non un truc simple: une pile, une ampoule. Qu'est ce qui se passe vraiment au niveau physique dans les fils électriques ?

D. Louapre, ScienceEtonnante (3/11/2024)

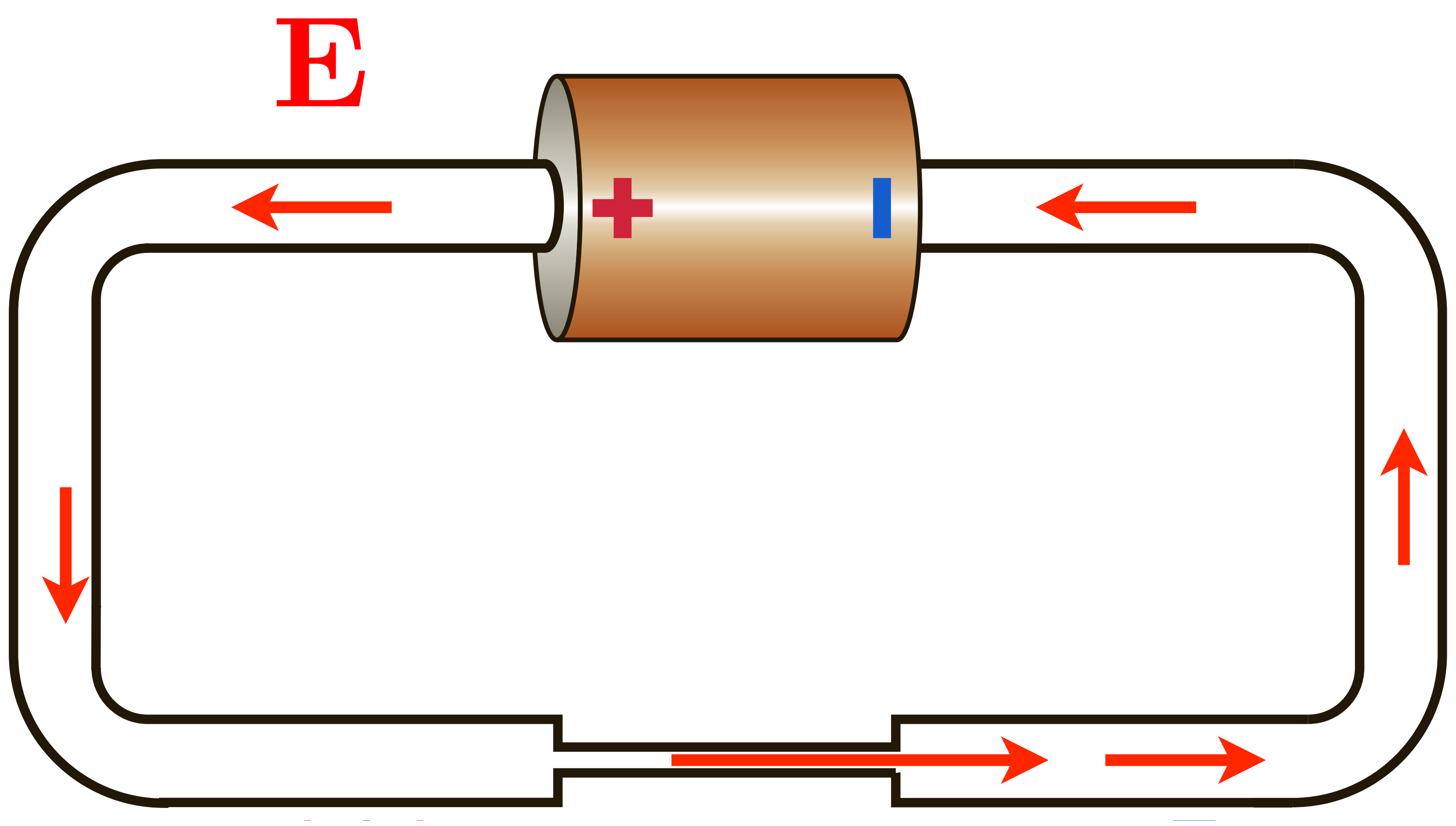
G. Kirchhoff, *On the deduction of Ohm's laws in connexion with the theory of electrostatics* Philosophical Magazine **37**, 463-468 (1850)

The *charges* side of electricity



$$R = \frac{\rho\, L}{S}$$

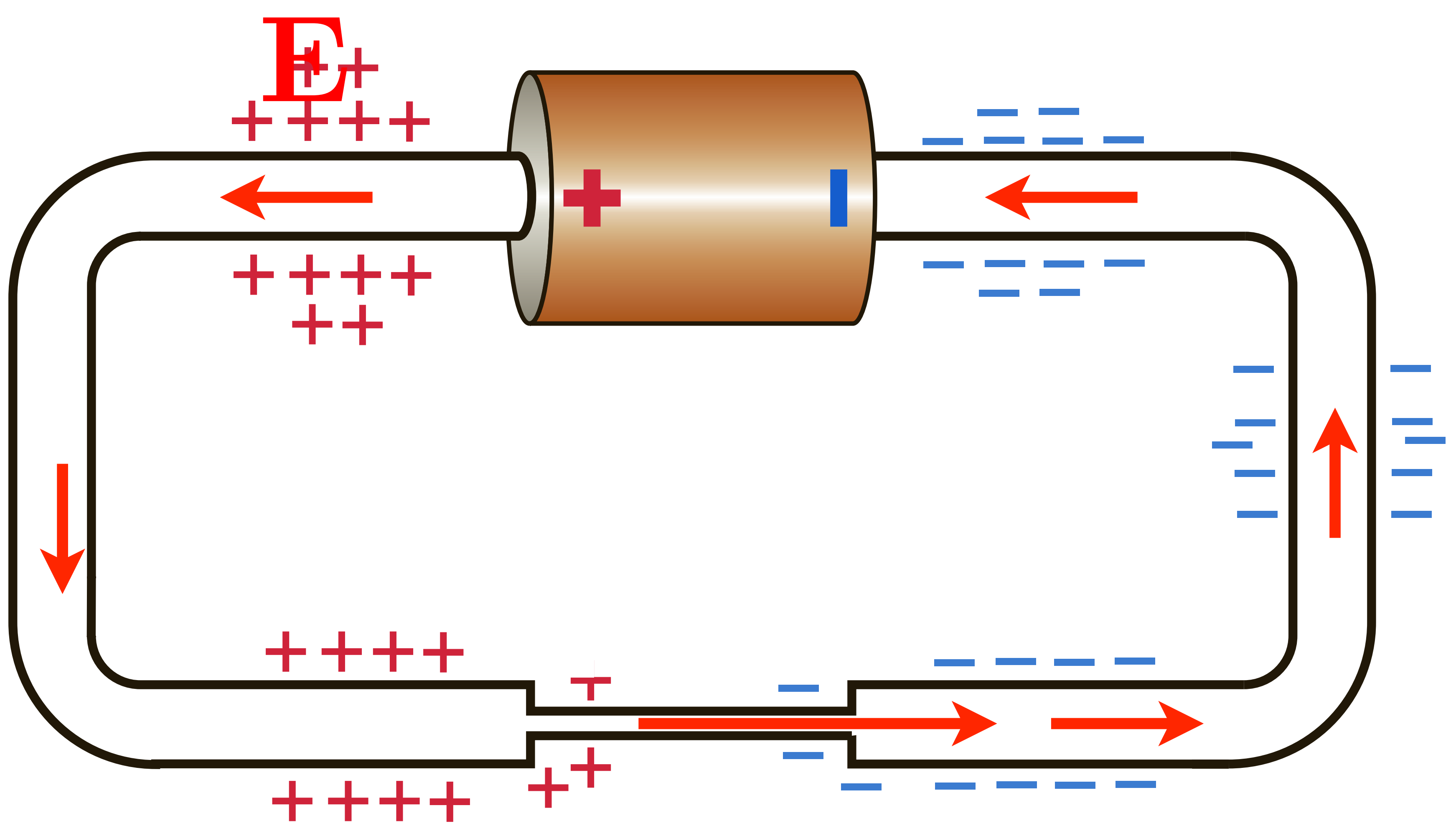
The *charges* side of electricity



$$R = \frac{\rho L}{S}$$

$$|\mathbf{E}| = \frac{\rho I}{S}$$

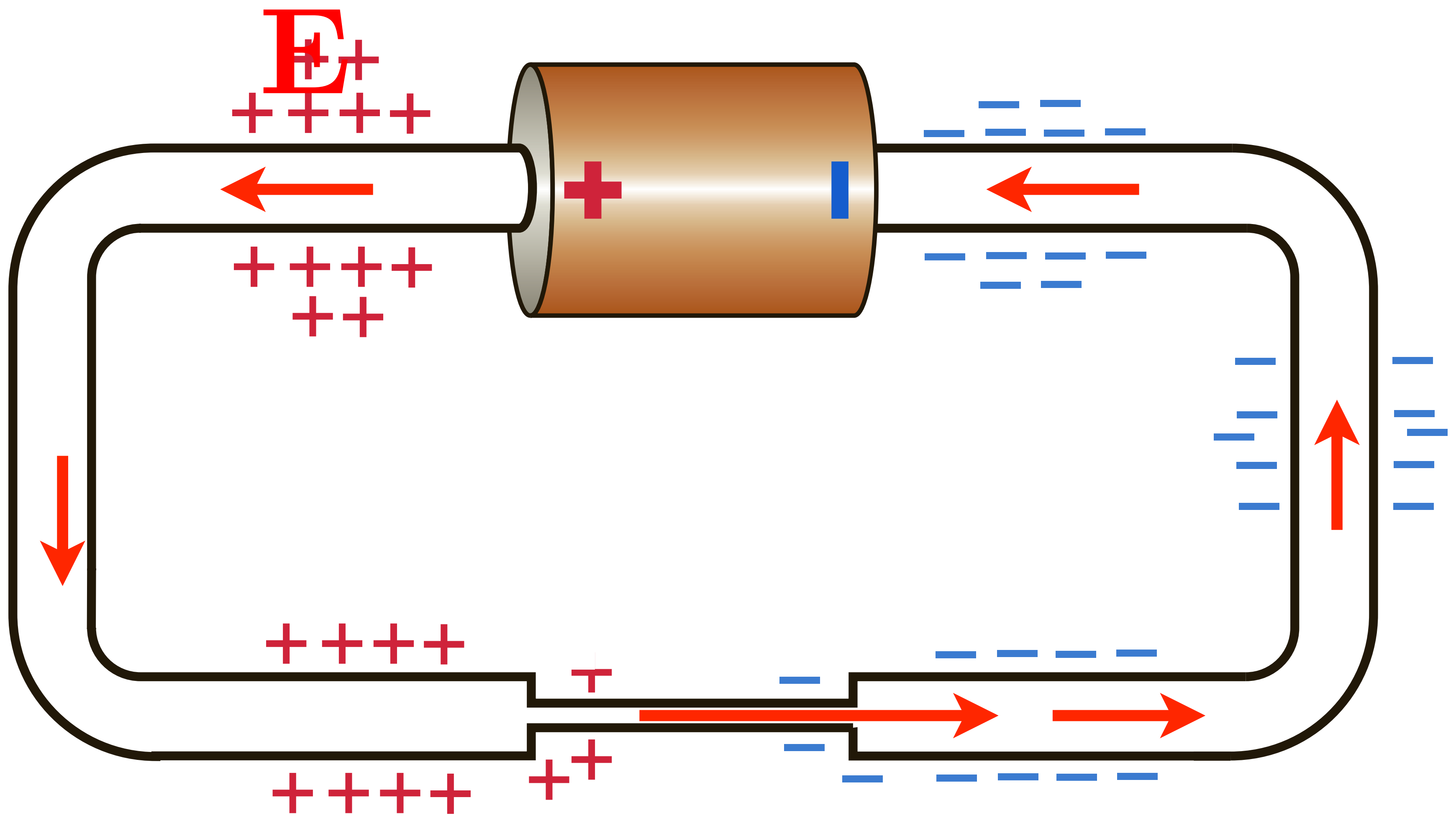
The *charges* side of electricity



$$R = \frac{\rho\, L}{S}$$

$$|\mathbf{E}| = \frac{\rho\, I}{S}$$

The *charges* side of electricity



Copper $\rho = 1.72 \times 10^{-8}\ \Omega\,\mathrm{m}$

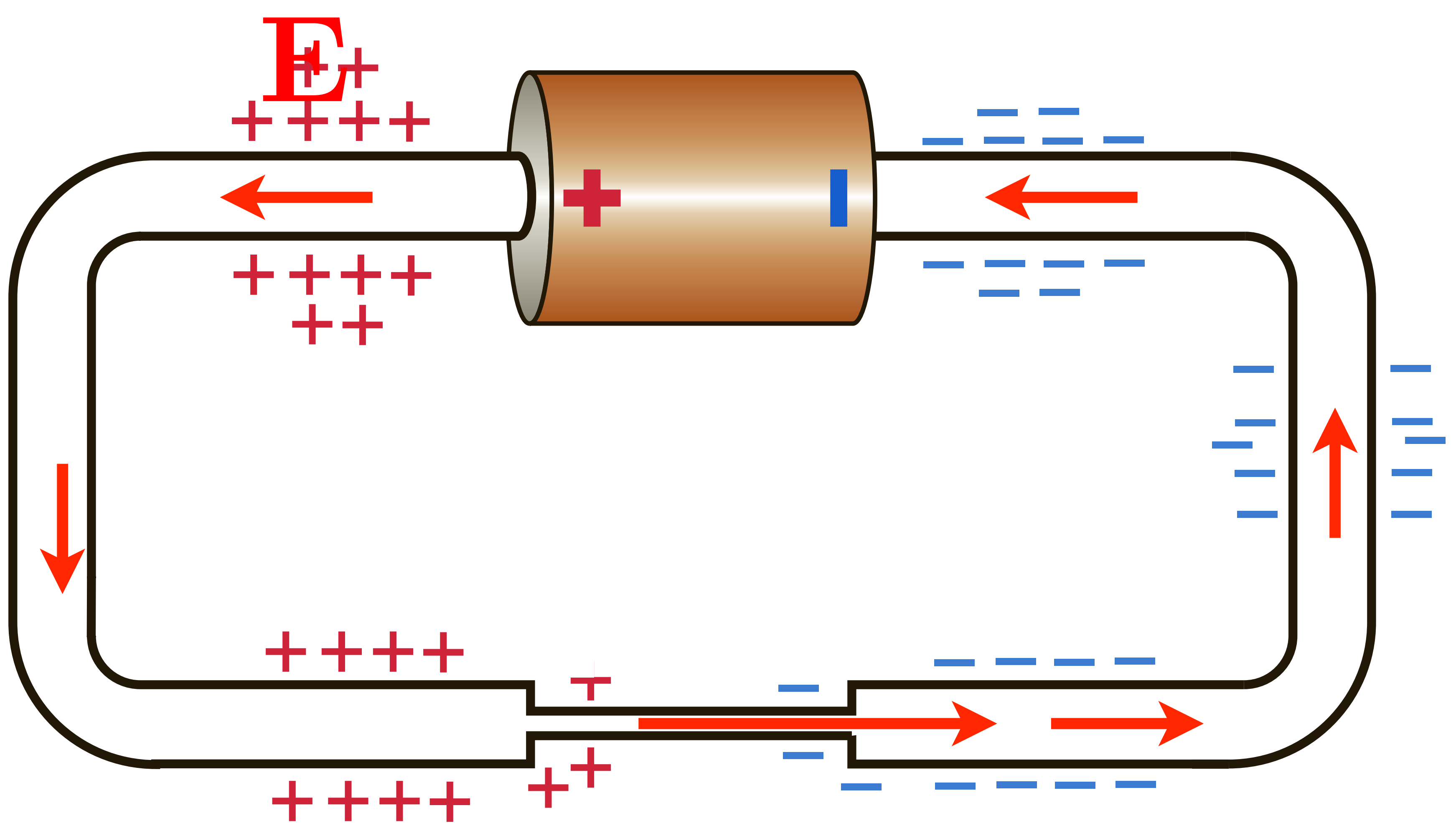
$I = 1\ \mathrm{A}$ $\qquad S = 1\ \mathrm{mm}^2$

$E = 1.72 \times 10^{-2}\ \mathrm{V/m}$

$$R = \frac{\rho\, L}{S}$$

$$|\mathbf{E}| = \frac{\rho\, I}{S}$$

The *charges* side of electricity



$$R = \frac{\rho\, L}{S}$$

$$|\mathbf{E}| = \frac{\rho\, I}{S}$$

Copper $\rho = 1.72 \times 10^{-8}\ \Omega\,\mathrm{m}$

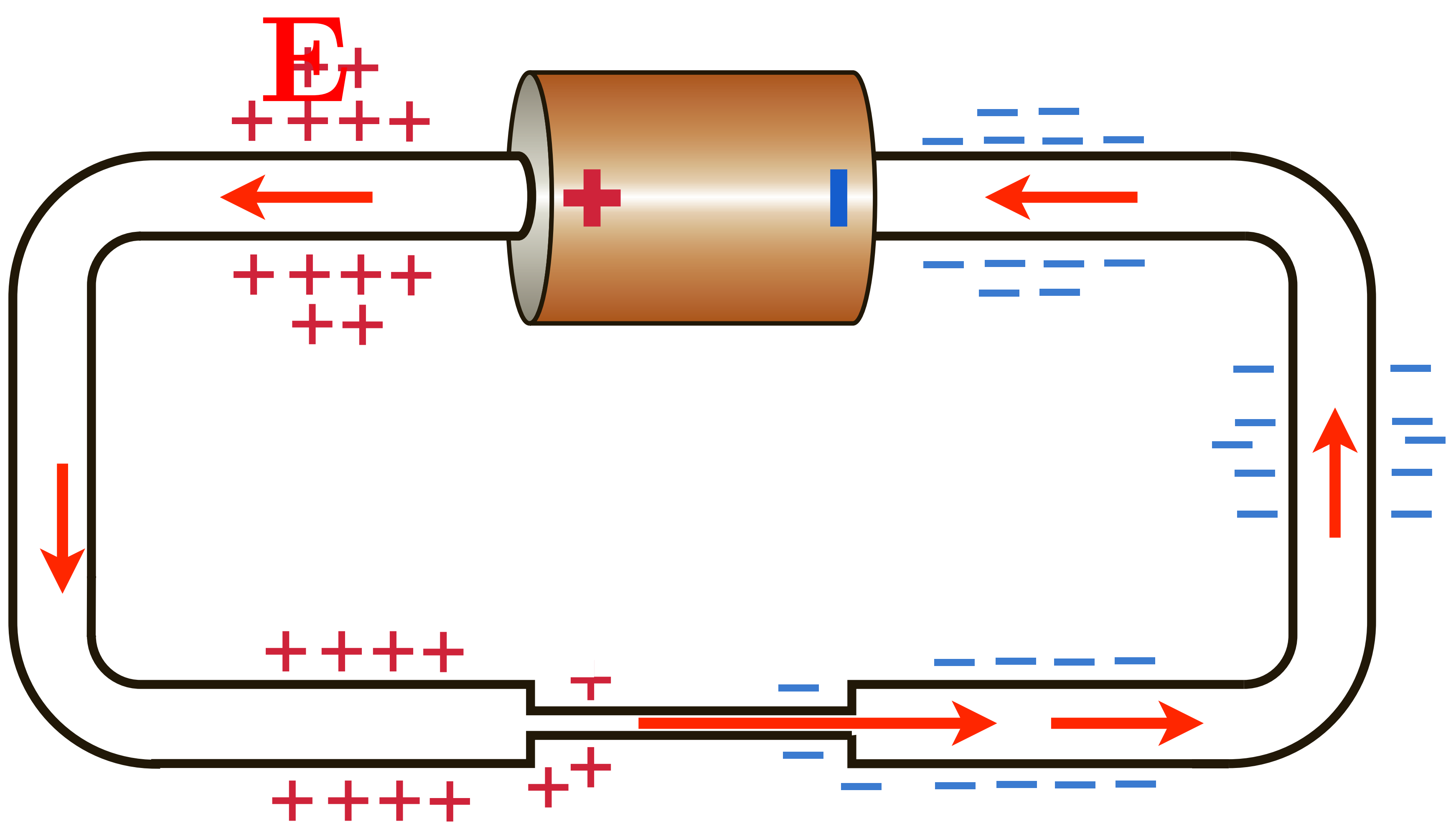
$I = 1\ \mathrm{A}$ $\quad S = 1\ \mathrm{mm}^2$

$E = 1.72 \times 10^{-2}\ \mathrm{V/m}$

Point charge estimate:

For $d = 0.28$ mm $\quad E = \dfrac{qe}{4\pi\varepsilon_0 d^2} \to q = 0.94$

The *charges* side of electricity



$$R = \frac{\rho\, L}{S}$$

$$|\mathbf{E}| = \frac{\rho\, I}{S}$$

Copper $\quad \rho = 1.72 \times 10^{-8}\ \Omega\,\mathrm{m}$

$$I = 1\ \mathrm{A} \qquad S = 1\ \mathrm{mm}^2$$

$$E = 1.72 \times 10^{-2}\ \mathrm{V/m}$$

Point charge estimate:

For $d = 0.28$ mm $\qquad E = \dfrac{qe}{4\pi\varepsilon_0 d^2} \to q = 0.94$

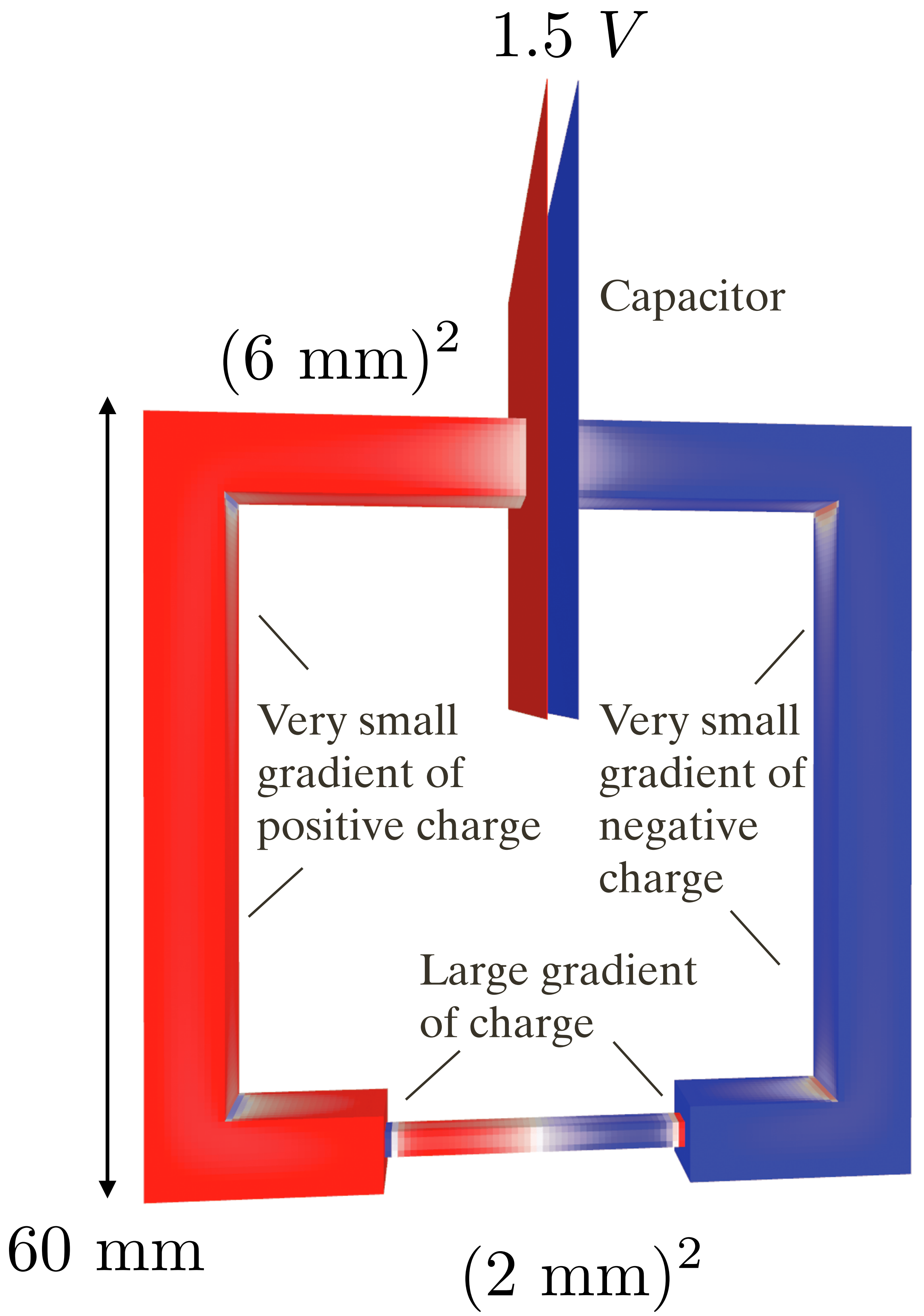
Surface charge estimate:

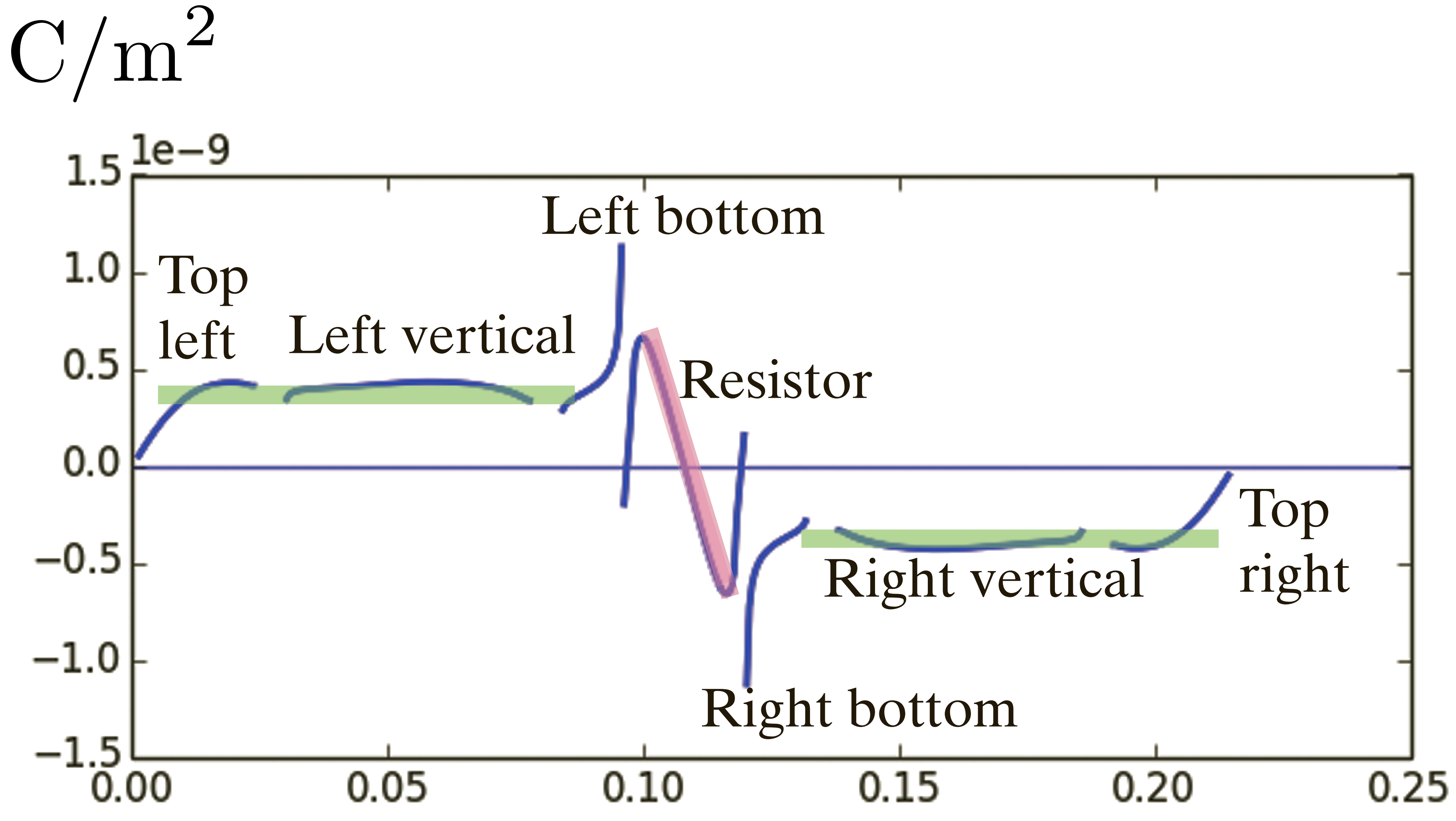
$$E = \frac{qe}{\varepsilon_0 S} = \frac{\rho I}{S} \longrightarrow q = \frac{\varepsilon_0 \rho}{e} I$$

$$\frac{\varepsilon_0 \rho}{e} = 0.95\ \mathrm{A}^{-1}$$

W. G. V. Rosser, Am. J. Phys. **38**, (1970)

The *charges* side of electricity





R. W. Chabay and B. A. Sherwood, *Matter & Interactions*, 4th ed., Wiley (2025)
Volume II, Chap. 18, sec. 18.5

Visualizing the electrical field around an electrical circuit

Demonstration of the Electric Fields of Current-Carrying Conductors

OLEG JEFIMENKO
Physics Department, West Virginia University, Morgantown, West Virginia

(Received July 31, 1961)

The making of the two-dimensional printed circuit type models of current-carrying conducting systems and the use of these models for demonstrating the electric fields of current-carrying conductors is described. The models are produced by drawing the systems under consideration on glass plates using a transparent conducting ink. The electric lines of force inside and outside the elements of these models are demonstrated with the aid of grass seeds strewed upon them.

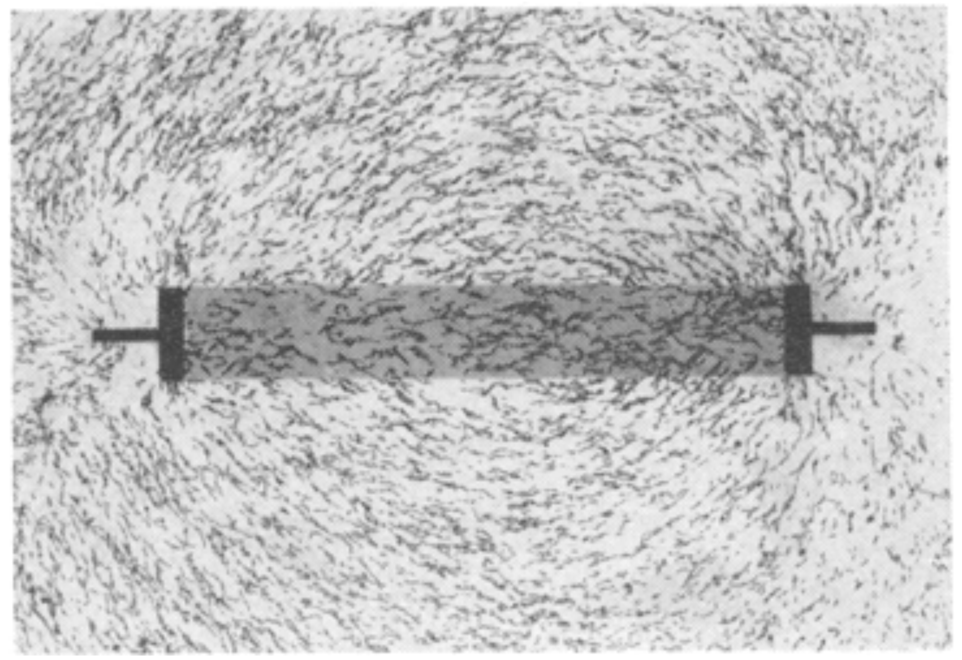


FIG. 1. Electric field of a straight conductor.

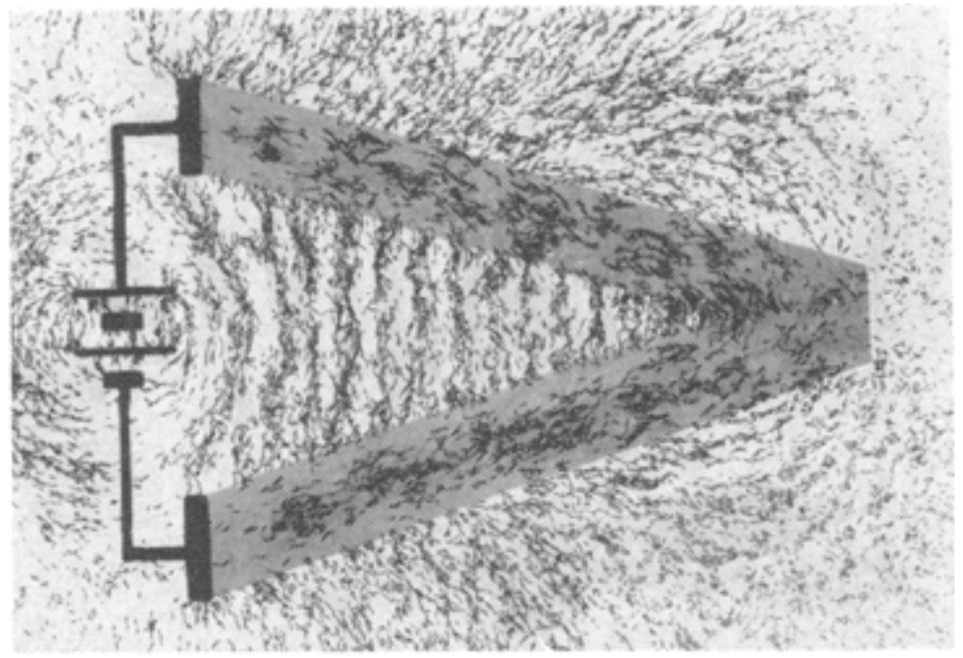


FIG. 2. Electric field of two intersecting straight conductors.

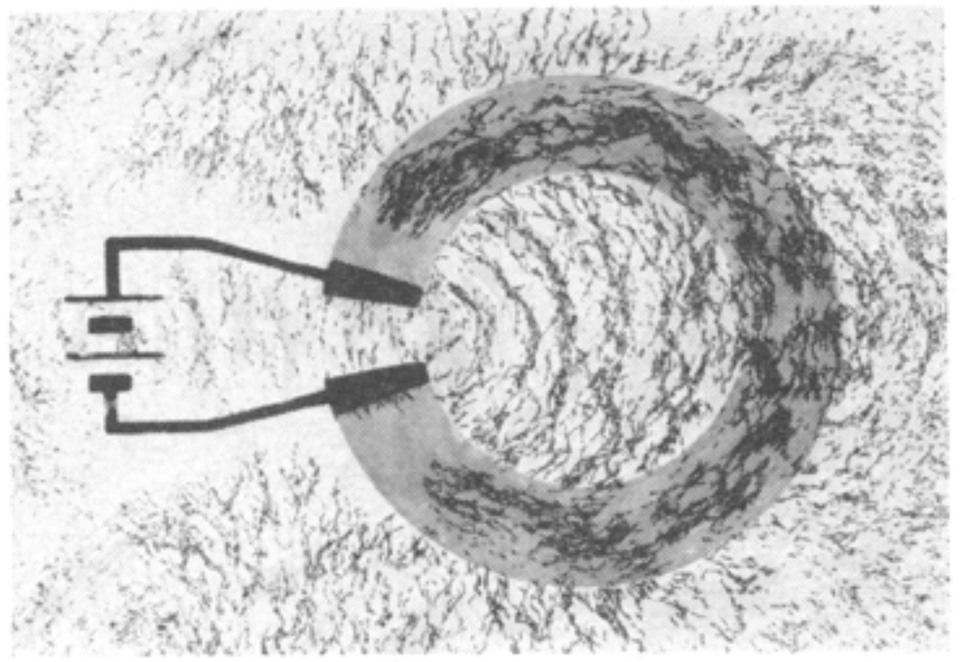


FIG. 3. Electric field of a circular conducting ring (hollow cylinder).

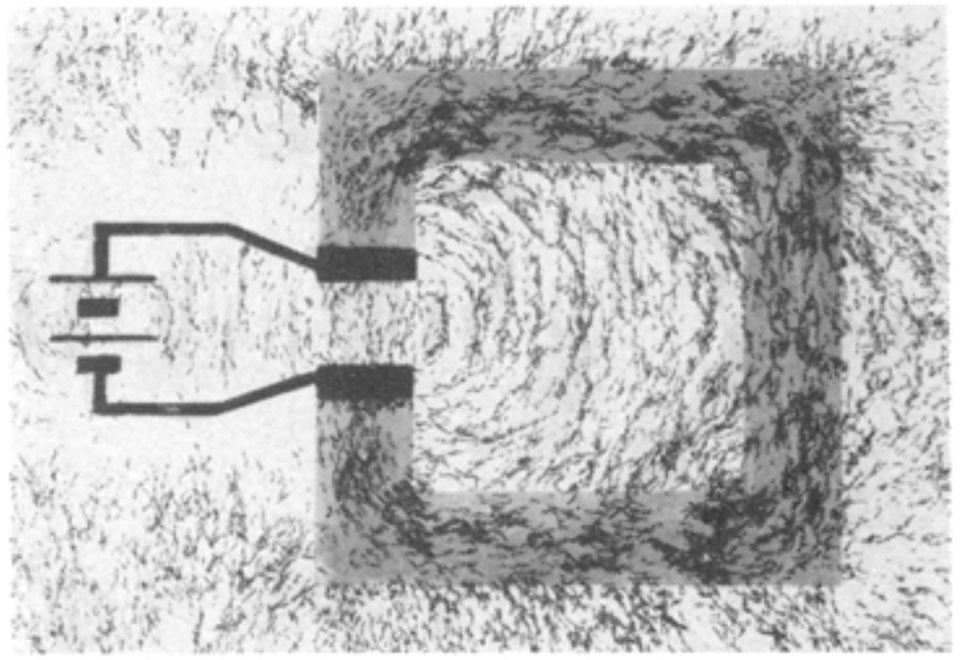


FIG. 4. Electric field of a square-shaped conducting ring (box).

O. Jefimenko, Am. J. Phys. **30**, 19 (1962)

A. K. Torres Assis and J. Akashi Hernandes, *The electric force of a curren*t (2007)

Energy flow and conservation in Maxwell's theory

Energy flow and conservation in Maxwell’s theory

Energy conservation equation

$$\mathcal{E} = \frac{\varepsilon_0}{2}\left(\mathbf{E}^2 + c^2\mathbf{B}^2\right)$$

$$\mathbf{S} = \varepsilon_0 c^2 \mathbf{E} \wedge \mathbf{B}$$

$$\frac{\partial \mathcal{E}}{\partial t} + \mathrm{div}(\mathbf{S}) = -\mathbf{j}\cdot\mathbf{E}$$

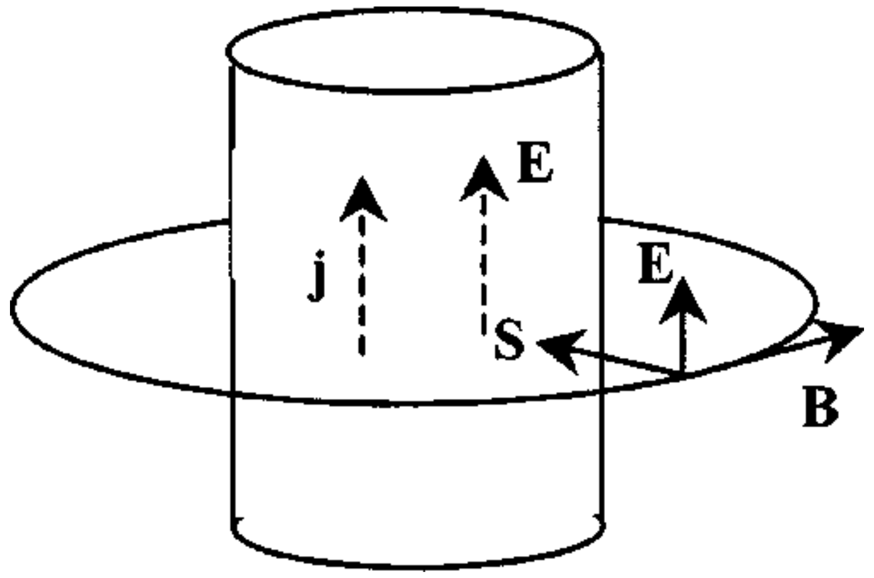
Energy flow and conservation in Maxwell's theory

Energy conservation equation

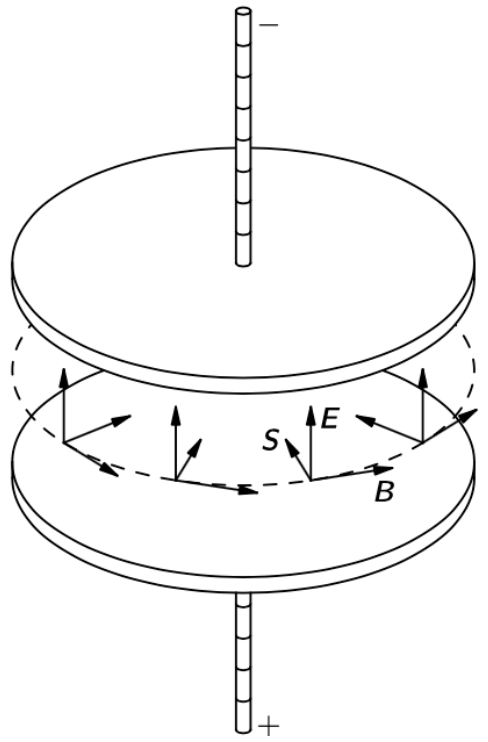
$$\mathcal{E} = \frac{\varepsilon_0}{2}\left(\mathbf{E}^2 + c^2\mathbf{B}^2\right)$$

$$\mathbf{S} = \varepsilon_0 c^2 \mathbf{E} \wedge \mathbf{B}$$

$$\frac{\partial \mathcal{E}}{\partial t} + \mathrm{div}(\mathbf{S}) = -\mathbf{j}\cdot\mathbf{E}$$



Close to a resistive wire



Close to a charging capacitor

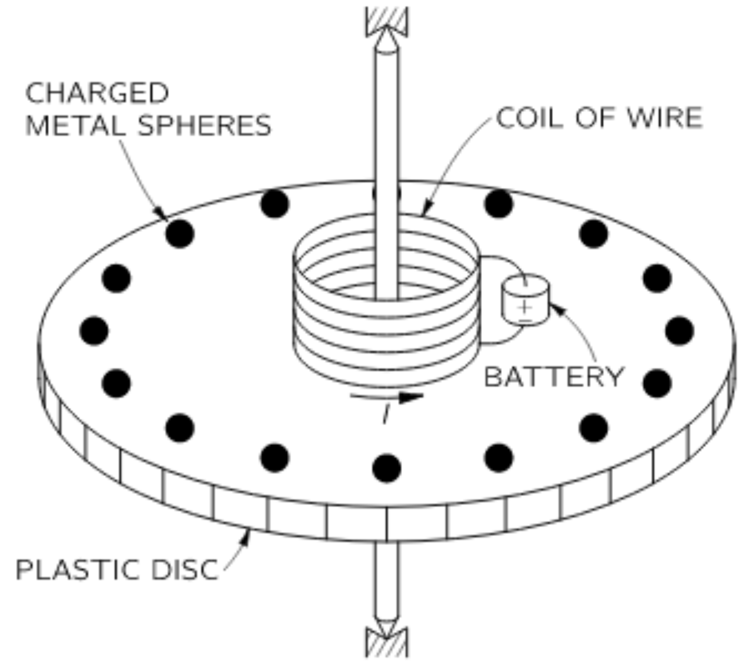


Fig. 17–5. Will the disc rotate if the current I is stopped?

R.P. Feynman *et al*, Lectures on Physics, chap. 27 (1965)

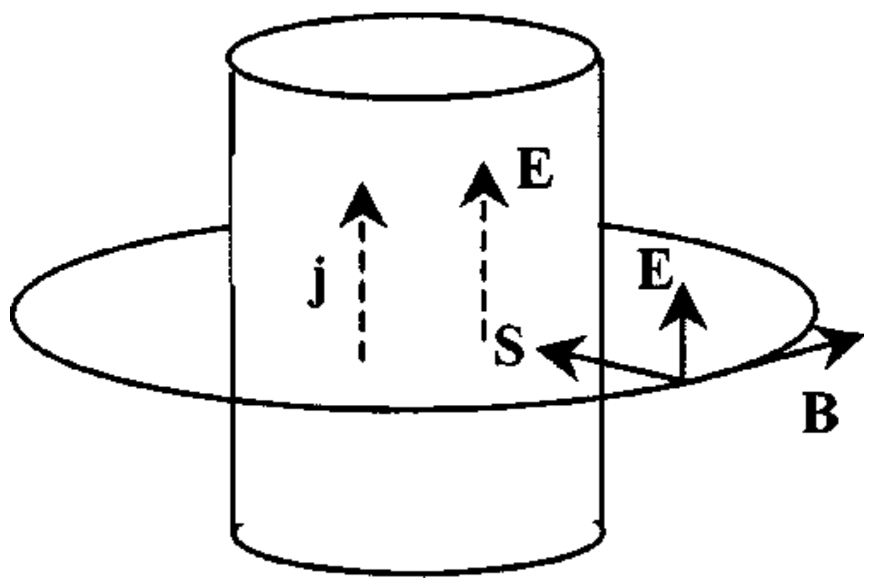
Energy flow and conservation in Maxwell's theory

Energy conservation equation

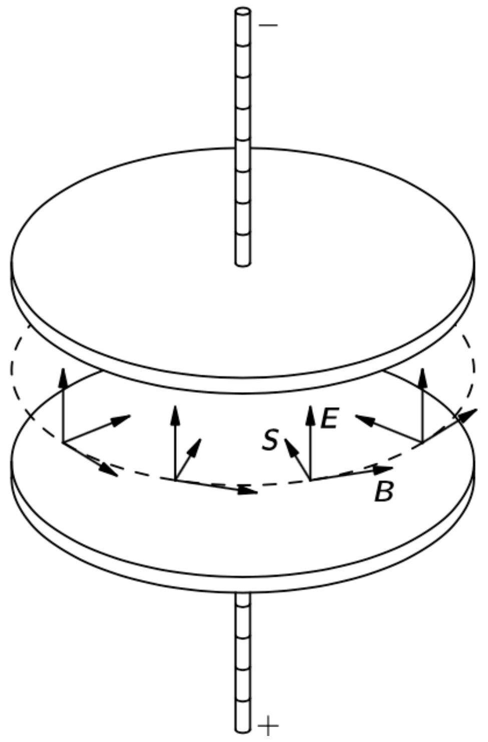
$$\mathcal{E} = \frac{\varepsilon_0}{2}\left(\mathbf{E}^2 + c^2\mathbf{B}^2\right)$$

$$\mathbf{S} = \varepsilon_0 c^2 \mathbf{E} \wedge \mathbf{B}$$

$$\frac{\partial \mathcal{E}}{\partial t} + \mathrm{div}(\mathbf{S}) = -\mathbf{j} \cdot \mathbf{E}$$



Close to a resistive wire



Close to a charging capacitor

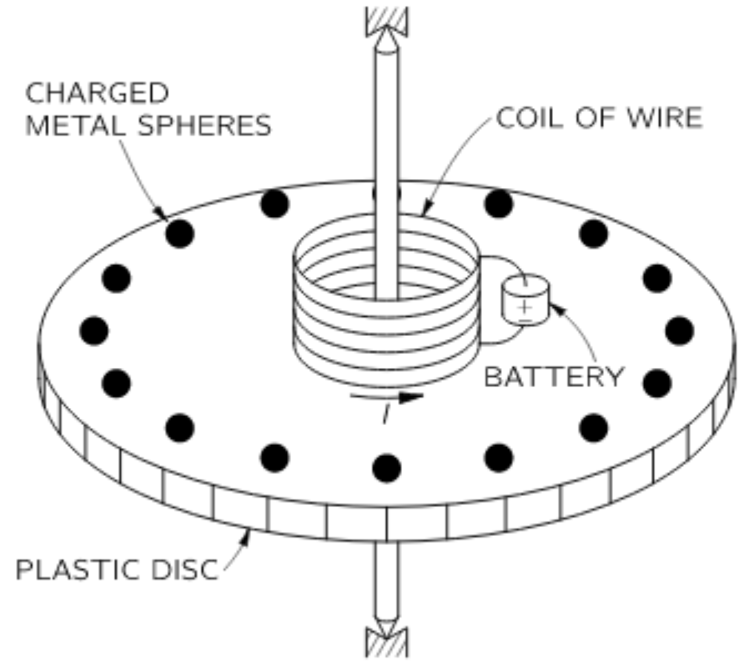


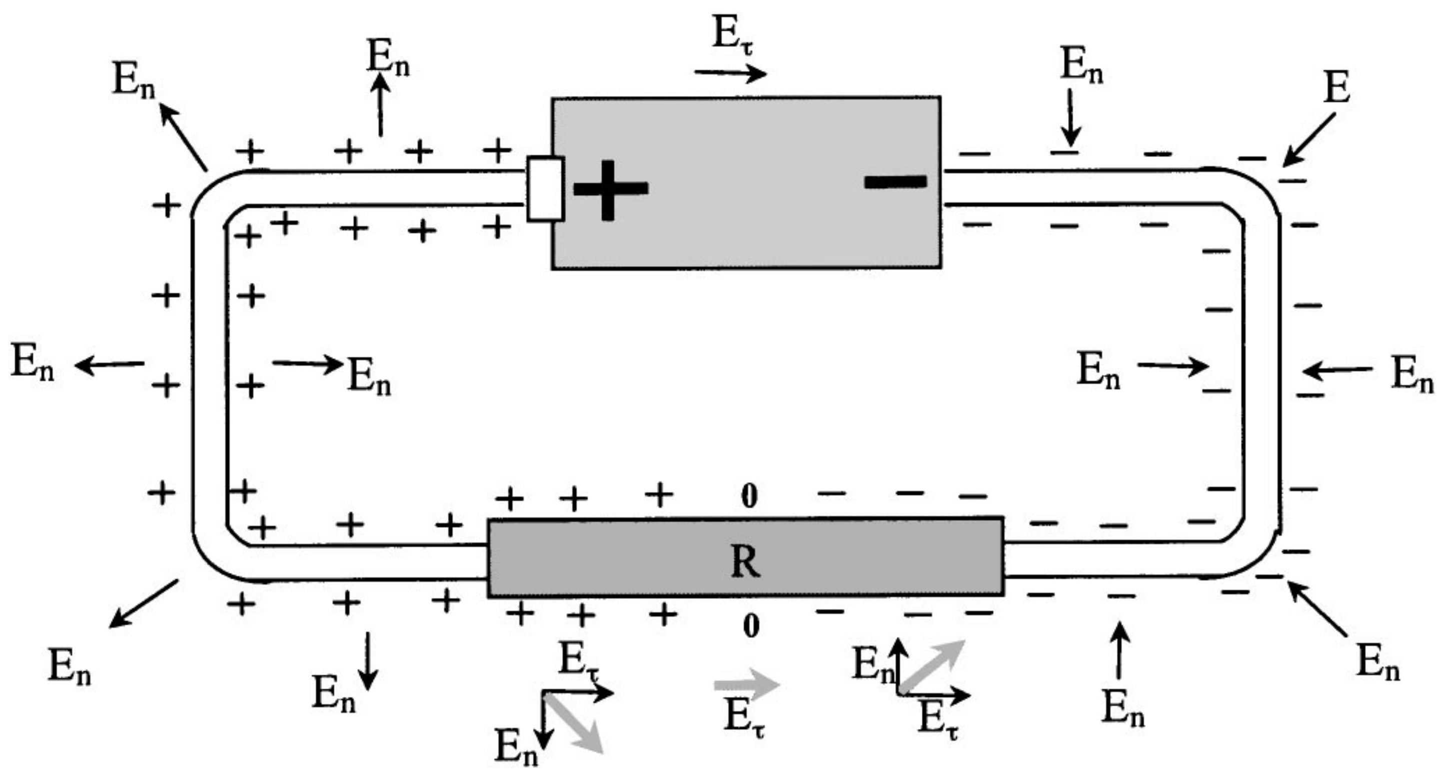
Fig. 17–5. Will the disc rotate if the current I is stopped?

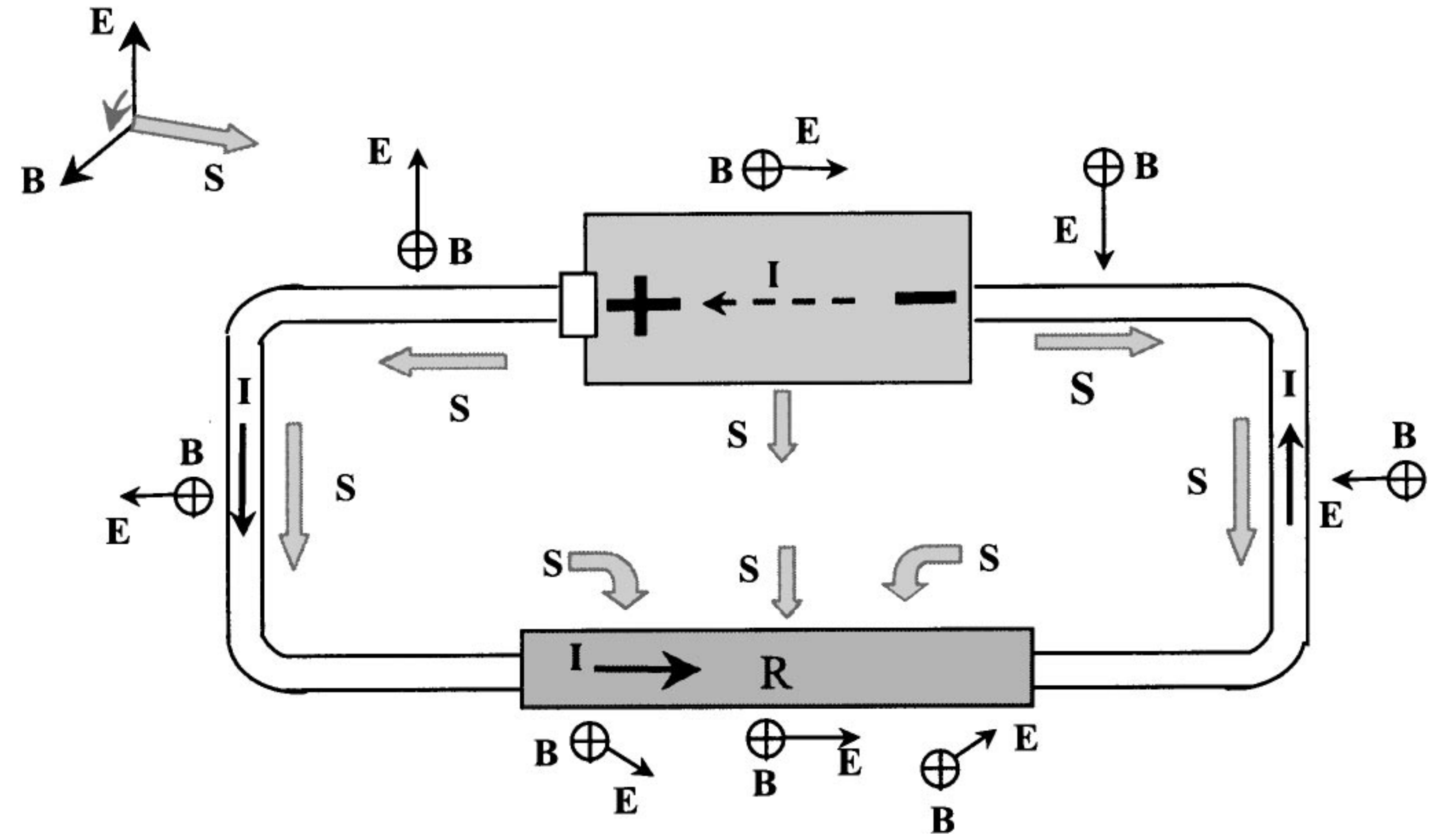
R.P. Feynman *et al*, Lectures on Physics, chap. 27 (1965)

Question: what about an electrical circuit ?

How energy flows in stationary circuits ?

Question: **what about an electrical circuit ?**





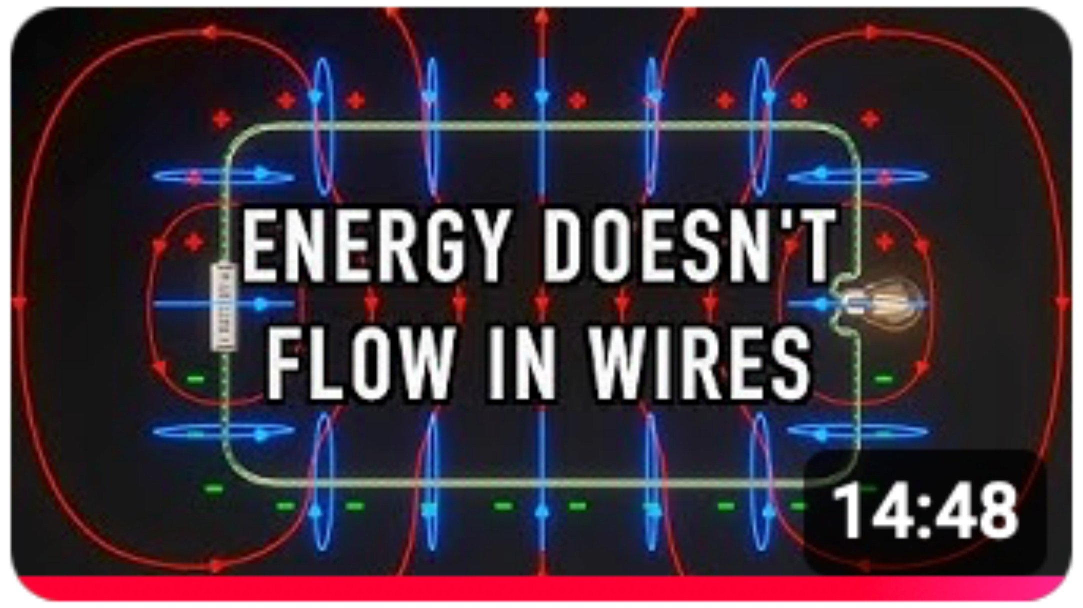
Surface charges
Electrical current

Electric field
Magnetic field

Energy flow (Poynting vector)

I. Galli and E. Goihbarg, Am. J. Phys. **73**, 141 (2005)

Electrical circuits and EM waves

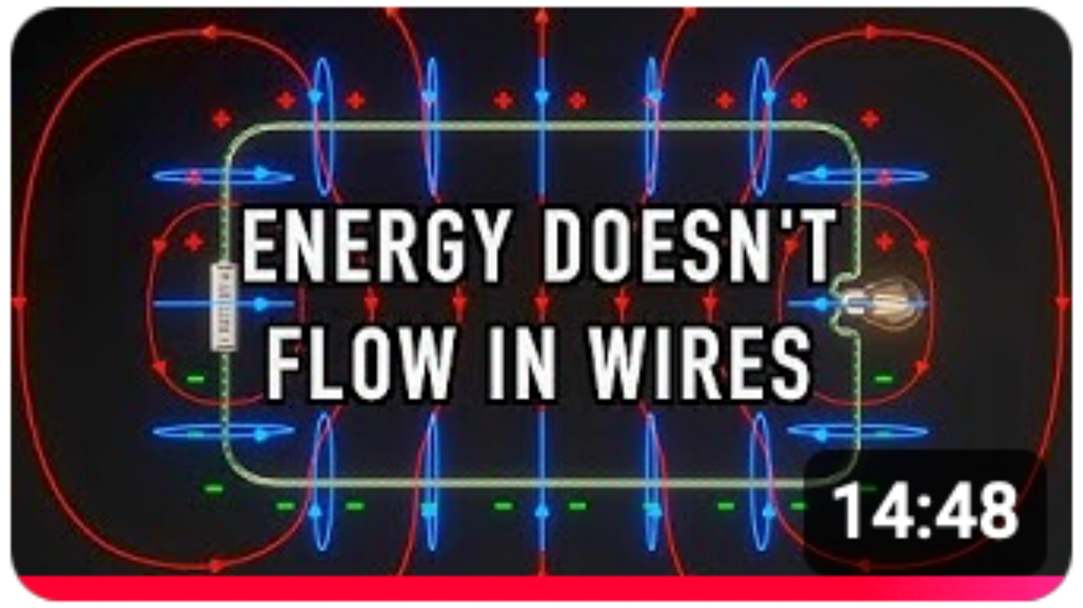


The Big Misconception About Electricity

Veritasium

24 M de vues • il y a 3 ans

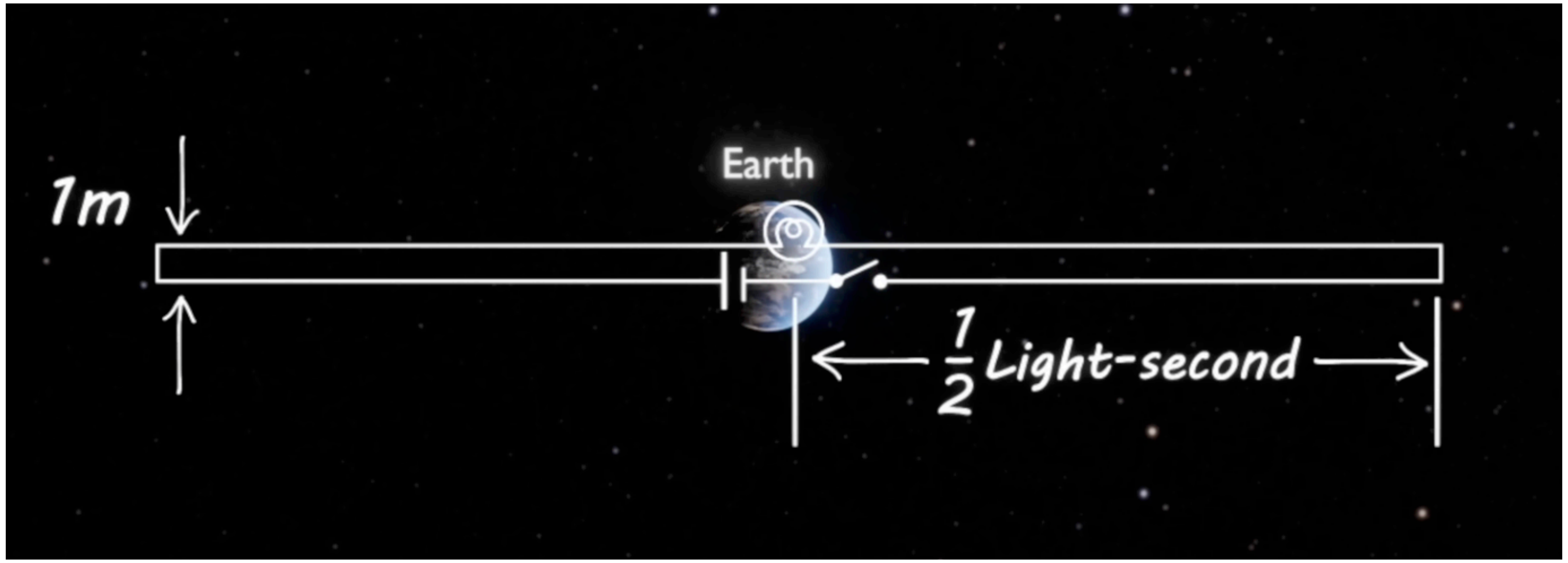
Electrical circuits and EM waves



The Big Misconception About Electricity

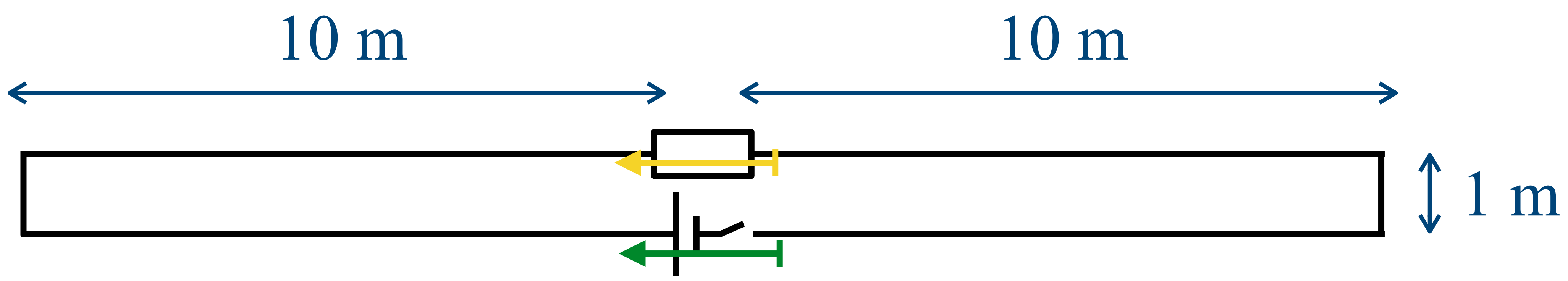
Veritasium

24 M de vues • il y a 3 ans

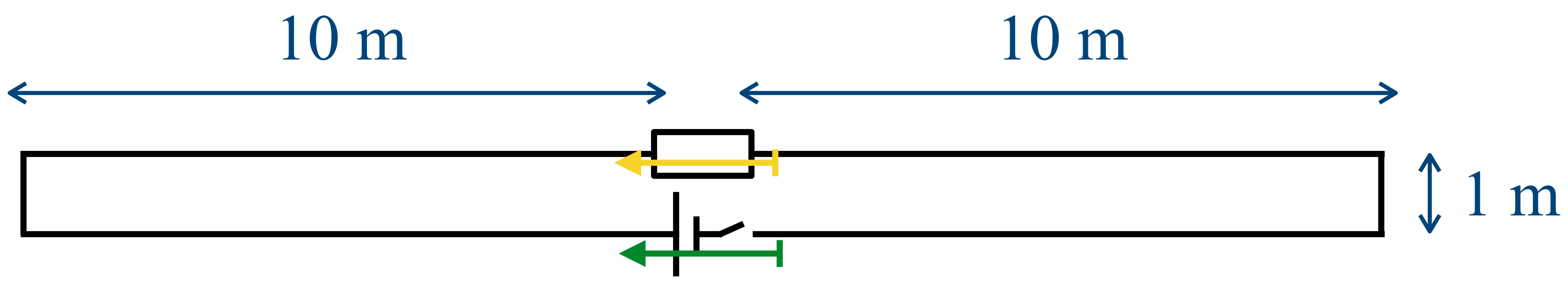


1 s or 3 ns ?

Experimental results !



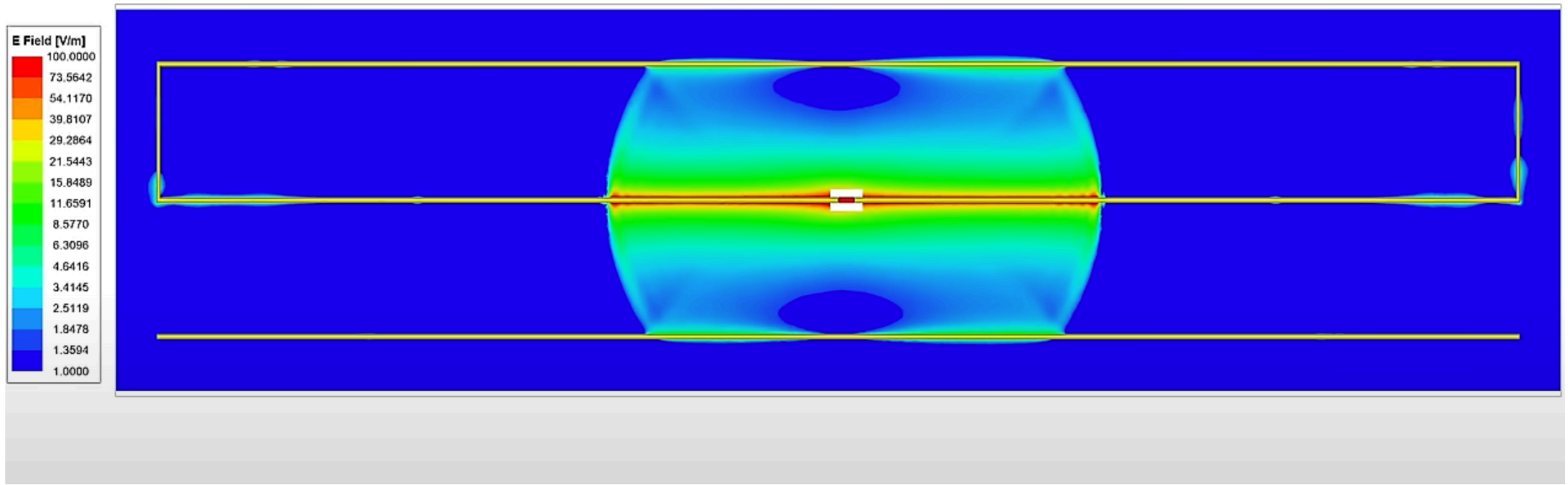
Experimental results !



Ansys

Ansys

Electric field simulation



Outgoing wave at the speed of light

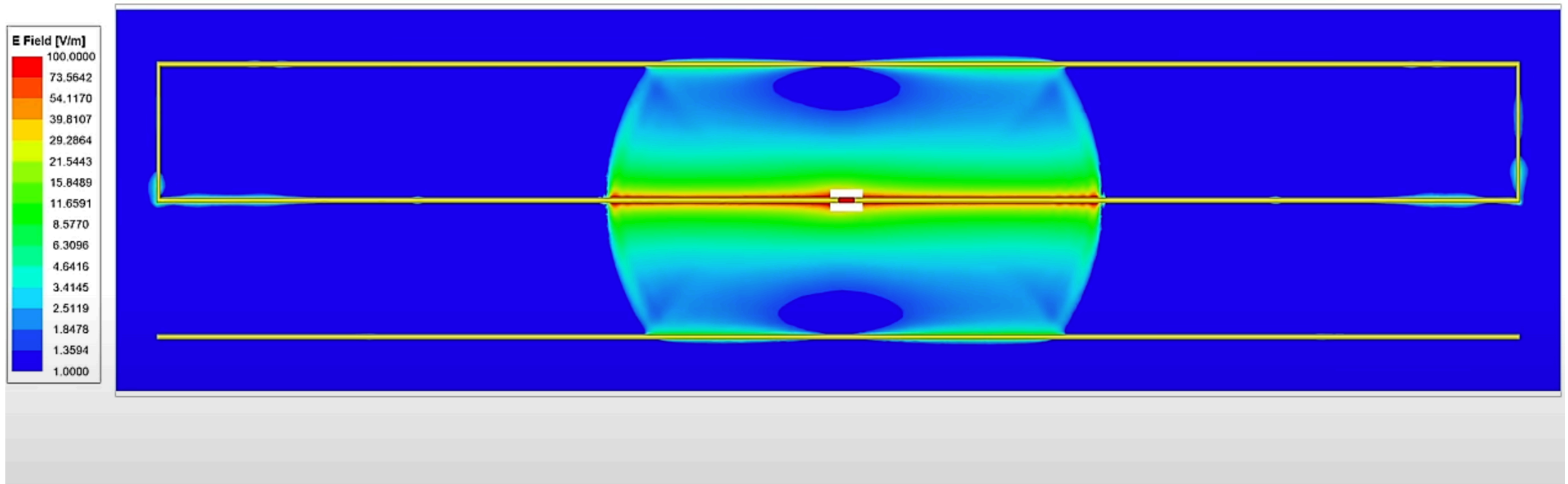
Transient regime: the propagating electric field can induce current in the resistor !

Electrical circuits and EM waves

Lab PHYS ENS de LYON

Ansys

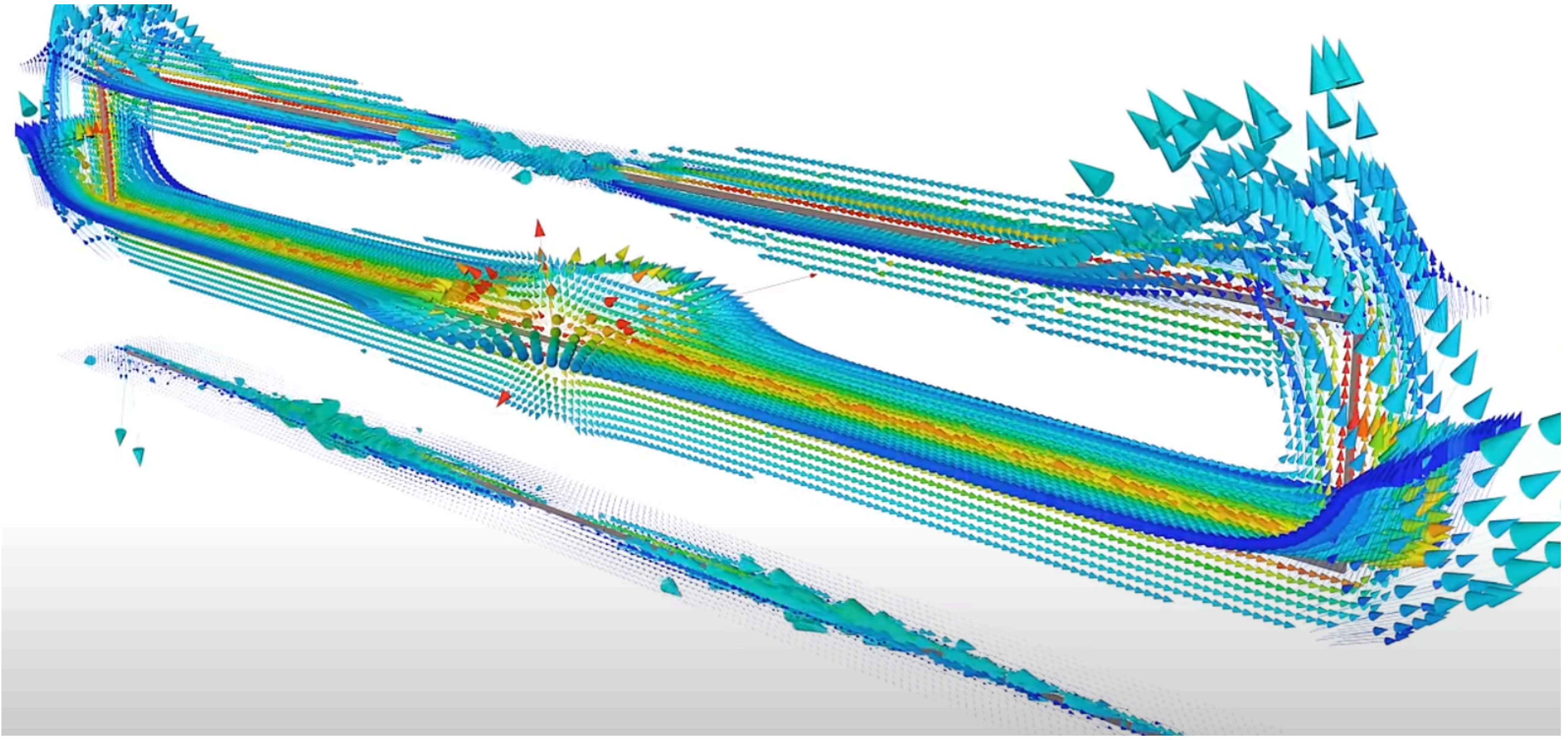
Electric field simulation



Outgoing wave at the speed of light

Transient regime: the propagating electric field can induce current in the resistor !

Poynting vector simulation



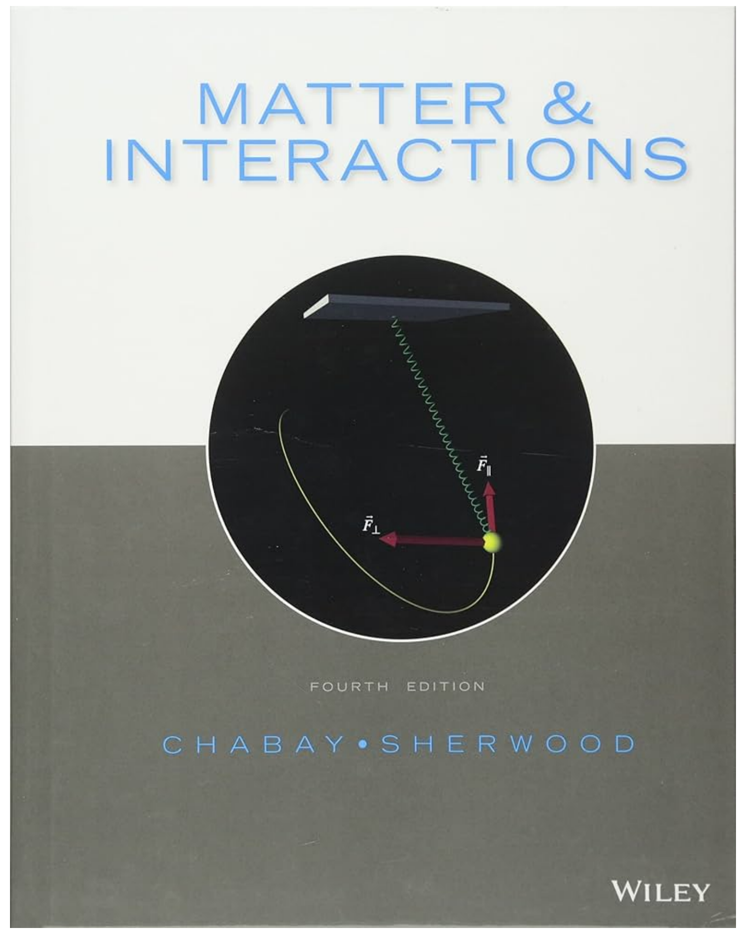
Close to stationary regime:
the energy flows is located **around the wires !**

Key messages on classical macroscopic circuits

Key messages on classical macroscopic circuits

Electrical circuits exhibit surface charges that:

- Maintain the potential around the circuit
- Generate the electrical field outside the conductors
- Ensure the confinement of the electrical current within the conductors

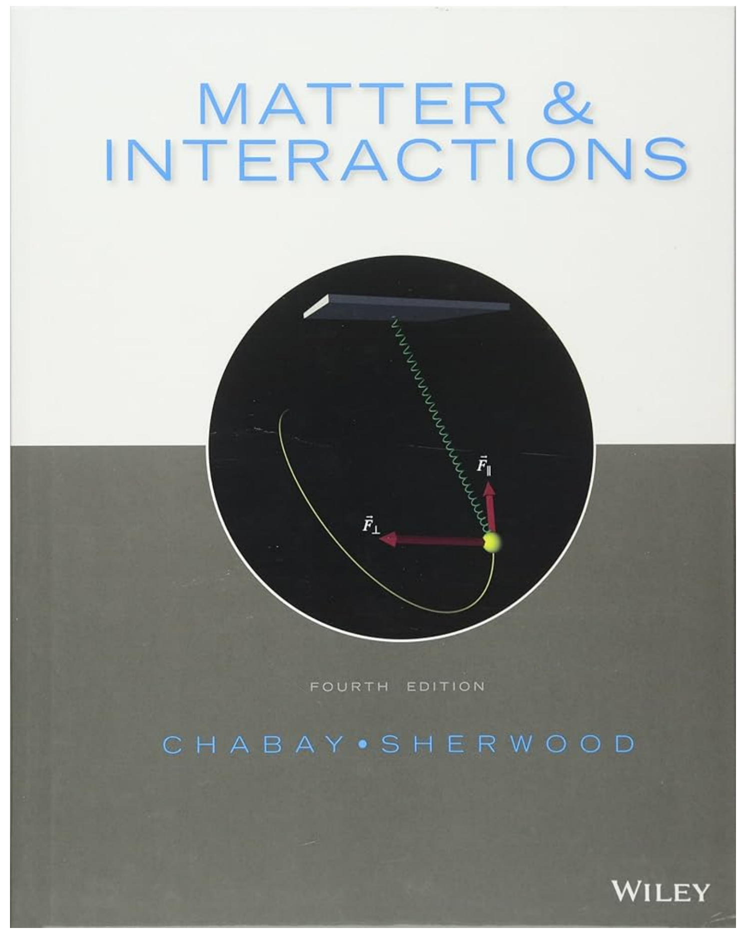


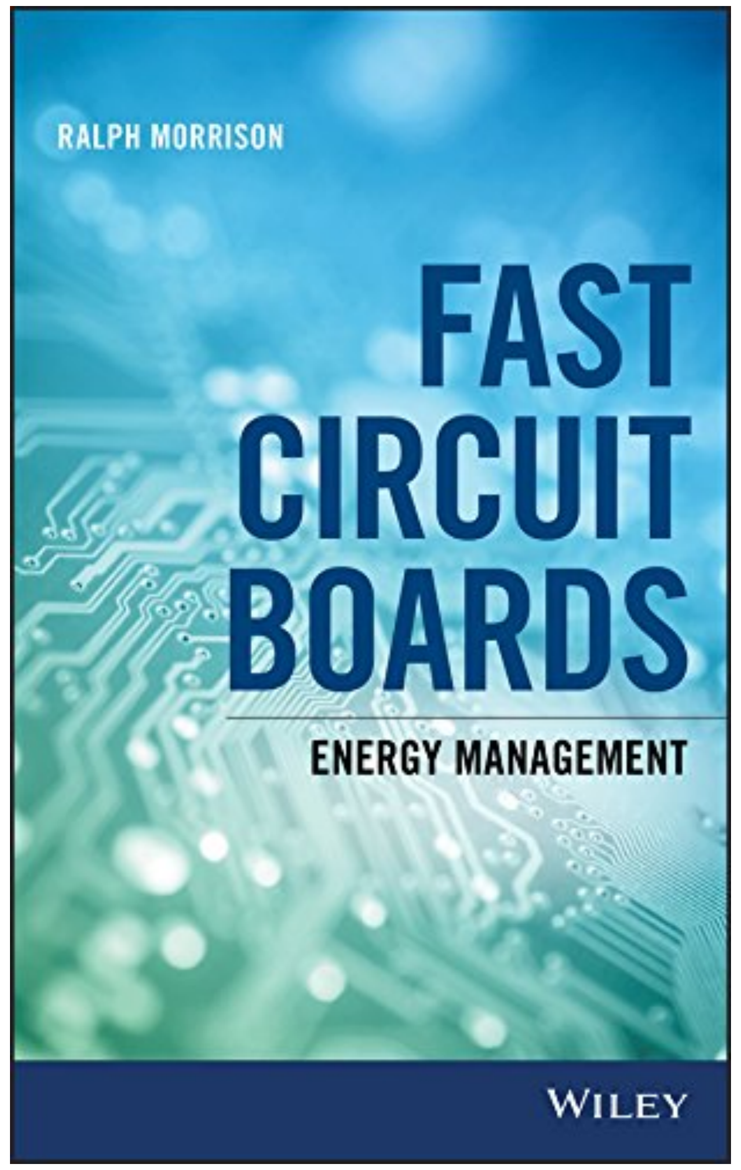
Key messages on classical macroscopic circuits

Electrical circuits exhibit surface charges that:

- Maintain the potential around the circuit
- Generate the electrical field outside the conductors
- Ensure the confinement of the electrical current within the conductors

Buildings have walls and halls.
People travel in the halls, not the walls.
Circuits have traces and spaces.
Energy travels in the spaces, not the traces.
Ralph Morrison





Outline

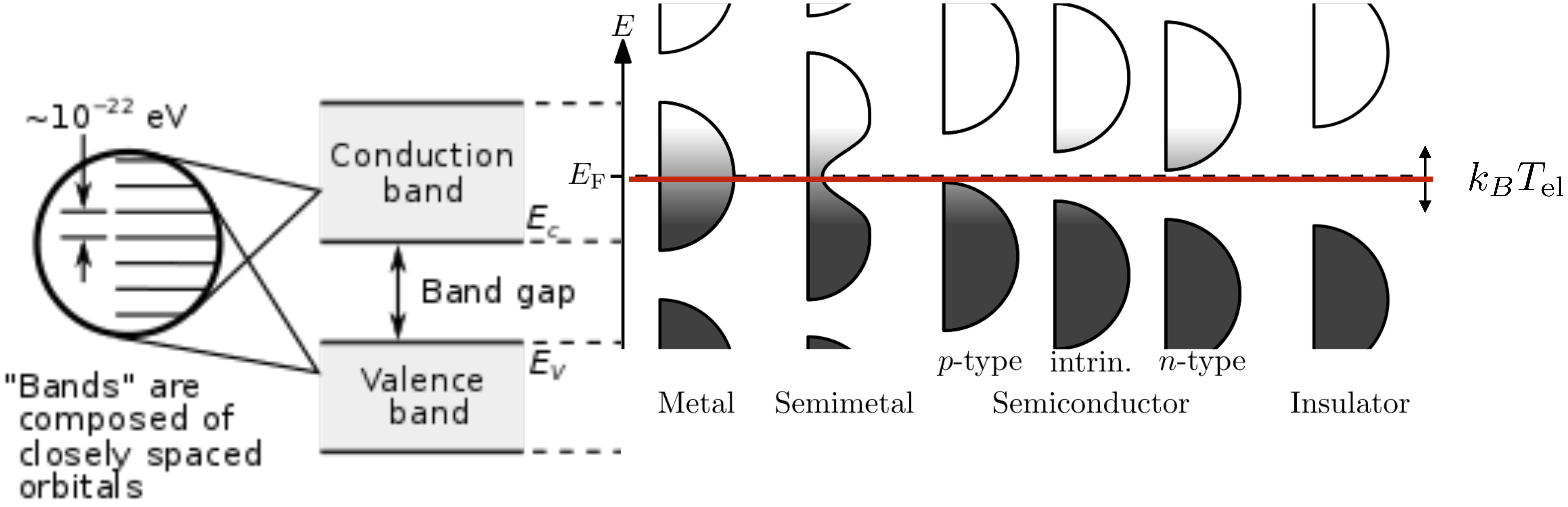
- From Ampère to electrical circuits
- Quantum physics within electricity
- Towards quantum coherent electronics
- Presentation of the PCP 2025 cycle

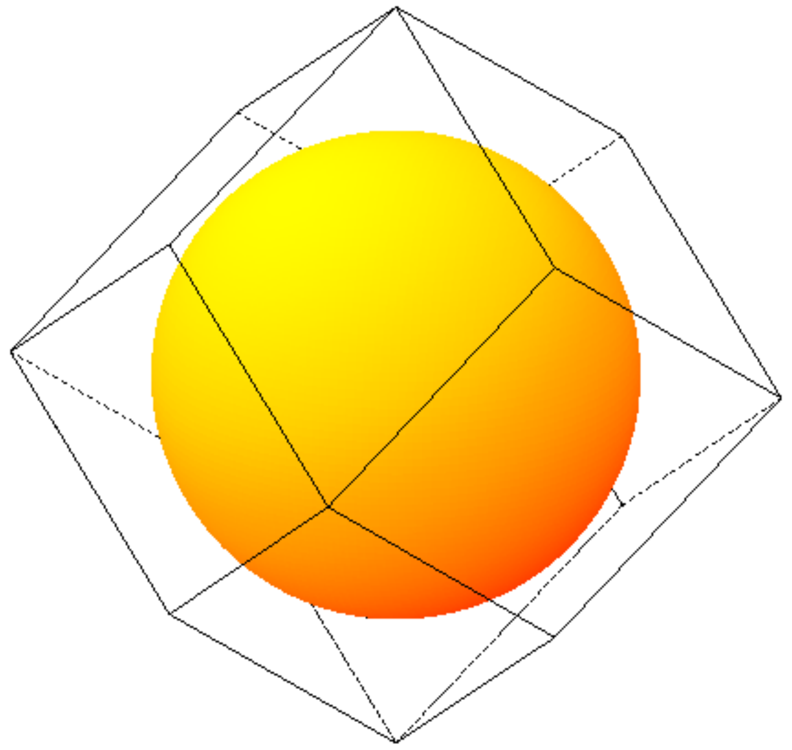
Part II: The quantum within electricity

- Quantum electrons in solids
- Quantum capacitances and inductances

Electrons in « ideal » solids

K



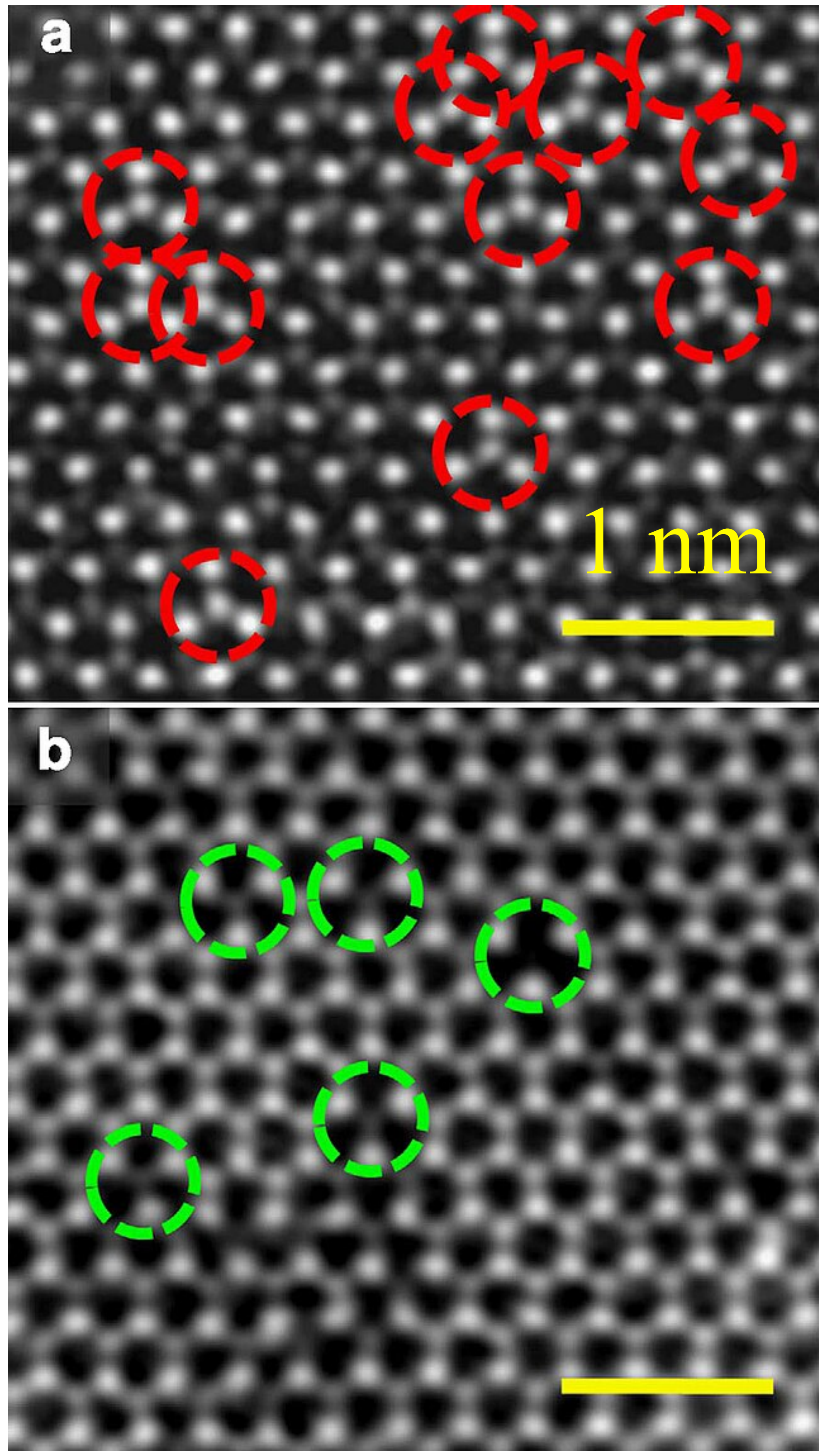


C. Kittel, *Quantum theory of solids*, chap 9.

Ideal crystal: macroscopic Slater determinant of delocalized electrons

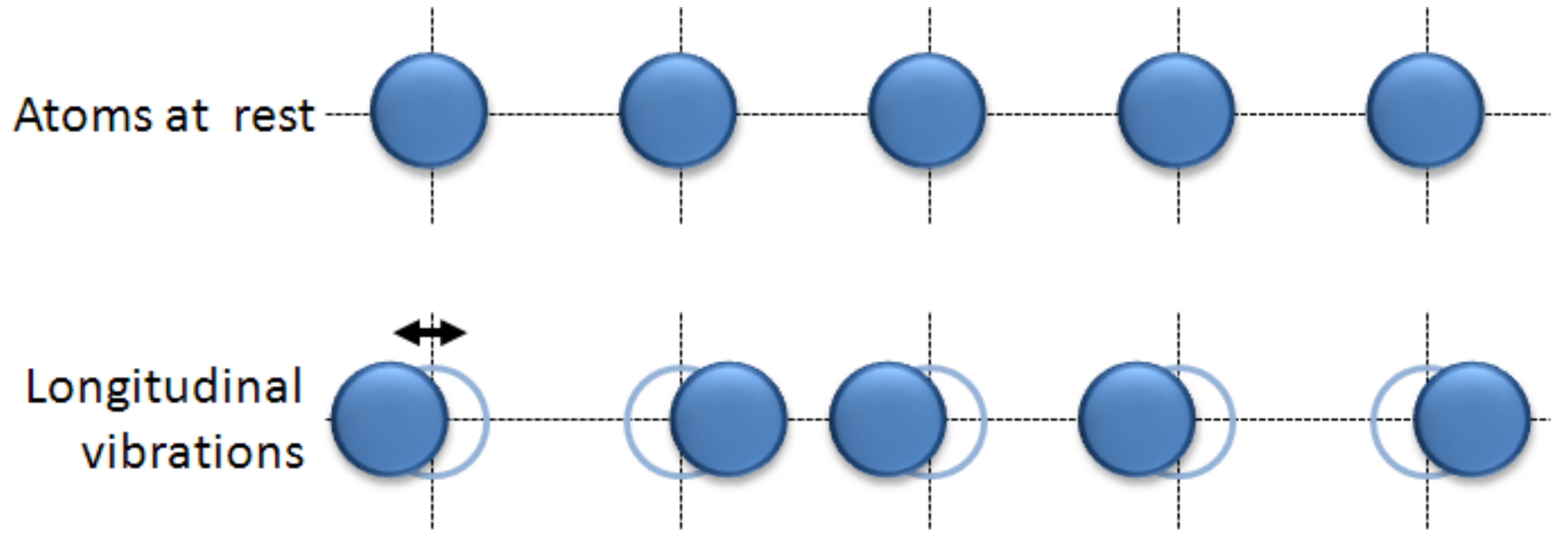
Electrons in « real » solids

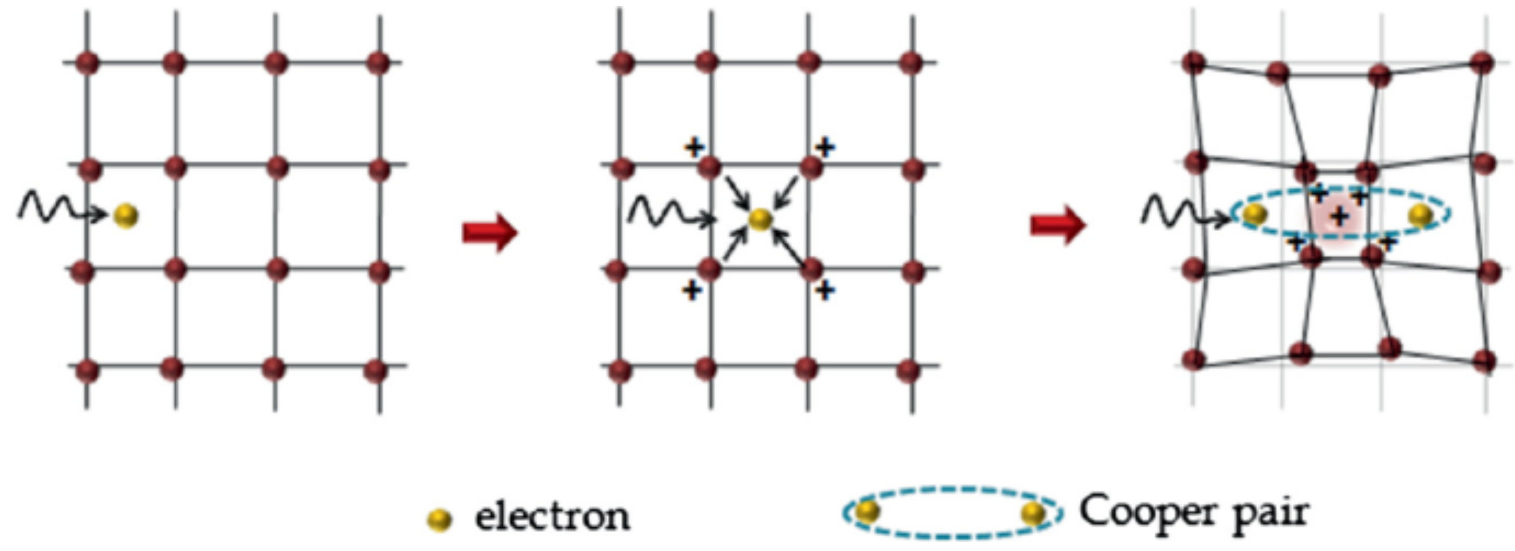
Crystal defects (*elastic collisions*)



Dynamical degrees of freedom (*inelastic collisions*)

Phonons





See H. Pothier's talk (19/03/2025)

Others: TLS, magnetic impurities, etc

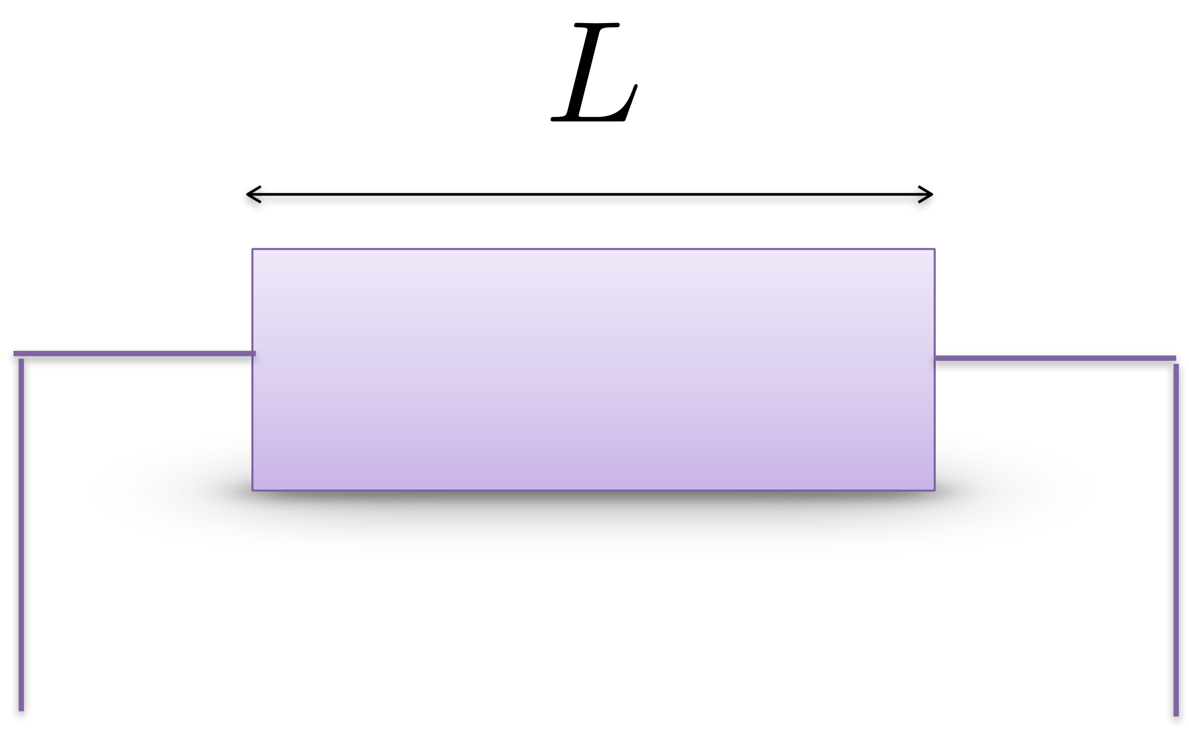
Electron / electron interactions !

Transport of electrons

Relevant length scales

l_e elastic scattering length

l_ϕ inelastic scattering length



Coherent transport is defined by: $L \lesssim l_\phi$

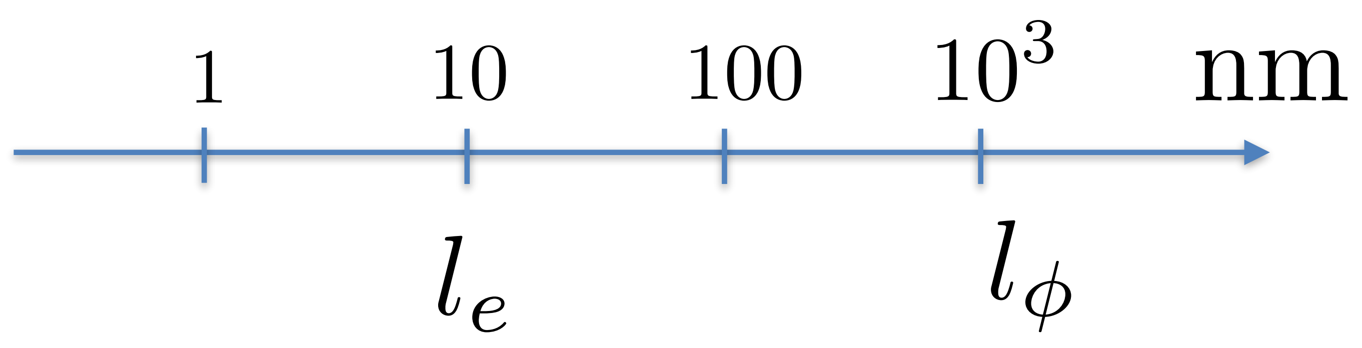
$$l_e \ll L \lesssim l_\phi$$

Diffusive transport

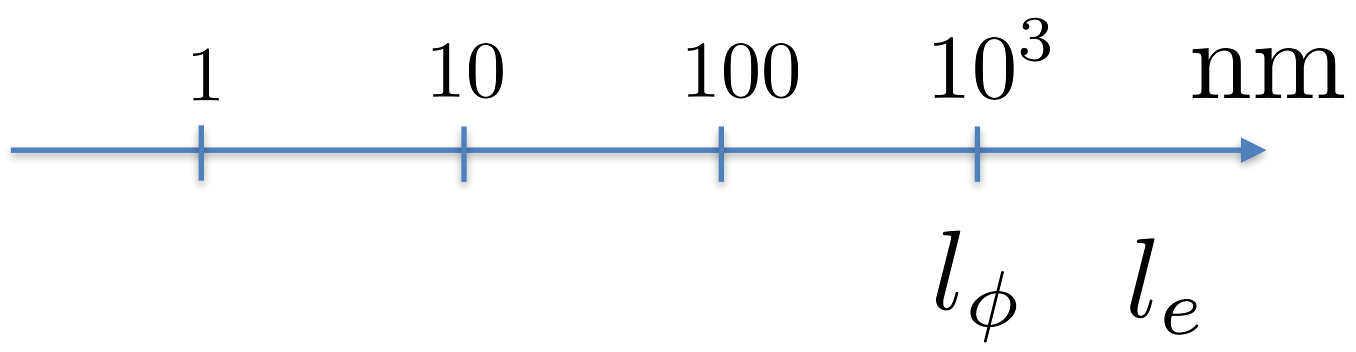
$$L \lesssim l_e \text{ and } l_\phi$$

Ballistic transport

Metal at 1 K



2DEG at 1 K

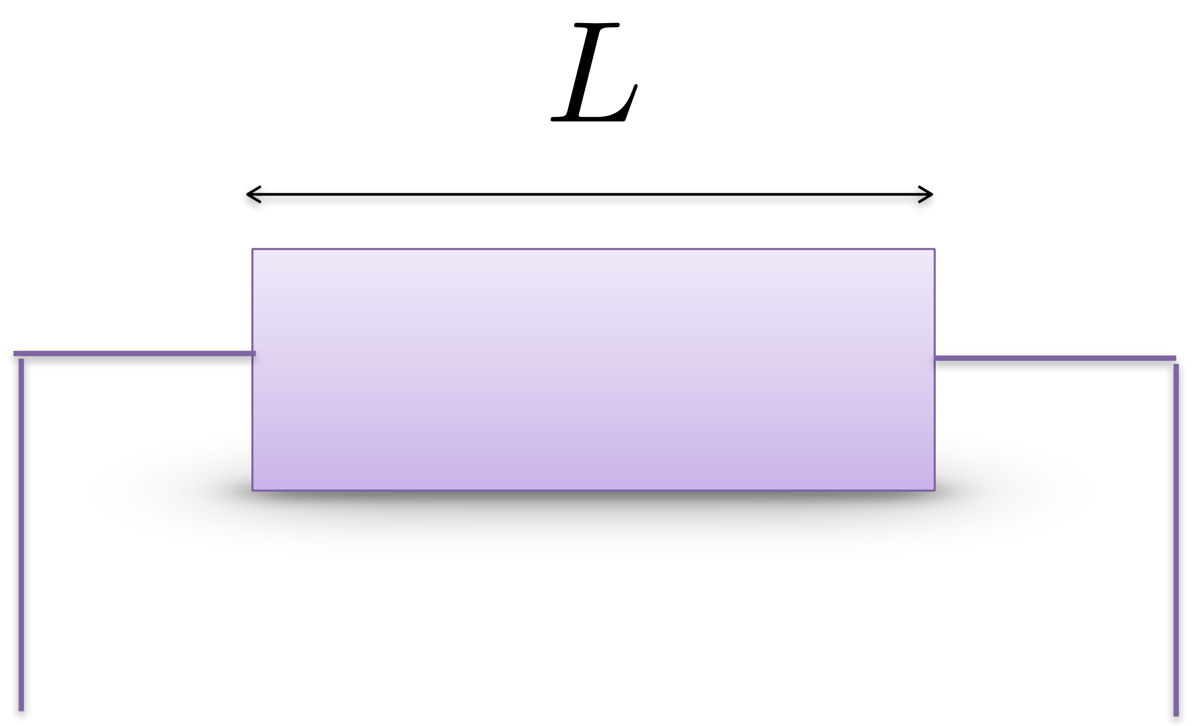


Transport of electrons

Relevant length scales

l_e elastic scattering length

l_ϕ inelastic scattering length

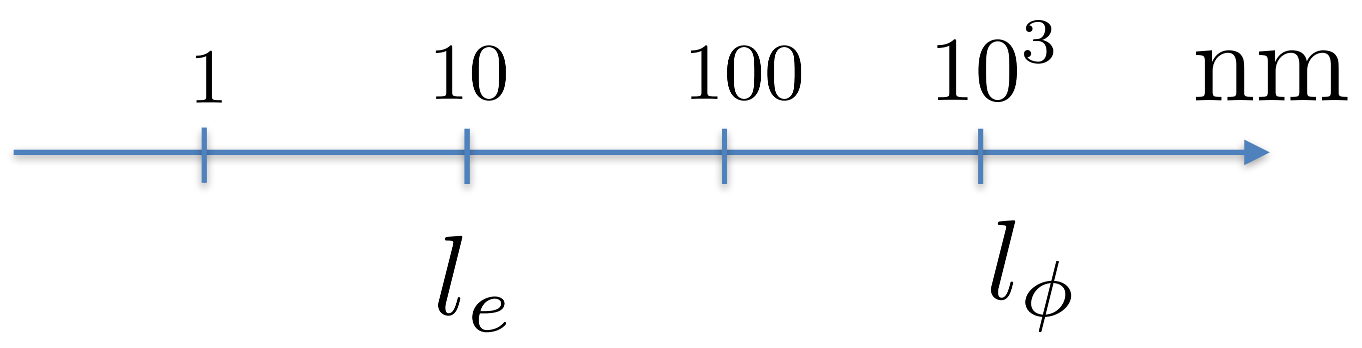


Coherent transport is defined by: $L \lesssim l_\phi$

$$l_e \ll L \lesssim l_\phi$$

Diffusive transport

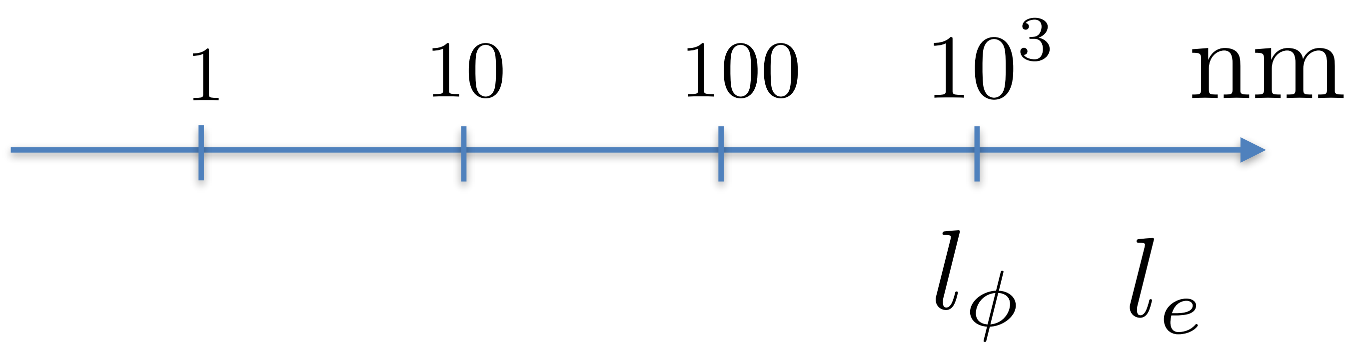
Metal at 1 K



$$L \lesssim l_e \text{ and } l_\phi$$

Ballistic transport

2DEG at 1 K



Conductivity in the semi-classical approach

First order electronic coherence at time t

$$\mathcal{G}_t^{(e)}(\mathbf{r},\mathbf{r}') = \mathrm{Tr}\left(\psi(\mathbf{r})\,\rho(t)\,\psi^\dagger(\mathbf{r}')\right)$$

Contains information on single particle quantities:

$$\langle \mathbf{j}(t,r)\rangle = \int \langle \mathbf{r}_+|\hat{j}(r)|\mathbf{r}_-\rangle\,\mathcal{G}_t^{(e)}(\mathbf{r}_+,\mathbf{r}_-)\,\mathrm{d}^d\mathbf{r}_+\,\mathrm{d}^d\mathbf{r}_-$$

Conductivity in the semi-classical approach

First order electronic coherence at time *t*

$$\mathcal{G}_t^{(e)}(\mathbf{r},\mathbf{r}') = \mathrm{Tr}\left(\psi(\mathbf{r})\,\rho(t)\,\psi^\dagger(\mathbf{r}')\right)$$

Contains information on single particle quantities:

$$\langle \mathbf{j}(t,r)\rangle = \int \langle \mathbf{r}_+|\hat{\jmath}(r)|\mathbf{r}_-\rangle\,\mathcal{G}_t^{(e)}(\mathbf{r}_+,\mathbf{r}_-)\,\mathrm{d}^d\mathbf{r}_+\,\mathrm{d}^d\mathbf{r}_-$$

Problem: evolution of $\mathcal{G}_t^{(e)}(\mathbf{r},\mathbf{r}')$?

Conductivity in the semi-classical approach

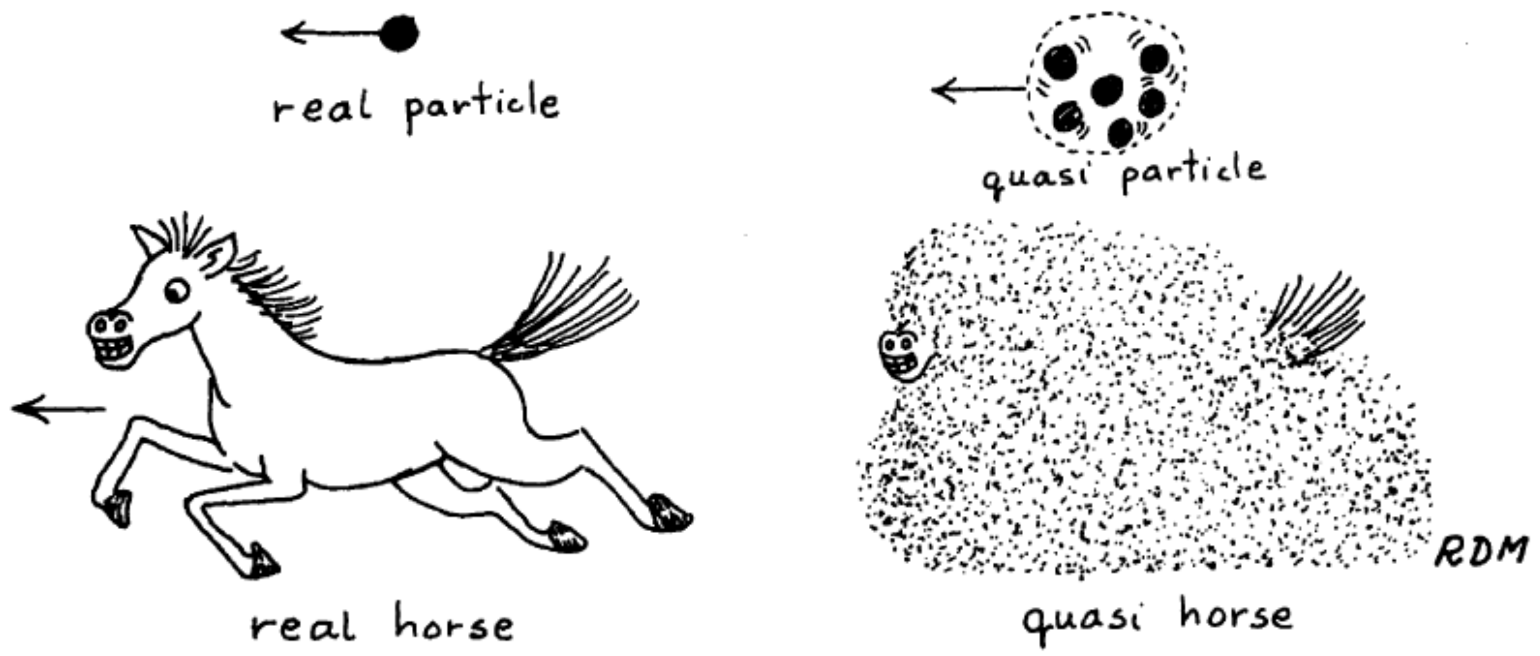
First order electronic coherence at time t

$$\mathcal{G}_t^{(e)}(\mathbf{r},\mathbf{r}') = \mathrm{Tr}\left(\psi(\mathbf{r})\,\rho(t)\,\psi^\dagger(\mathbf{r}')\right)$$

Contains information on single particle quantities:

$$\langle \mathbf{j}(t,r)\rangle = \int \langle \mathbf{r}_+|\hat{j}(r)|\mathbf{r}_-\rangle\, \mathcal{G}_t^{(e)}(\mathbf{r}_+,\mathbf{r}_-)\,\mathrm{d}^d\mathbf{r}_+\,\mathrm{d}^d\mathbf{r}_-$$

Problem: evolution of $\mathcal{G}_t^{(e)}(\mathbf{r},\mathbf{r}')$?



Conductivity in the semi-classical approach

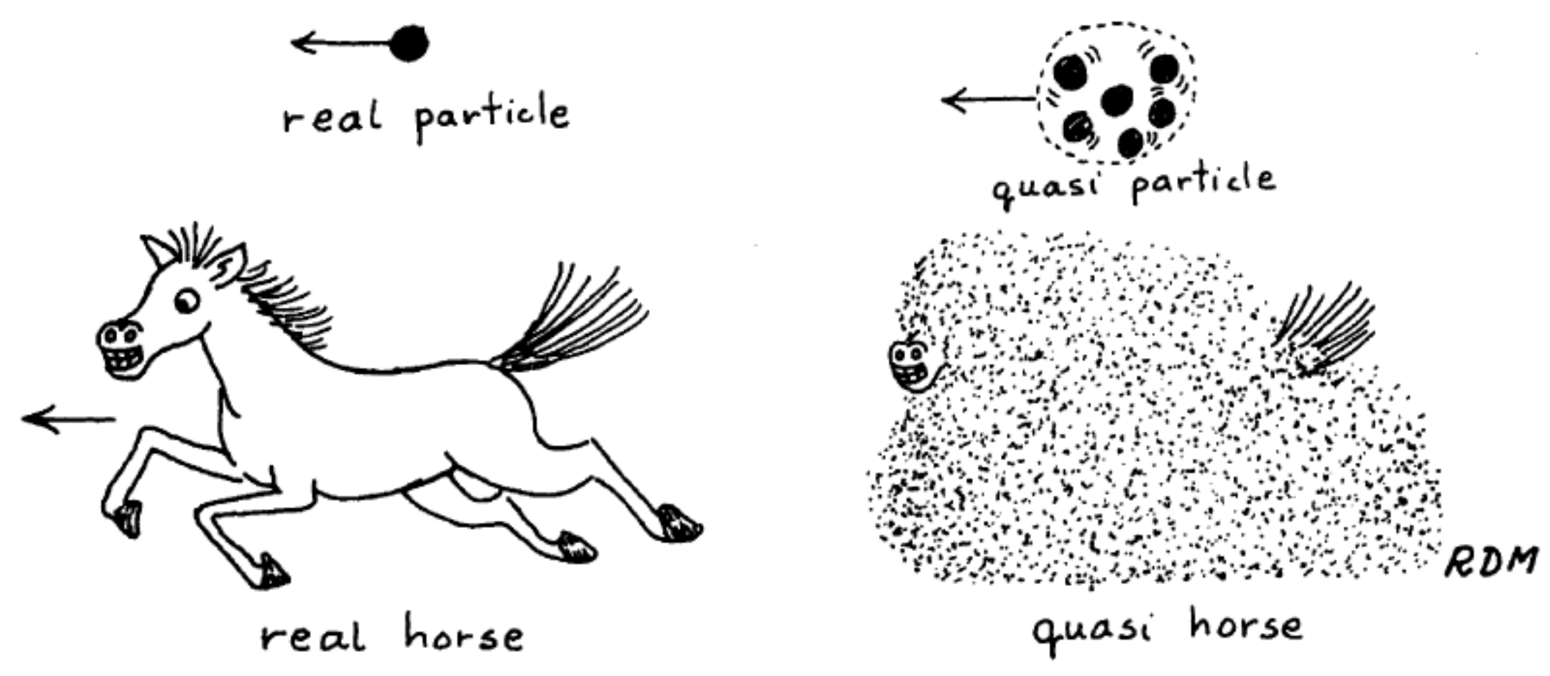
First order electronic coherence at time t

$$\mathcal{G}_t^{(e)}(\mathbf{r},\mathbf{r}') = \mathrm{Tr}\left(\psi(\mathbf{r})\,\rho(t)\,\psi^\dagger(\mathbf{r}')\right)$$

Contains information on single particle quantities:

$$\langle \mathbf{j}(t,r)\rangle = \int \langle \mathbf{r}_+|\hat{\jmath}(r)|\mathbf{r}_-\rangle\,\mathcal{G}_t^{(e)}(\mathbf{r}_+,\mathbf{r}_-)\,\mathrm{d}^d\mathbf{r}_+\,\mathrm{d}^d\mathbf{r}_-$$

Problem: evolution of $\mathcal{G}_t^{(e)}(\mathbf{r},\mathbf{r}')$?



Some 1D chiral systems with interactions → See G. Fève's talk (16/04/2025)

Conductivity in the semi-classical approach

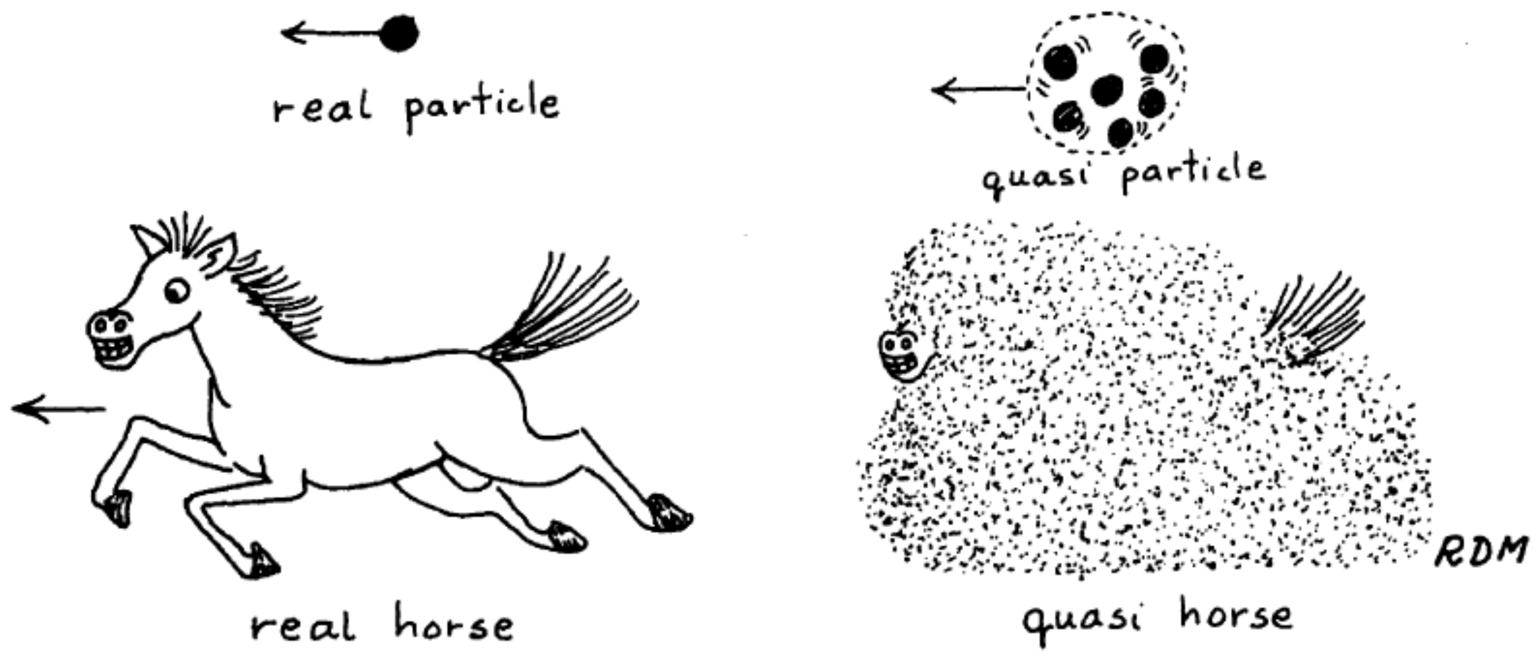
First order electronic coherence at time *t*

$$\mathcal{G}_t^{(e)}(\mathbf{r},\mathbf{r}') = \mathrm{Tr}\left(\psi(\mathbf{r})\,\rho(t)\,\psi^\dagger(\mathbf{r}')\right)$$

Contains information on single particle quantities:

$$\langle \mathbf{j}(t,r)\rangle = \int \langle \mathbf{r}_+|\hat{j}(r)|\mathbf{r}_-\rangle\,\mathcal{G}_t^{(e)}(\mathbf{r}_+,\mathbf{r}_-)\,\mathrm{d}^d\mathbf{r}_+\,\mathrm{d}^d\mathbf{r}_-$$

Problem: evolution of $\mathcal{G}_t^{(e)}(\mathbf{r},\mathbf{r}')$?



Some 1D chiral systems with interactions → See G. Fève's talk (16/04/2025)

Normal metals with not too strong disorder and interactions

Semi-classical propagation of electrons

Disordered but not too much

$k_F l_e \gg 1$

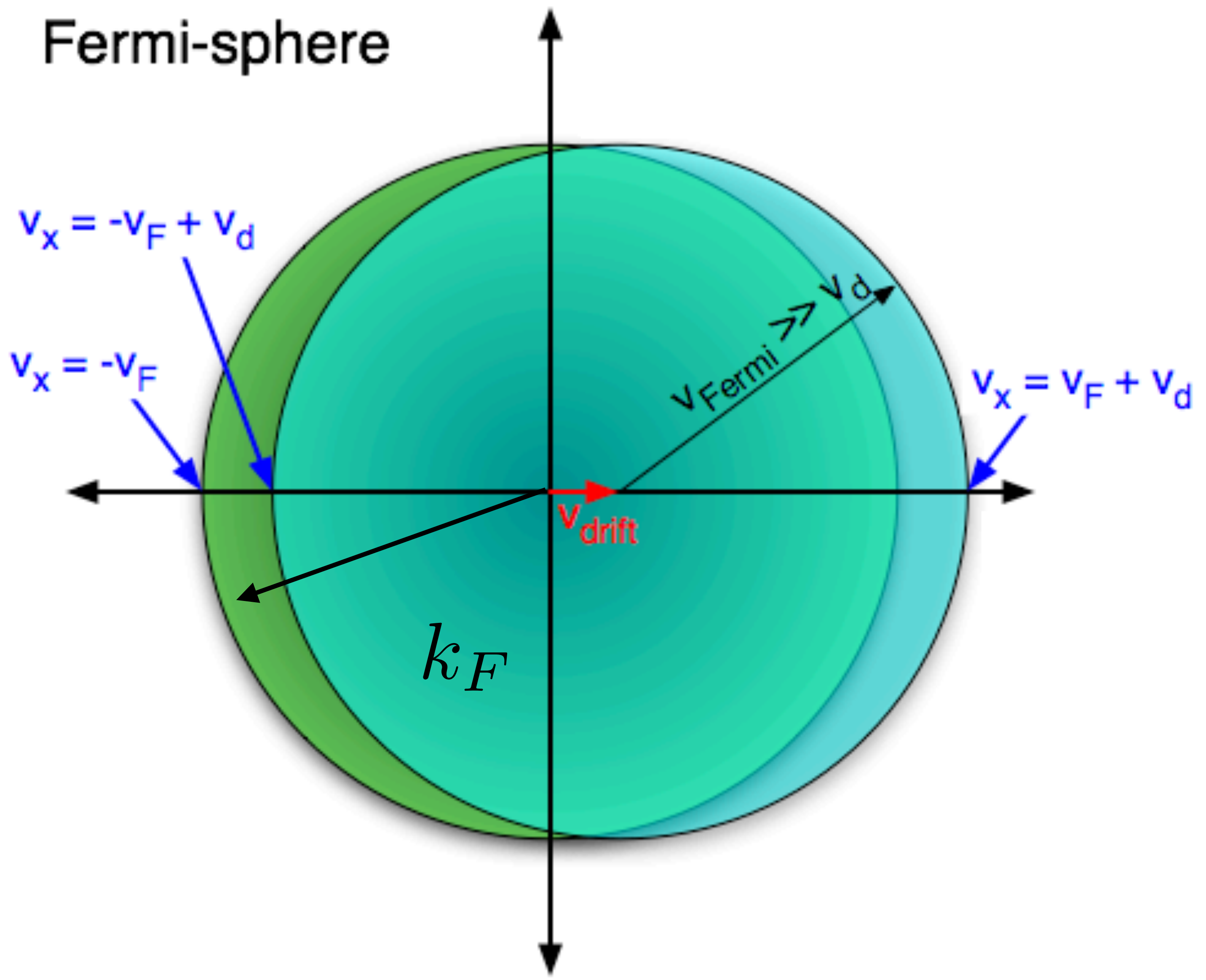
Classical motion with multiple collisions

$$D = \frac{v_F l_e}{d} = \frac{v_F^2 \tau}{d} \qquad l_e = v_F \tau \qquad \tau \simeq 10^{-14}\ \mathrm{s} \quad (300\ \mathrm{K})$$

Drude model

$$E(\mathbf{k}) = \frac{(\hbar \mathbf{k})^2}{2m_*}$$

$$v_F = \frac{\hbar k_F}{m_*}$$



$$\mathbf{j} = \frac{ne^2\tau}{m_*}\mathbf{E}$$

$$\sigma = \frac{ne^2\tau}{m_*} = \frac{ne^2 D}{dv_F^2}$$

$$v_{\mathrm{drift}} = \mu E$$

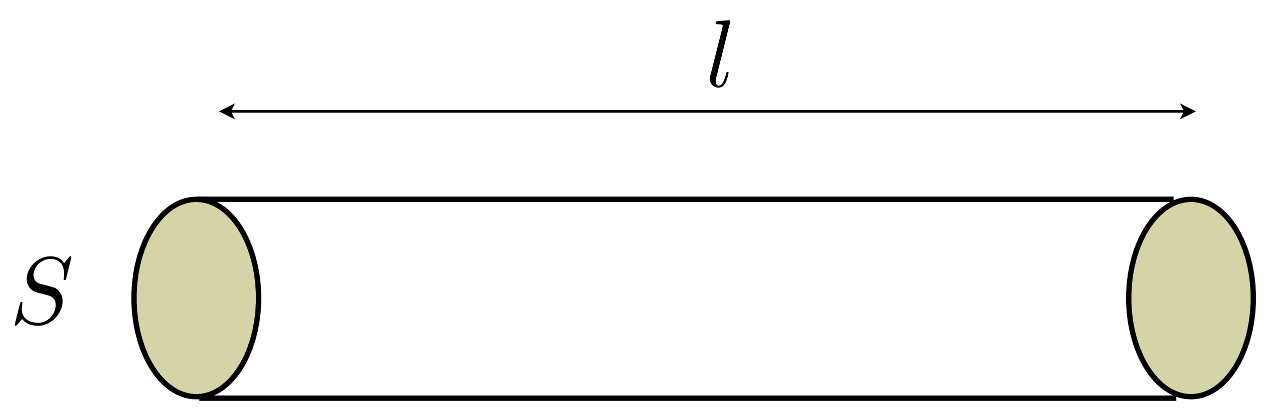
$$\mu \simeq 10 - 50\ \mathrm{cm}^2\,\mathrm{s}^{-1}\,\mathrm{V}^{-1} \quad (300\ \mathrm{K})$$

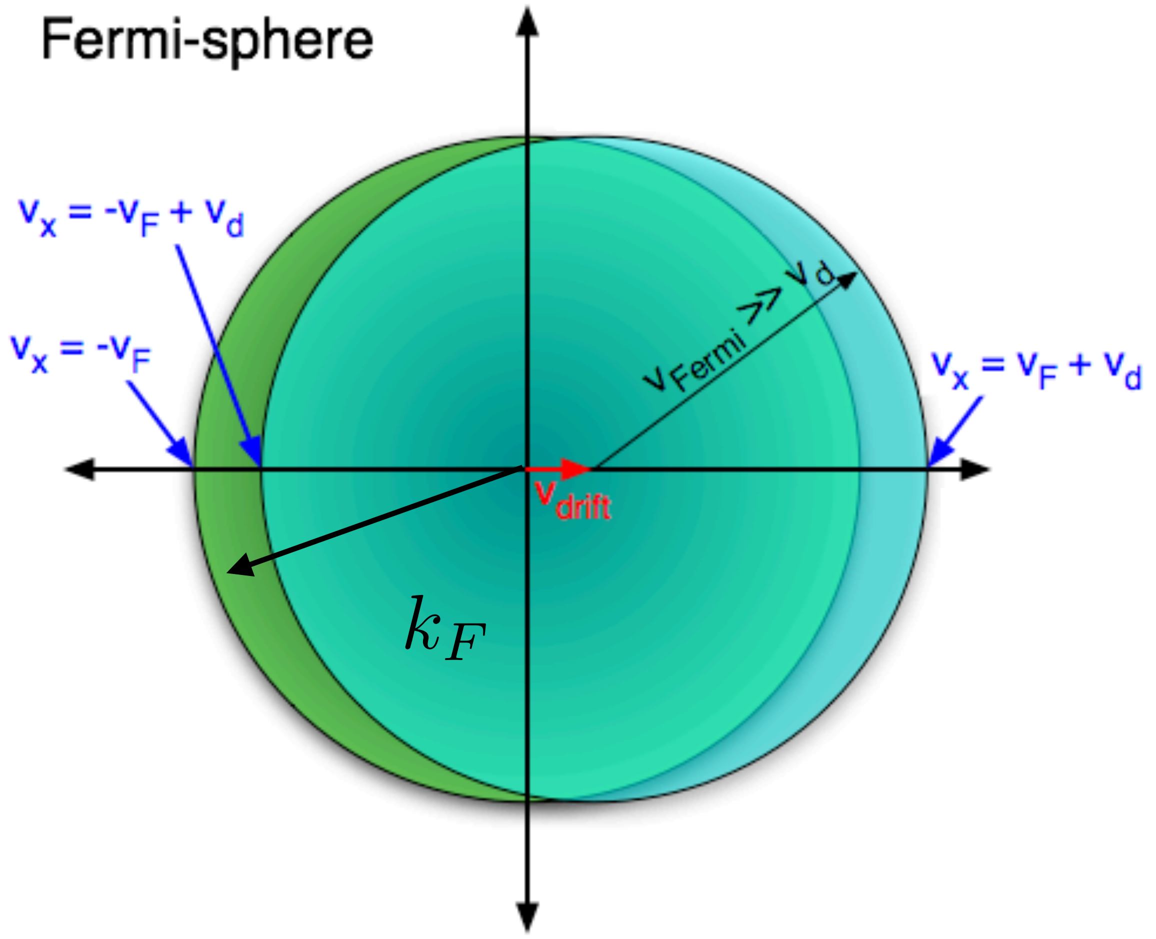
Cu: $E_F = 7.05\mathrm{eV} \simeq 82 \times 10^4\ \mathrm{K}$

$v_F \simeq 1.57 \times 10^6 \mathrm{m\,s}^{-1}$

G. Montambeaux, Ecole du GDR Physique Quantique Mésoscopique (2012)
G. M et E. Akkermans, *Physique mésoscopique des électrons et des photons*, Savoirs Actuels

Energy flow and stock





Magnetic energy

$$E_{\mathrm{mag}} = \frac{L_{\mathrm{m}} I^2}{2}$$

Relaxation times of RL circuit

$$\frac{L_{\mathrm{m}}}{R} = \frac{Z_0}{R}\,\frac{l}{c}\,\mathcal{F}_{\mathrm{geom}}$$

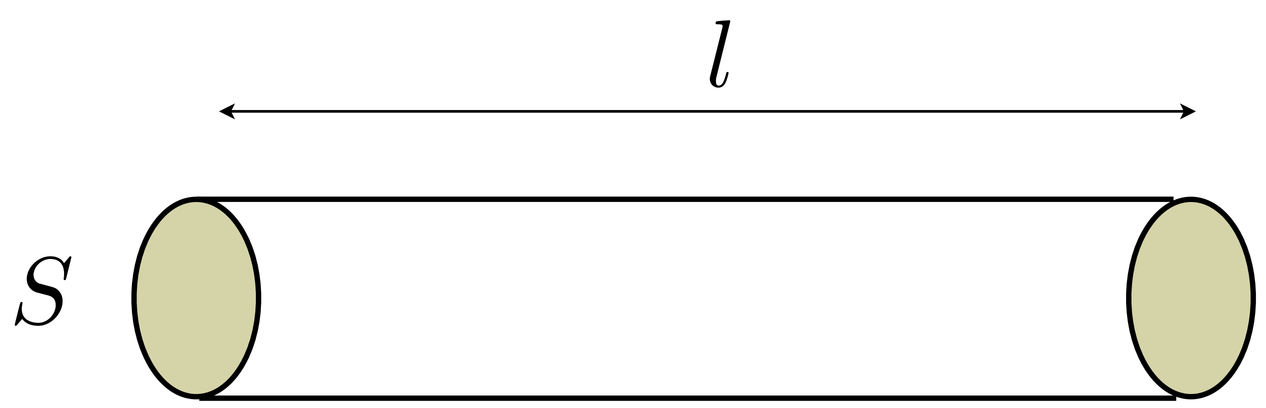
$$Z_0 = \frac{1}{\varepsilon_0 c} \simeq 377\ \Omega$$

Kinetic energy of electrons

$$E_{\mathrm{cin}} = \frac{L_K I^2}{2}$$

$$\frac{L_K}{R} = \frac{\tau}{2}$$

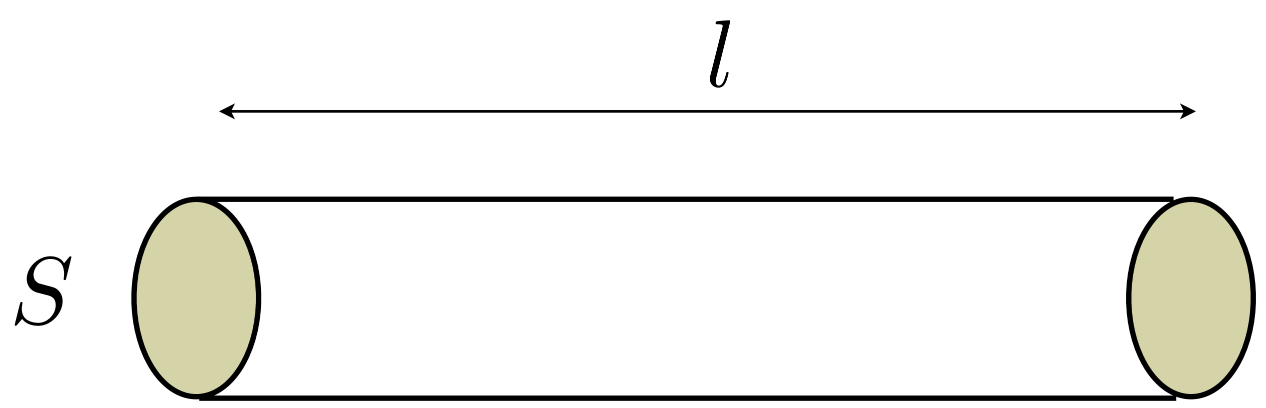
Energy stock and flow



Energy stocks

$$\frac{E_{\mathbf{B}}}{E_{\text{kin}}} \sim \frac{Z_0}{R}\,\frac{l}{c\tau}$$

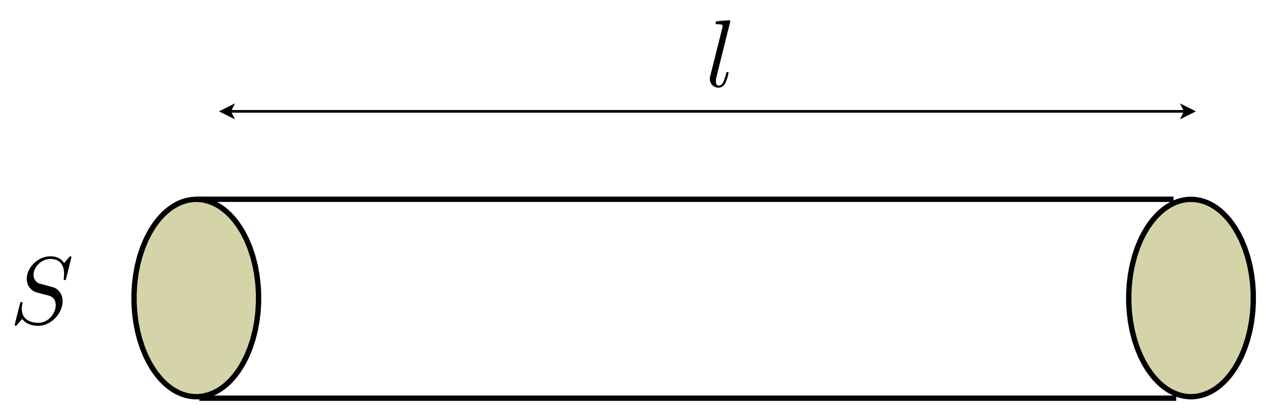
Energy stock and flow



Energy stocks

$$\frac{E_{\mathbf{B}}}{E_{\text{kin}}} \sim \frac{Z_0}{R}\frac{l}{c\tau} \qquad \frac{E_{\mathbf{E}}}{E_{\text{kin}}} \sim \frac{R}{Z_0}\frac{l}{c\tau}$$

Energy stock and flow



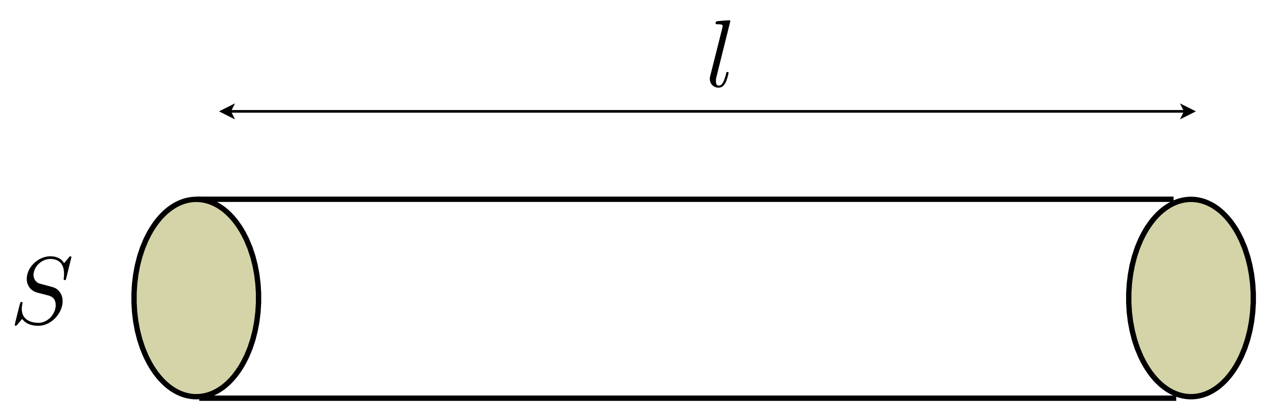
Energy stocks

$$\frac{E_{\mathbf{B}}}{E_{\text{kin}}} \sim \frac{Z_0}{R}\,\frac{l}{c\tau} \qquad \frac{E_{\mathbf{E}}}{E_{\text{kin}}} \sim \frac{R}{Z_0}\,\frac{l}{c\tau}$$

$c\tau \simeq 3\ \mu\text{m}$

$$l \gg c\tau \implies E_{\mathbf{B}},\, E_{\mathbf{E}} \gg E_{\text{kin}}$$

Energy stock and flow



Energy stocks

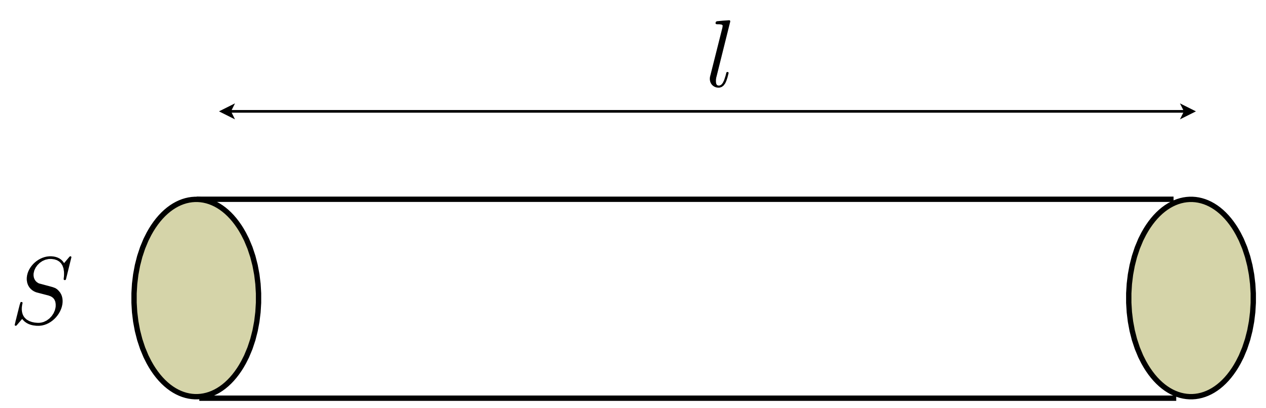
$c\tau \simeq 3\ \mu\mathrm{m}$

$$\frac{E_{\mathbf{B}}}{E_{\mathrm{kin}}} \sim \frac{Z_0}{R}\,\frac{l}{c\tau} \qquad \frac{E_{\mathbf{E}}}{E_{\mathrm{kin}}} \sim \frac{R}{Z_0}\,\frac{l}{c\tau}$$

$$l \gg c\tau \implies E_{\mathbf{B}},\, E_{\mathbf{E}} \gg E_{\mathrm{kin}}$$

$$\frac{E_{\mathbf{B}}}{E_{\mathbf{E}}} \sim \frac{6}{\pi}\left(\frac{Z_0}{R}\right)^2$$

Energy stock and flow



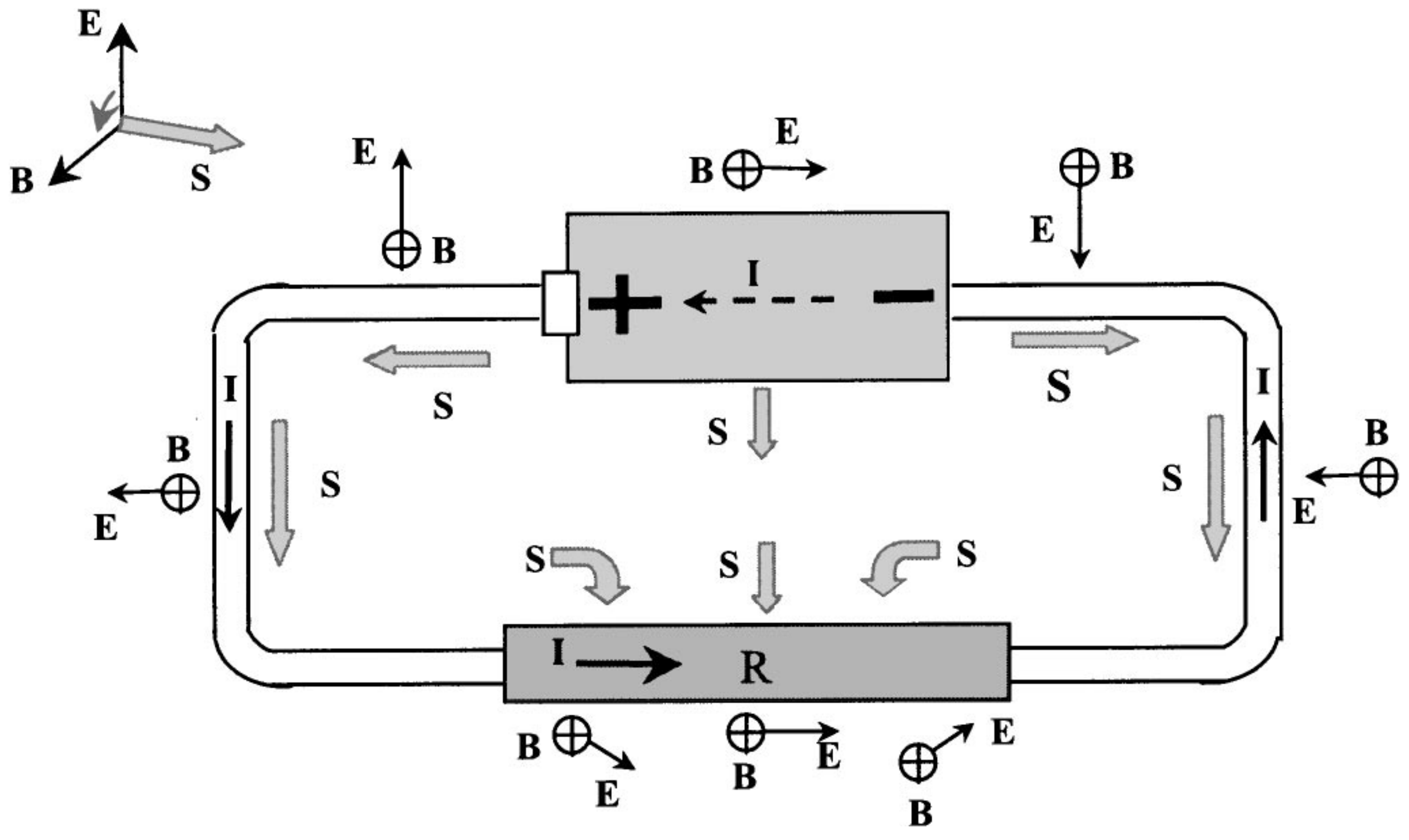
Energy stocks

$$\frac{E_{\mathbf{B}}}{E_{\text{kin}}} \sim \frac{Z_0}{R}\,\frac{l}{c\tau} \qquad \frac{E_{\mathbf{E}}}{E_{\text{kin}}} \sim \frac{R}{Z_0}\,\frac{l}{c\tau}$$

$c\tau \simeq 3\ \mu\text{m}$

$$l \gg c\tau \implies E_{\mathbf{B}},\, E_{\mathbf{E}} \gg E_{\text{kin}}$$

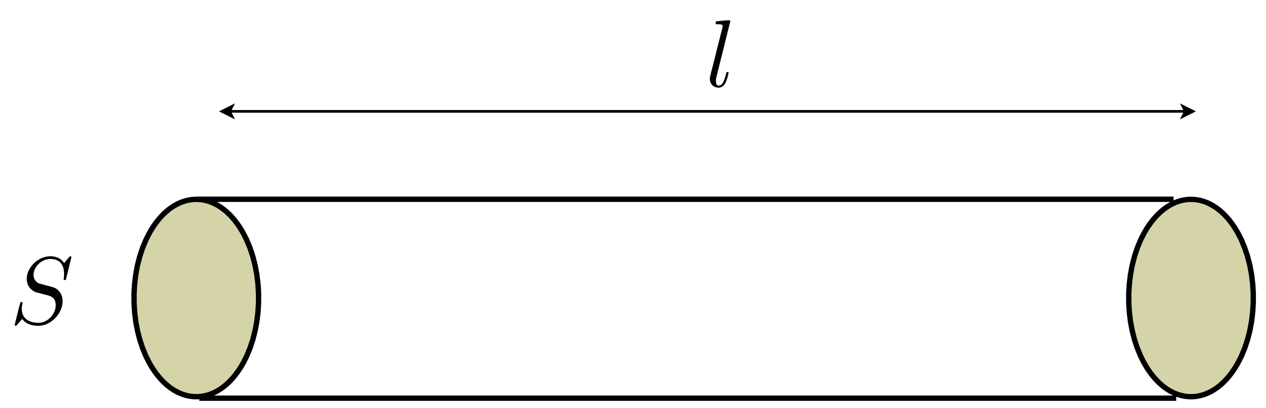
$$\frac{E_{\mathbf{B}}}{E_{\mathbf{E}}} \sim \frac{6}{\pi}\left(\frac{Z_0}{R}\right)^2$$



Energy flux

$$\Phi_{\mathbf{S}} = RI^2 \qquad \Phi_{\text{kin}} \sim \frac{L_K I^2}{l}\, v_{\text{drift}}$$

Energy stock and flow



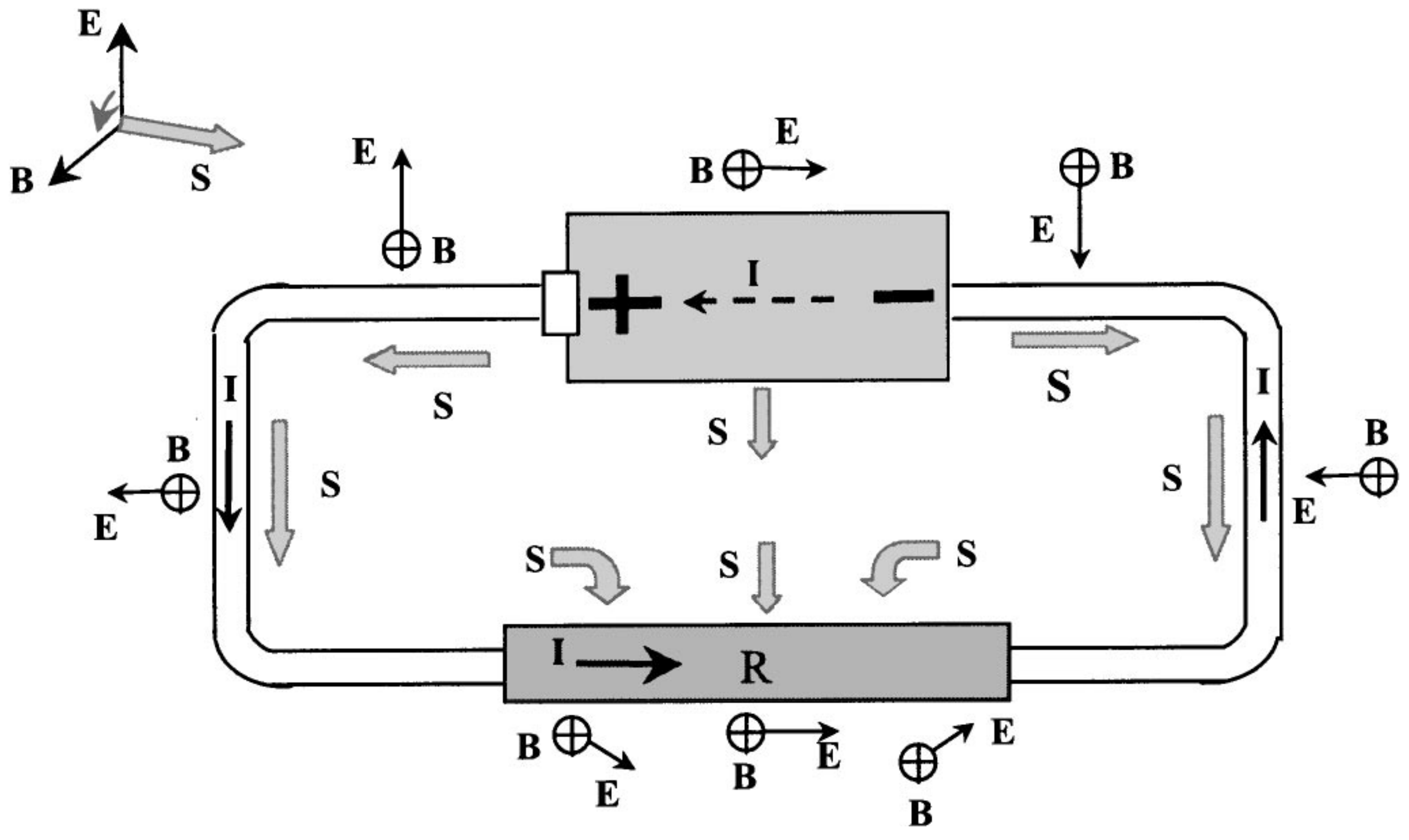
Energy stocks

$$\frac{E_{\mathbf{B}}}{E_{\mathrm{kin}}} \sim \frac{Z_0}{R}\,\frac{l}{c\tau} \qquad \frac{E_{\mathbf{E}}}{E_{\mathrm{kin}}} \sim \frac{R}{Z_0}\,\frac{l}{c\tau}$$

$c\tau \simeq 3\ \mu\mathrm{m}$

$$l \gg c\tau \implies E_{\mathbf{B}},\, E_{\mathbf{E}} \gg E_{\mathrm{kin}}$$

$$\frac{E_{\mathbf{B}}}{E_{\mathbf{E}}} \sim \frac{6}{\pi}\left(\frac{Z_0}{R}\right)^2$$

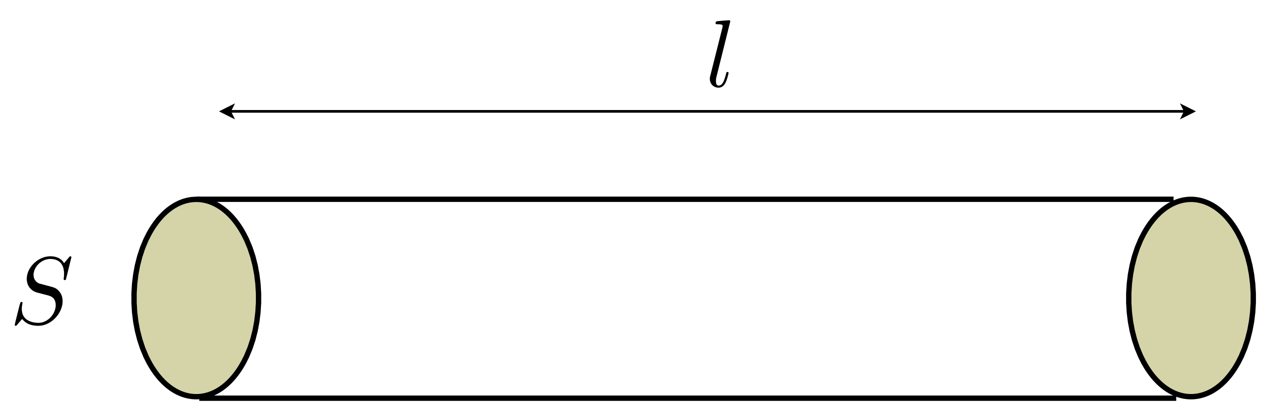


Energy flux

$$\Phi_{\mathbf{S}} = RI^2 \qquad \Phi_{\mathrm{kin}} \sim \frac{L_K I^2}{l}\, v_{\mathrm{drift}}$$

$$\frac{\Phi_{\mathbf{S}}}{\Phi_{\mathrm{kin}}} = \frac{R}{L_K}\,\frac{l}{v_{\mathrm{drift}}} \sim \frac{l}{l_e}\,\frac{v_F}{v_{\mathrm{drift}}}$$

Energy stock and flow



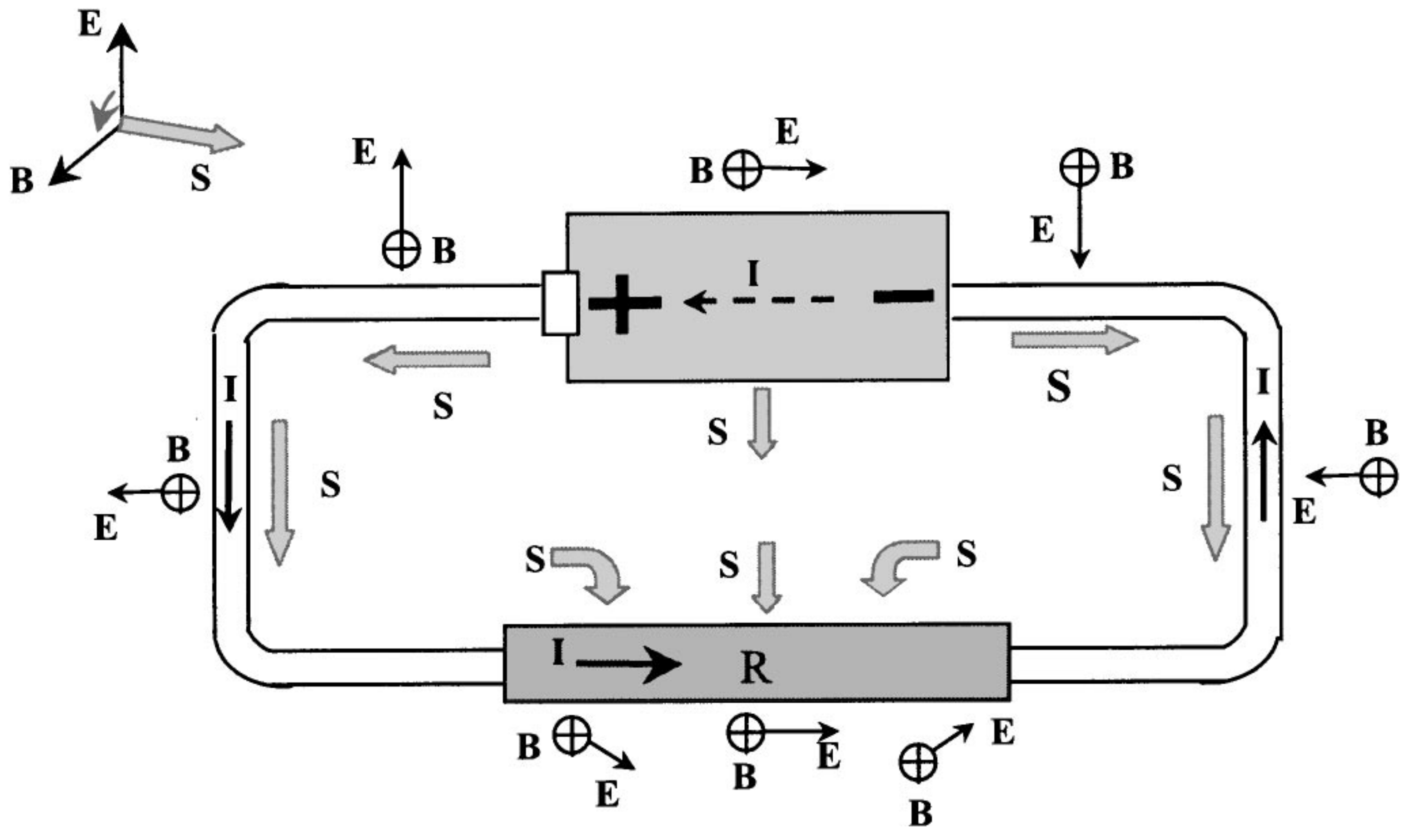
Energy stocks

$$\frac{E_{\mathbf{B}}}{E_{\text{kin}}} \sim \frac{Z_0}{R}\frac{l}{c\tau} \qquad \frac{E_{\mathbf{E}}}{E_{\text{kin}}} \sim \frac{R}{Z_0}\frac{l}{c\tau}$$

$c\tau \simeq 3\ \mu\text{m}$

$$l \gg c\tau \implies E_{\mathbf{B}},\, E_{\mathbf{E}} \gg E_{\text{kin}}$$

$$\frac{E_{\mathbf{B}}}{E_{\mathbf{E}}} \sim \frac{6}{\pi}\left(\frac{Z_0}{R}\right)^2$$



Energy flux

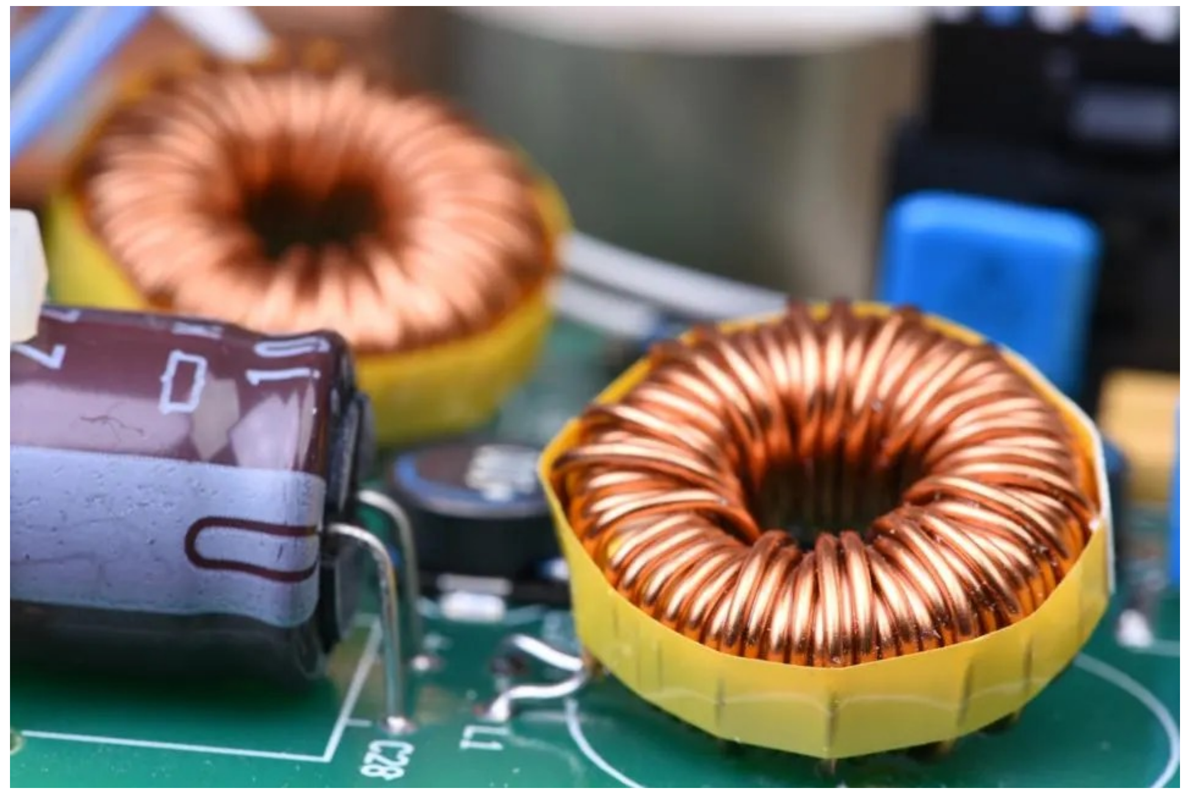
$$\Phi_{\mathbf{S}} = RI^2 \qquad \Phi_{\text{kin}} \sim \frac{L_K I^2}{l} v_{\text{drift}}$$

$$\frac{\Phi_{\mathbf{S}}}{\Phi_{\text{kin}}} = \frac{R}{L_K}\frac{l}{v_{\text{drift}}} \sim \frac{l}{l_e}\frac{v_F}{v_{\text{drift}}} \gg 1$$

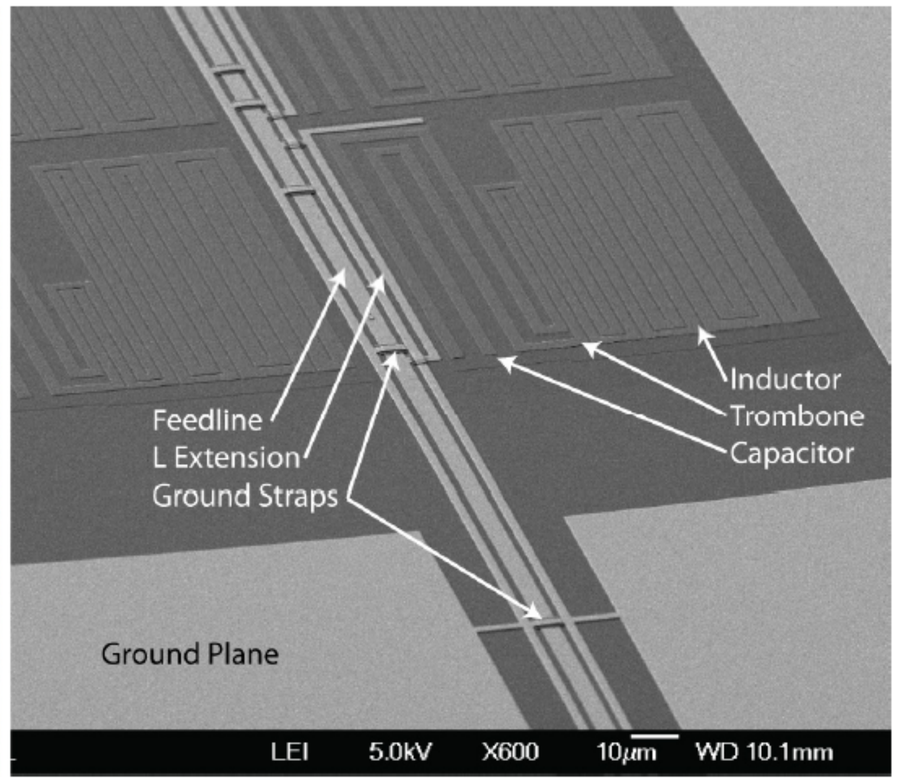
$l \gg l_e$ and $v_F \gg v_{\text{drift}}$

How electricity flows in « simpler » conductors ?

Usual conductors:
« big », 3D



Simpler conductors:
« small », low dimensional

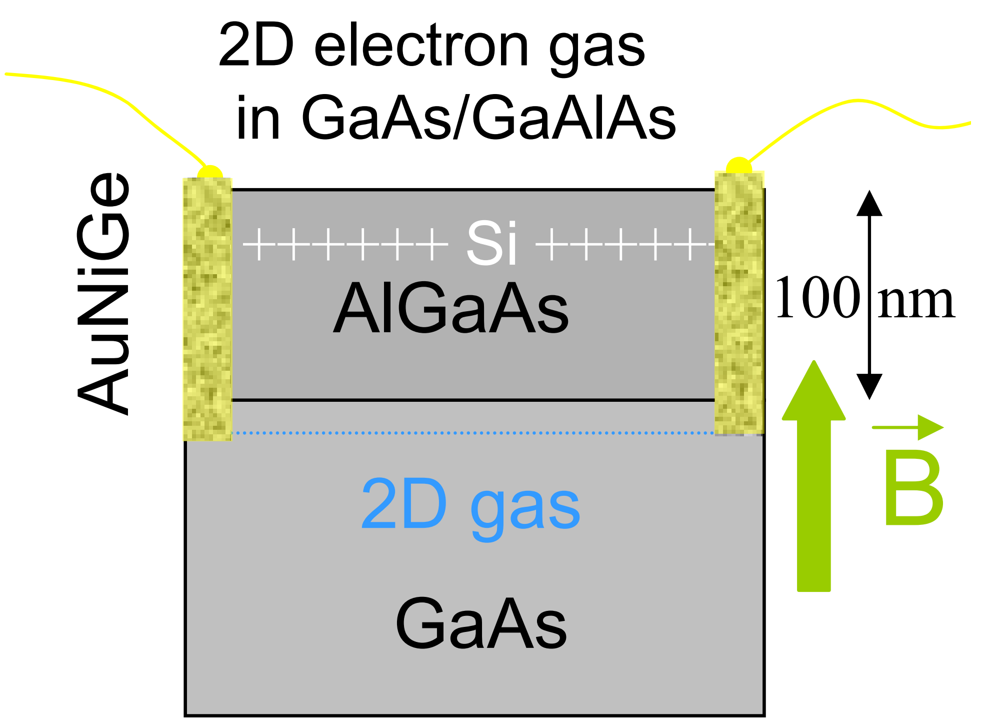


Role of quantum effects ?

Quantum Hall devices

Quantum Hall devices

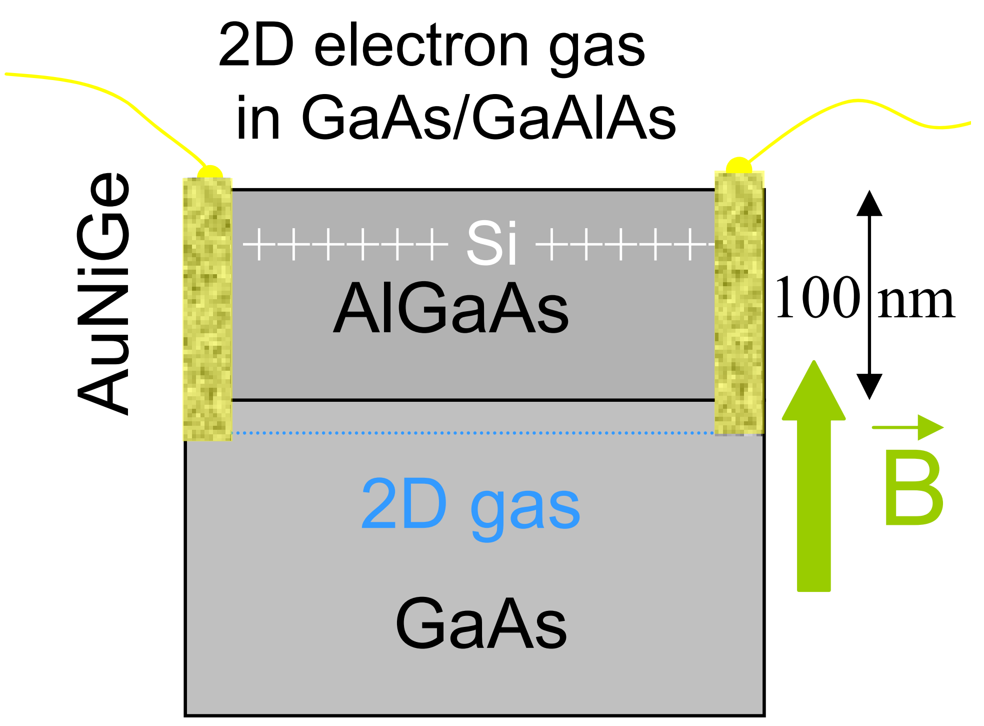
2DEG



$$n \simeq 10^{11}\ \mathrm{cm}^{-2}$$
$$\mu \simeq 10^{6}\ \mathrm{cm}^2/\mathrm{VS}$$

Quantum Hall devices

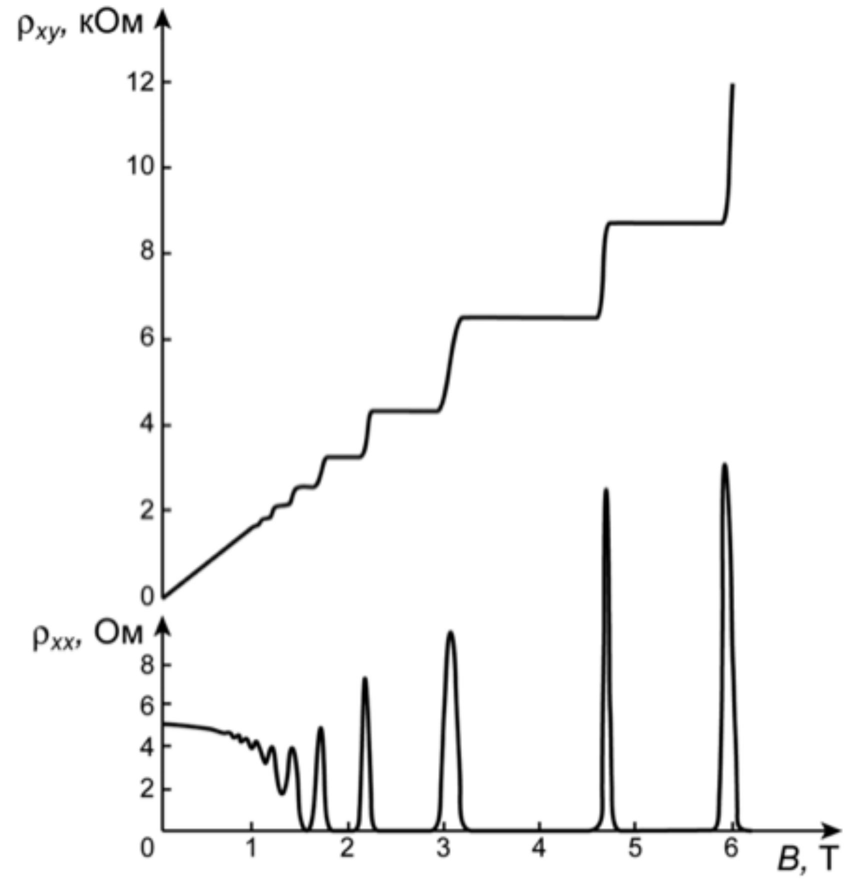
2DEG

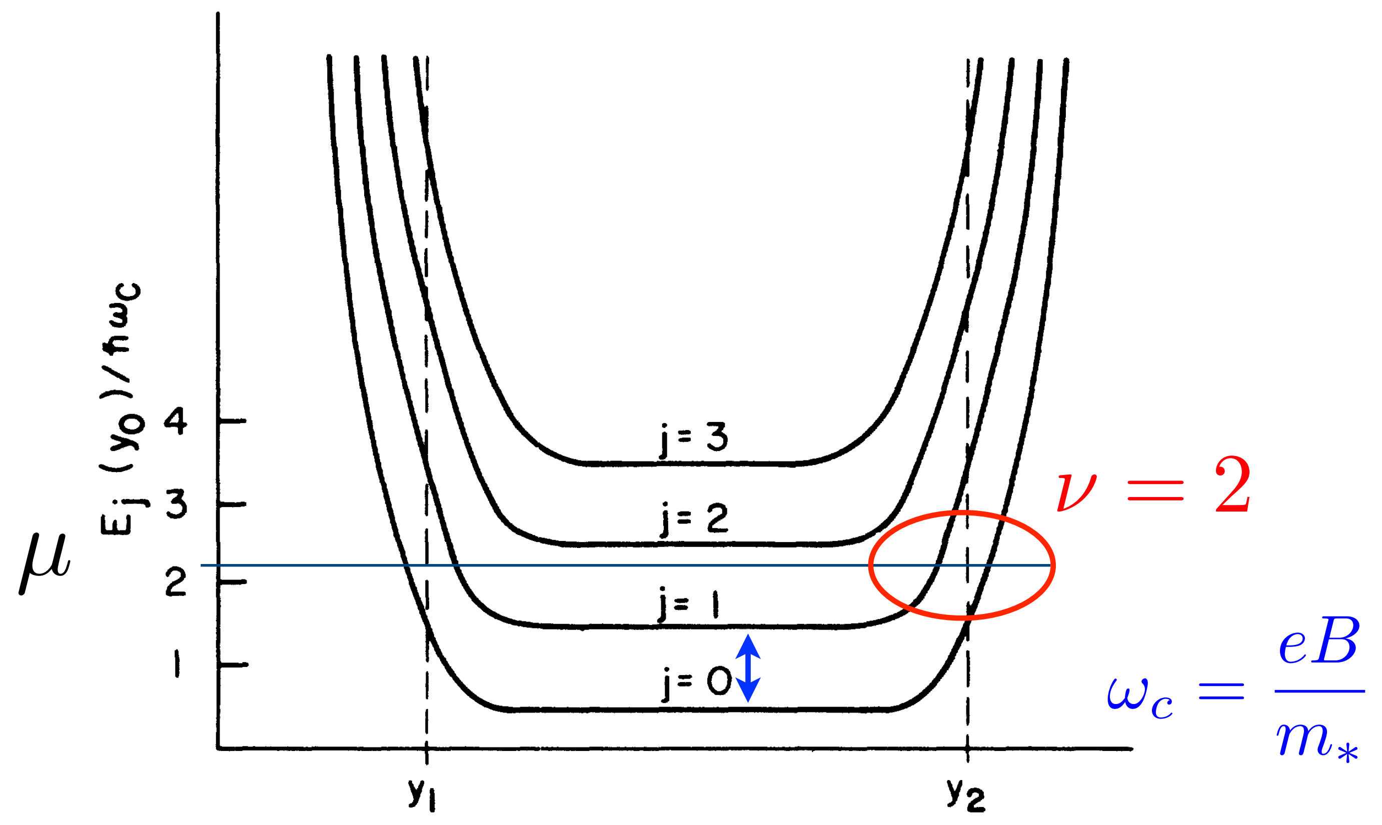


$n \simeq 10^{11}\ \mathrm{cm}^{-2}$

$\mu \simeq 10^{6}\ \mathrm{cm}^2/\mathrm{VS}$

Edge channels: 1D chiral wires





Insulating 2D bulk. Conducting edge channels!

Chiral relativistic fermions

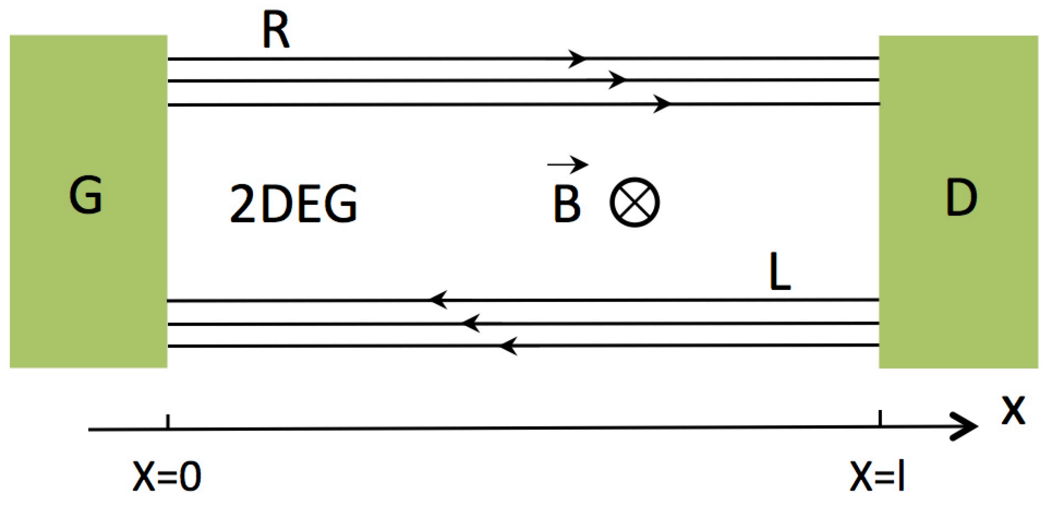
$$v_F \simeq 10^5 - 10^6\ \mathrm{m\,s}^{-1}$$

M. Büttiker, Phys. Rev. B. **88**, 9375 (1988)

Quantum Hall devices

Quantum Hall bar:
a very special wire

$\nu = 3$

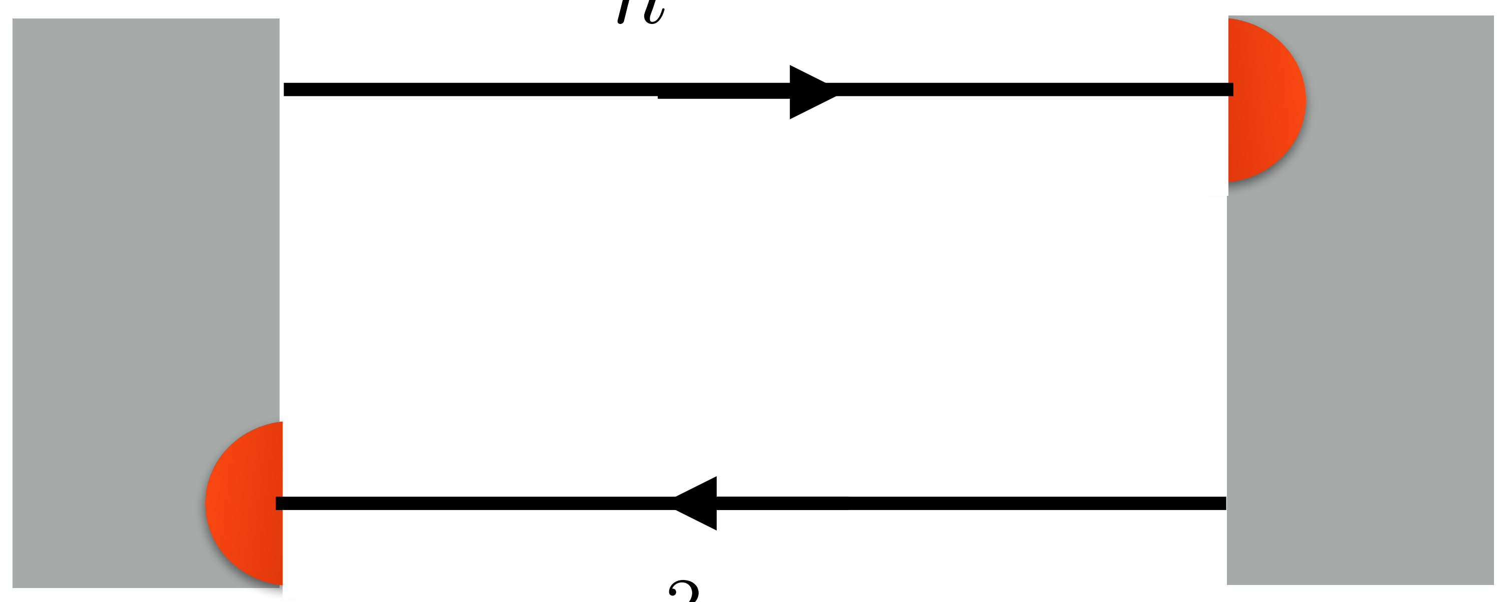


Two directions but
no backscattering!

$v_{\text{drift}} = v_F$

$$i_R = \frac{\nu e^2}{h} V_G = v_d \rho_R \qquad P_{\text{diss,D}} = \frac{e^2}{2h}(V_G - V_D)^2$$

V_G



V_D

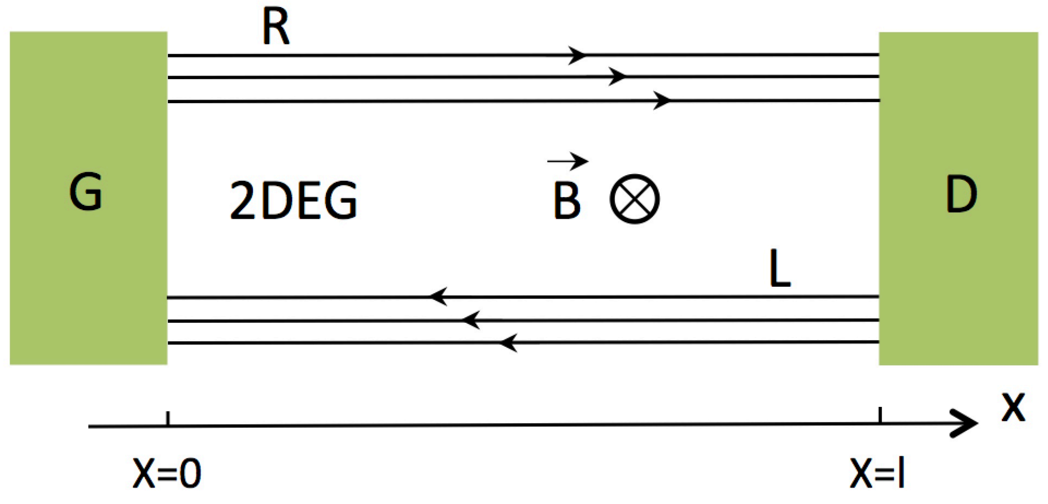
$$P_{\text{diss,D}} = P_{\text{diss,G}} \qquad i_L = \frac{\nu e^2}{h} V_D = v_d \rho_L$$

$$V_G - V_D = R_H(\nu)\, I \qquad P_{\text{diss}} = R_H(\nu)\, I^2 \qquad R_H(\nu) = \frac{R_K}{\nu}$$

Quantum Hall devices

Quantum Hall bar:
a very special wire

$\nu = 3$



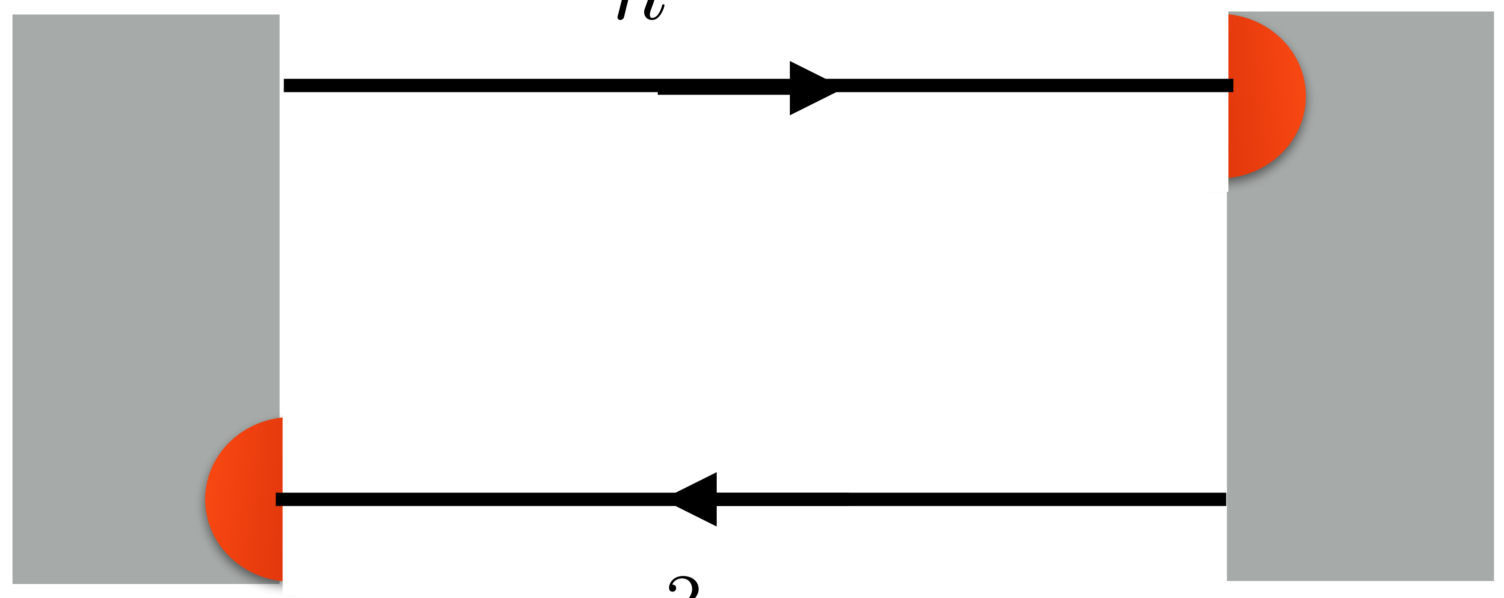
Two directions but
no backscattering!

$v_{\text{drift}} = v_F$

$$P_{\text{diss,D}} = \frac{e^2}{2h}(V_G - V_D)^2$$

$$i_R = \frac{\nu e^2}{h} V_G = v_d \rho_R$$

V_G



V_D

$$P_{\text{diss,D}} = P_{\text{diss,G}} \qquad i_L = \frac{\nu e^2}{h} V_D = v_d \rho_L$$

$$V_G - V_D = R_H(\nu)\, I \qquad P_{\text{diss}} = R_H(\nu)\, I^2 \qquad R_H(\nu) = \frac{R_K}{\nu}$$

Question: is it just a resistance ?

The quantum Hall bar: experiment

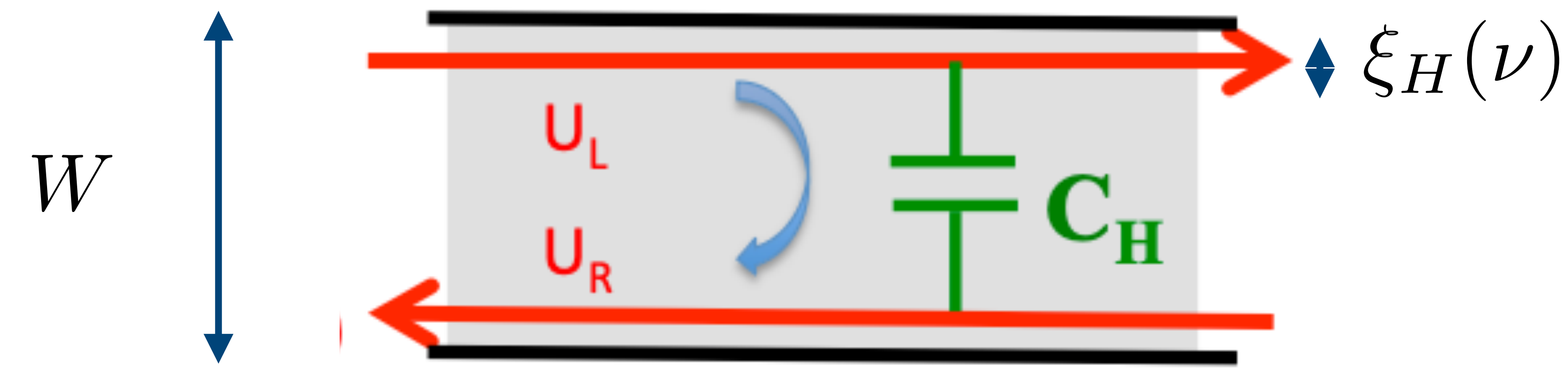
Impedance measurement

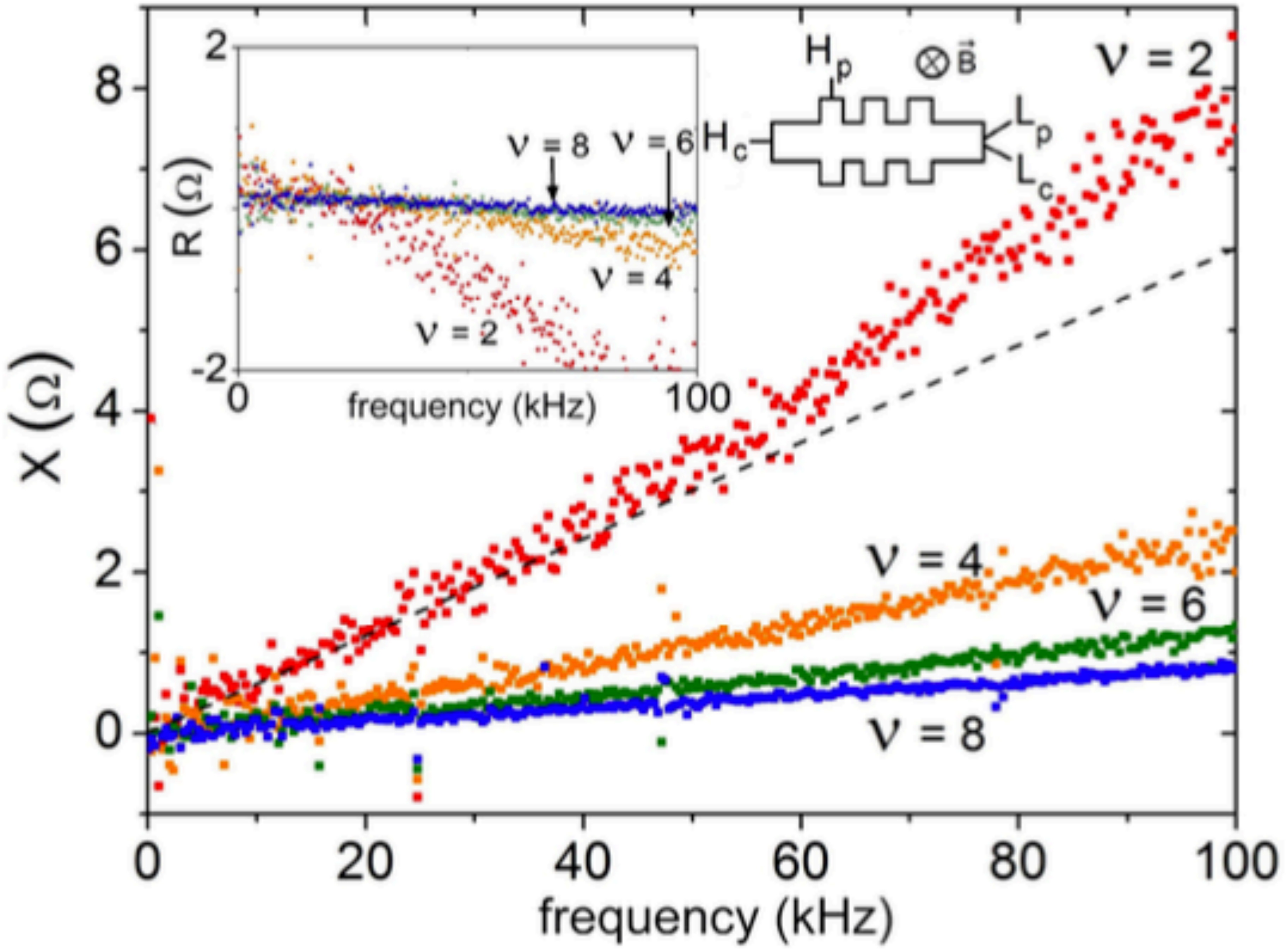
$$\frac{L}{R_H(\nu)} = \frac{l}{2\,v(\nu)}$$

$$v(\nu) = v_d(\nu) + \frac{\sigma_H(\nu)}{2\pi\varepsilon_0\varepsilon_r}\log\left(\frac{W}{\nu\xi_H(\nu)}\right)$$

bare drift velocity — Coulomb interaction correction

Charge density waves along edge channels





2×0.4 mm^2

$\nu = 2$ to 10

A. Delgard *et al*, Phys. Rev. B **104**, L121301 (2021)

Principle of microwaves circuits, McGraw Hill (1948)

M.K. Haldar *et al*, IEEE Access **10**, 79249 (2022)

I. Safi, Eur. Phys. J. D **12**, 451 (1999)

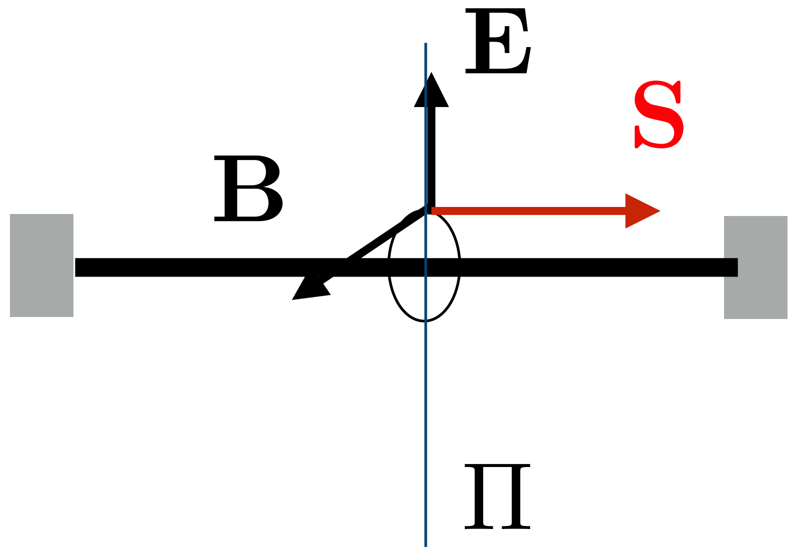
Energy stock and flow

	Normal wire	Quantum Hall bar (ν=1)
$E_{\mathbf{B}}/E_{\text{kin}}$	$\sim \frac{Z_0}{R}\frac{l}{c\tau}$	$\sim \frac{Z_0}{R_K}\frac{v_F}{c}$
$E_{\mathbf{E}}/E_{\text{kin}}$	$\sim \frac{R}{Z_0}\frac{l}{c\tau}$	$\sim \frac{R_K}{Z_0}\frac{v_F}{c}$
Energy flux ratios	$\frac{R I^2}{\Phi_{\text{kin}}(\Pi)} \sim \frac{l}{l_e}\frac{v_F}{v_{\text{drift}}}$	$\frac{\Phi_{\mathbf{S}}(\Pi)}{\Phi_{\text{kin}}(\Pi)} \sim \frac{\alpha_{\text{qed}}}{\pi\varepsilon_r}\frac{c}{v_F}$

$$\frac{Z_0}{R_K} = 2\,\alpha_{\text{qed}}$$

$$\alpha_{\text{qed}} = \frac{e^2}{4\pi\varepsilon_0\hbar c} \simeq \frac{1}{137}$$

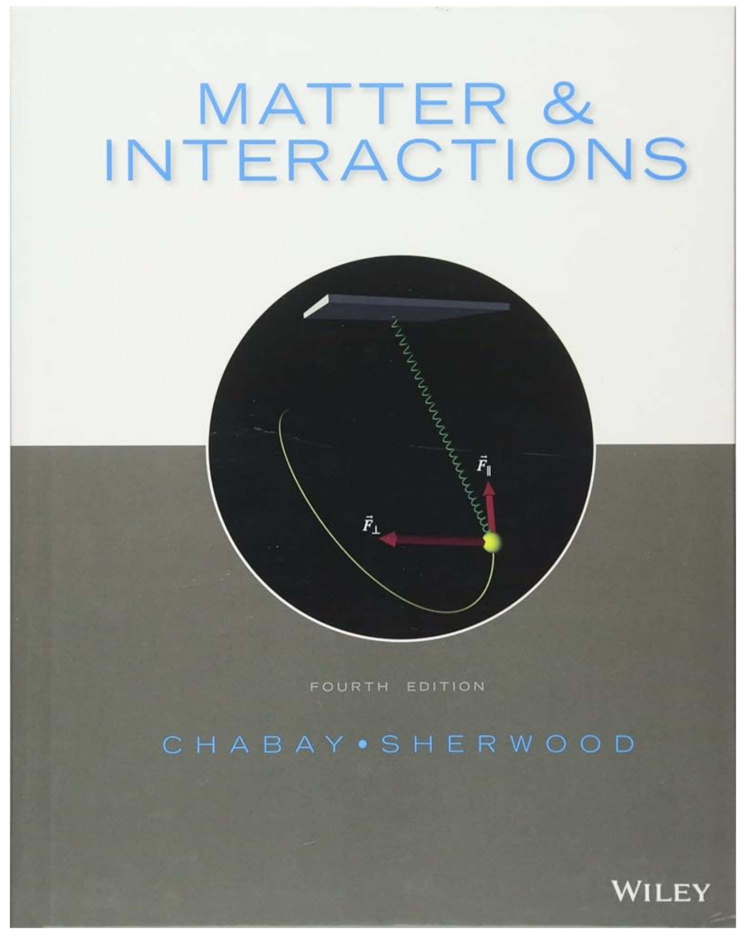
$$\alpha_{\text{eff}} = \frac{e^2}{4\pi\varepsilon_0\varepsilon_r v_F} = \frac{\alpha_{\text{qed}}}{\varepsilon_r}\frac{c}{v_F}$$



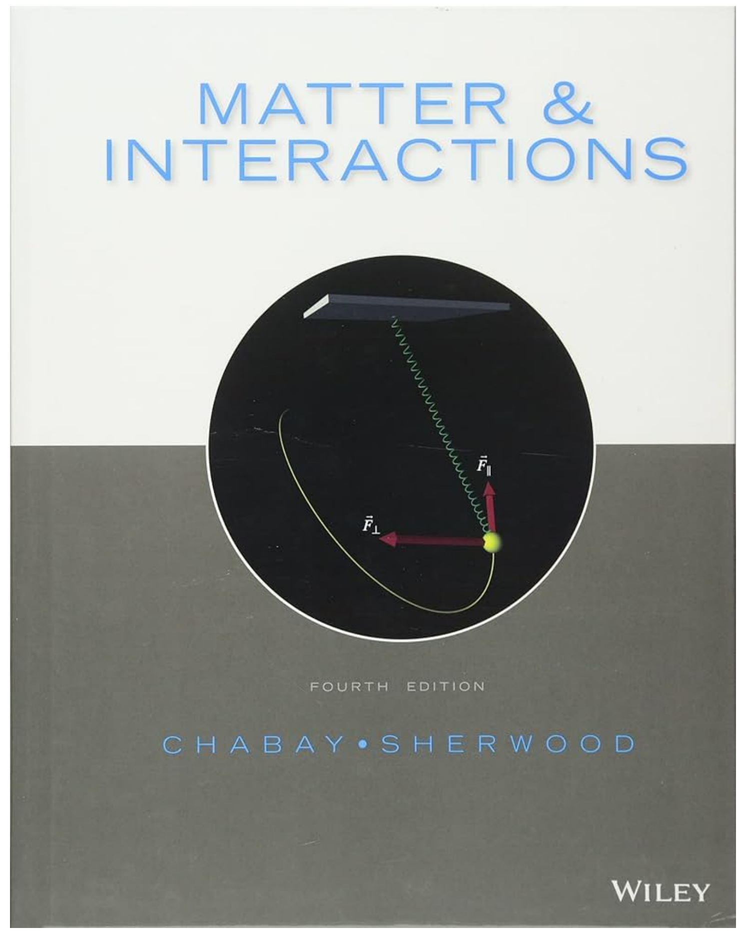
Key messages on simple, not fully quantum, circuits

Electrical circuits exhibit surface charges that:

- Maintain the potential around the circuit
- Generate the electrical field outside the conductors
- Ensure the confinement of the electrical current within the conductors



Key messages on simple, not fully quantum, circuits



Electrical circuits exhibit surface charges that:

- Maintain the potential around the circuit
- Generate the electrical field outside the conductors
- Ensure the confinement of the electrical current within the conductors

Energy is not necessarily carried by the EM field

Sometimes, dissipation takes place in the contacts !

Capacitance and inductances are renormalized because the electrical current is carried by Fermions !!!

Coulomb interaction effects can be strong !

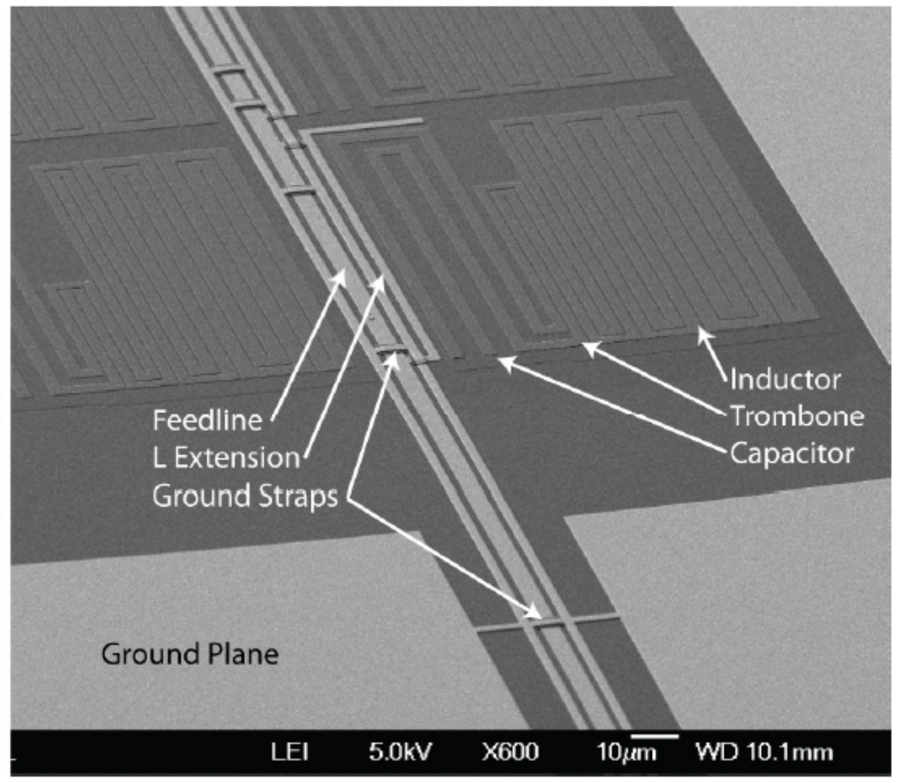
Outline

- From Ampère to electrical circuits
- Quantum physics within electricity
- Towards quantum coherent electronics
- Presentation of the PCP 2025 cycle

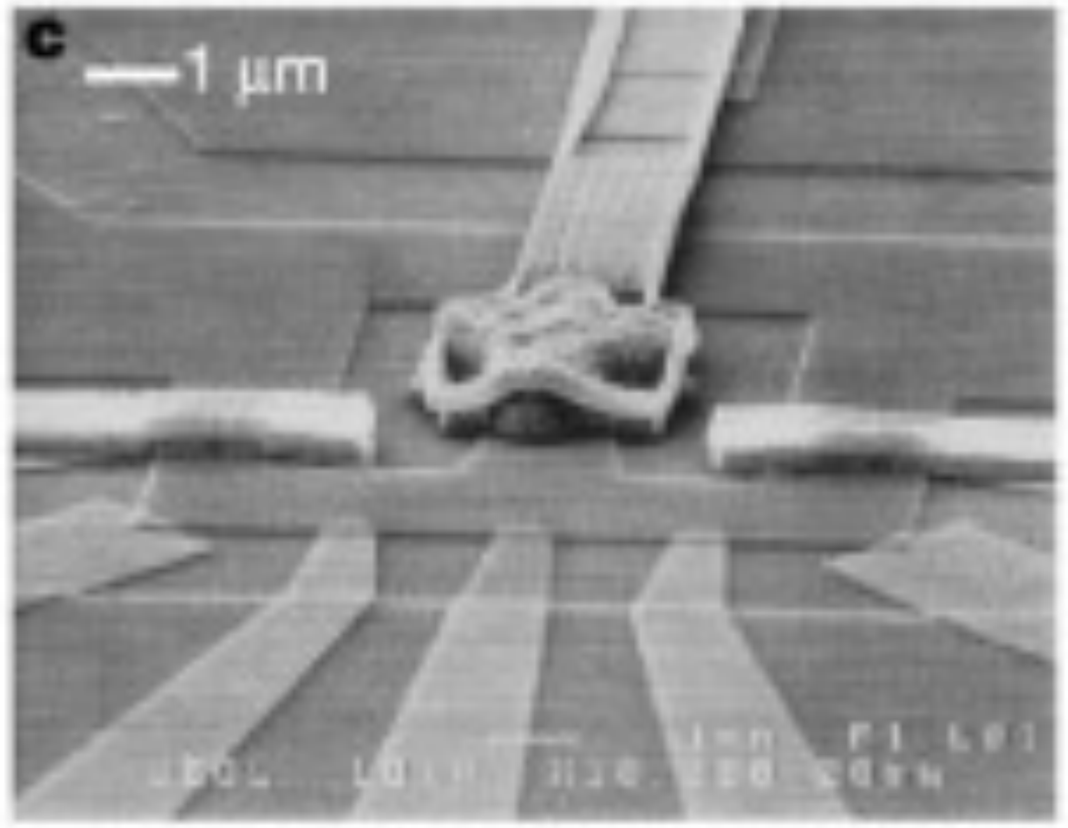
- Quantum transport
- AC transport and Coulomb interactions
- Quantum electrical currents and photons

How electricity flows in « quantum » conductors ?

Simpler conductors:
« small », but not yet quantum



Quantum conductors:
electronic interferences ?



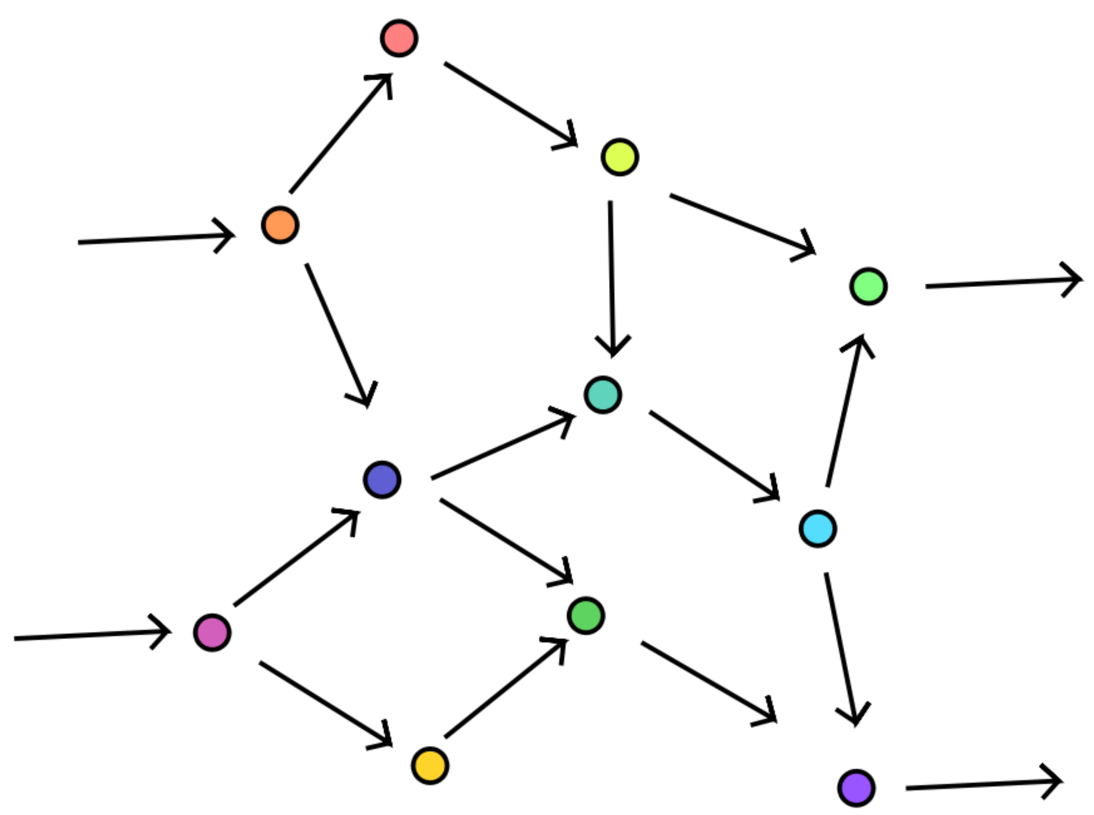
Electronic coherence ?

Quantum coherence for electrons in a conductor

Quantum coherence for electrons in a conductor

$$l_e \ll L \lesssim l_\phi$$

Diffusive transport

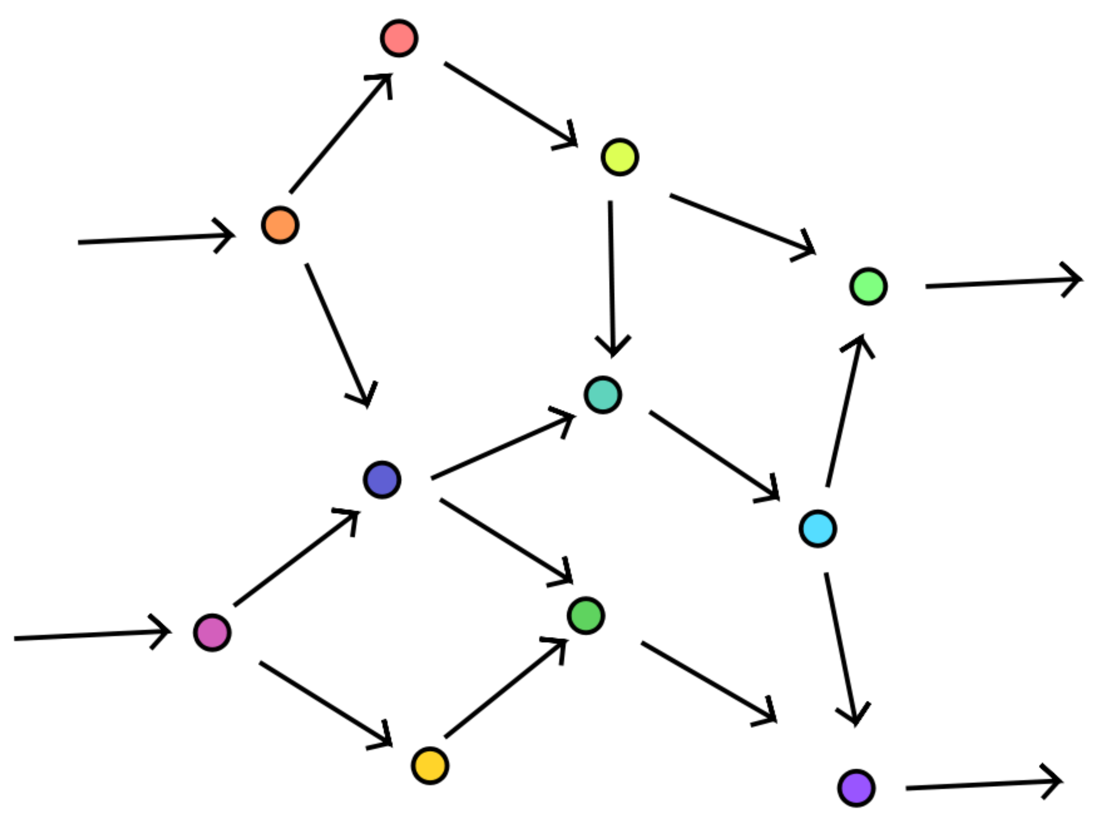


Deviations to classical diffusion:
weak localization, *etc*

Quantum coherence for electrons in a conductor

$$l_e \ll L \lesssim l_\phi$$

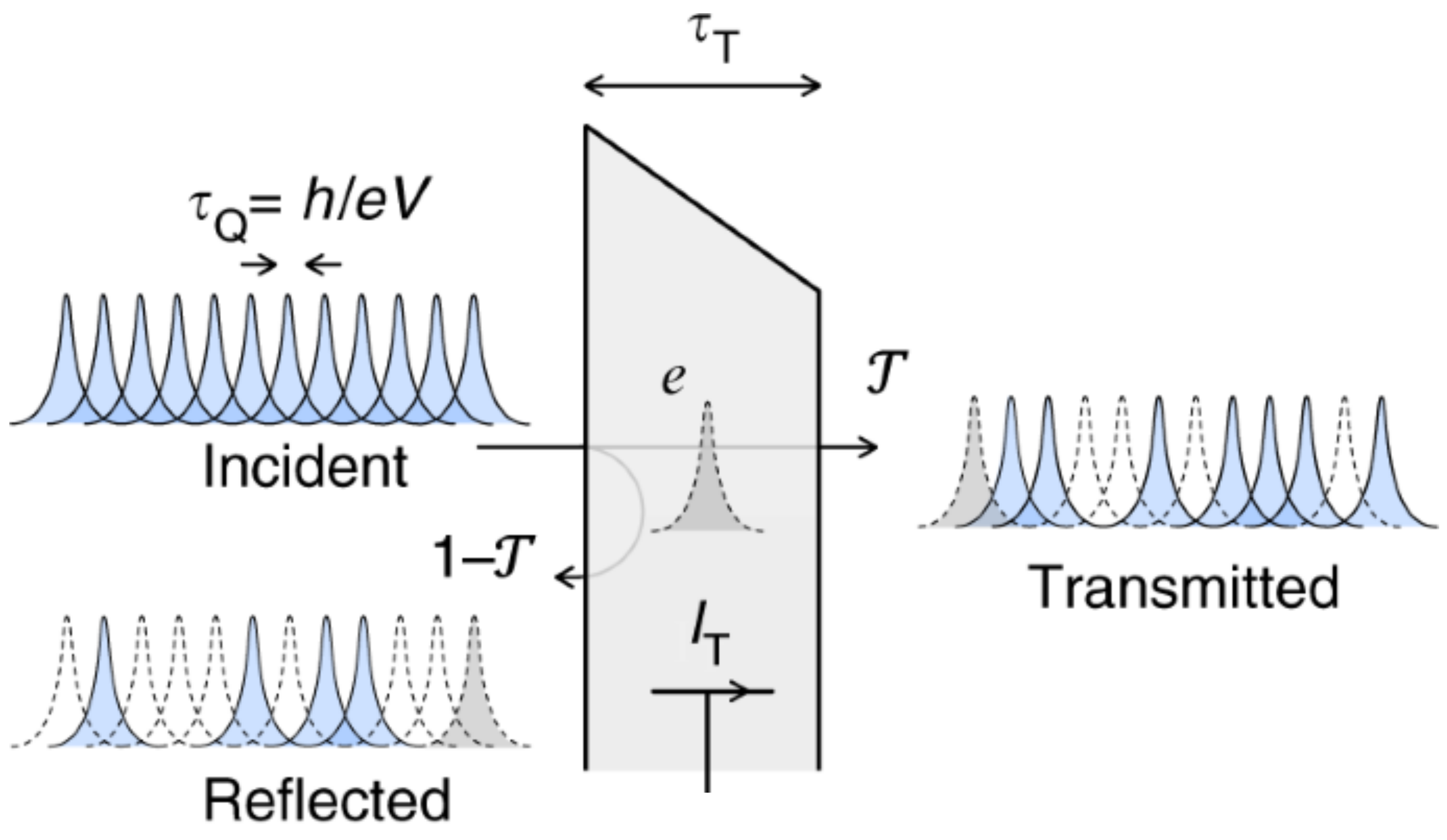
Diffusive transport



Deviations to classical diffusion:
weak localization, *etc*

$$L \lesssim l_e \text{ and } l_\phi$$

Ballistic transport

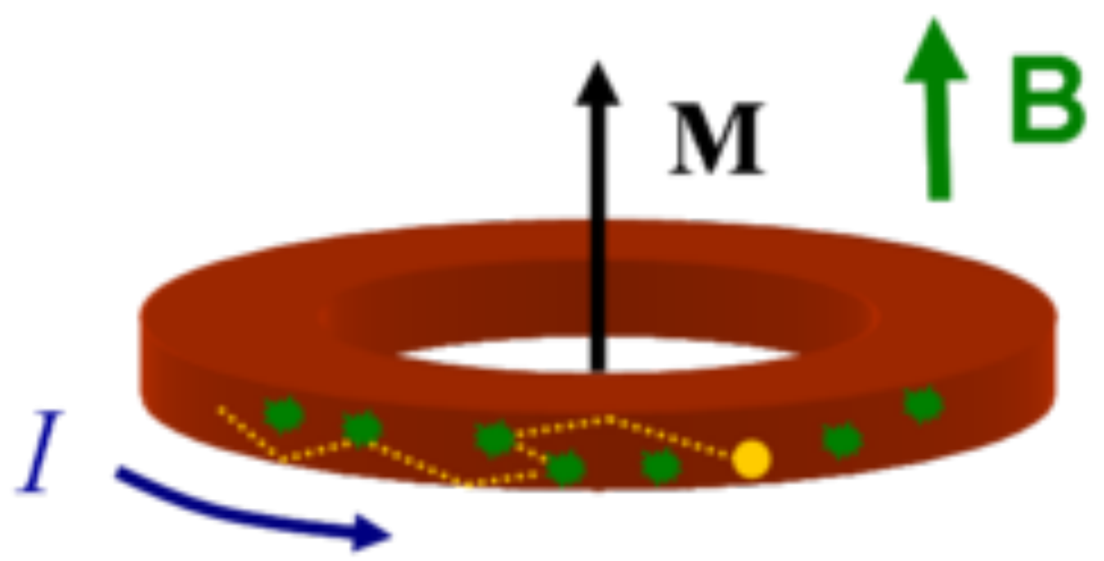


Conductors as electronic waveguides
and scatterers

Electronic coherence: permanent currents

Electronic coherence: permanent currents

$$\psi(x+L) = \mathrm{e}^{2\pi\mathrm{i}\frac{\Phi_{\mathbf{B}}}{h/e}}\psi(x)$$



M. Büttiker, Y. Imry and R. Landauer, Phys. Lett. A **96**, 365 (1983)

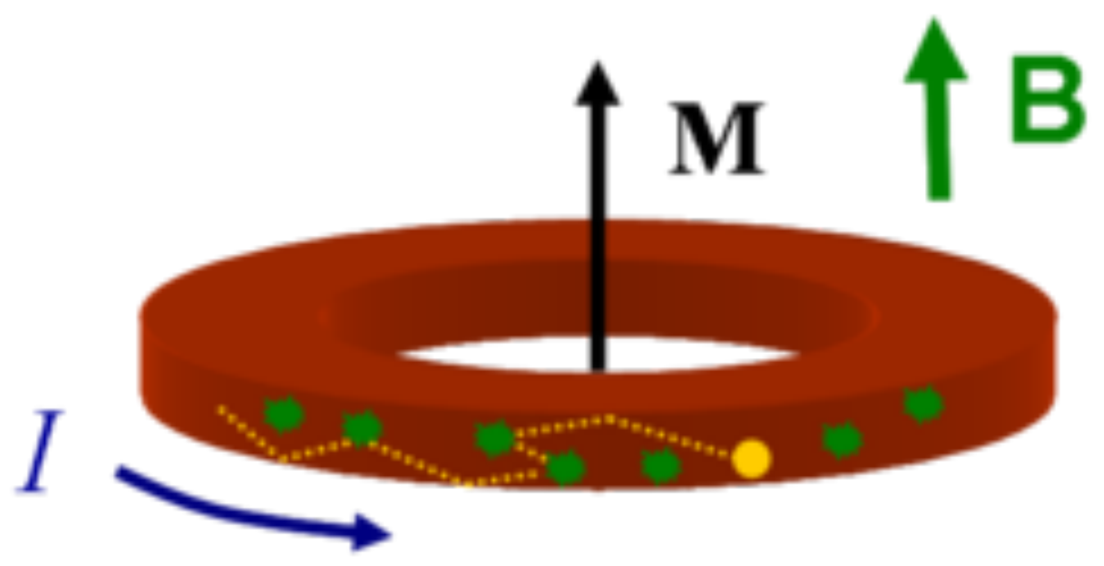
$$\psi(x) = \frac{1}{\sqrt{l}}\,\mathrm{e}^{\mathrm{i}kx} \qquad k_n = \frac{2\pi}{l}\left(n + \frac{\Phi_B}{h/e}\right)$$

$$i(k_n) = -e\frac{\hbar k_n}{m_*} = -\frac{2\pi e}{m_* l^2}\left(n + \frac{\Phi_{\mathbf{B}}}{h/e}\right)$$

current per level

Electronic coherence: permanent currents

$$\psi(x+L) = \mathrm{e}^{2\pi \mathrm{i}\frac{\Phi_{\mathbf{B}}}{h/e}}\psi(x)$$



M. Büttiker, Y. Imry and R. Landauer, Phys. Lett. A **96**, 365 (1983)

$$\psi(x) = \frac{1}{\sqrt{l}}\mathrm{e}^{\mathrm{i}kx} \qquad k_n = \frac{2\pi}{l}\left(n + \frac{\Phi_B}{h/e}\right)$$

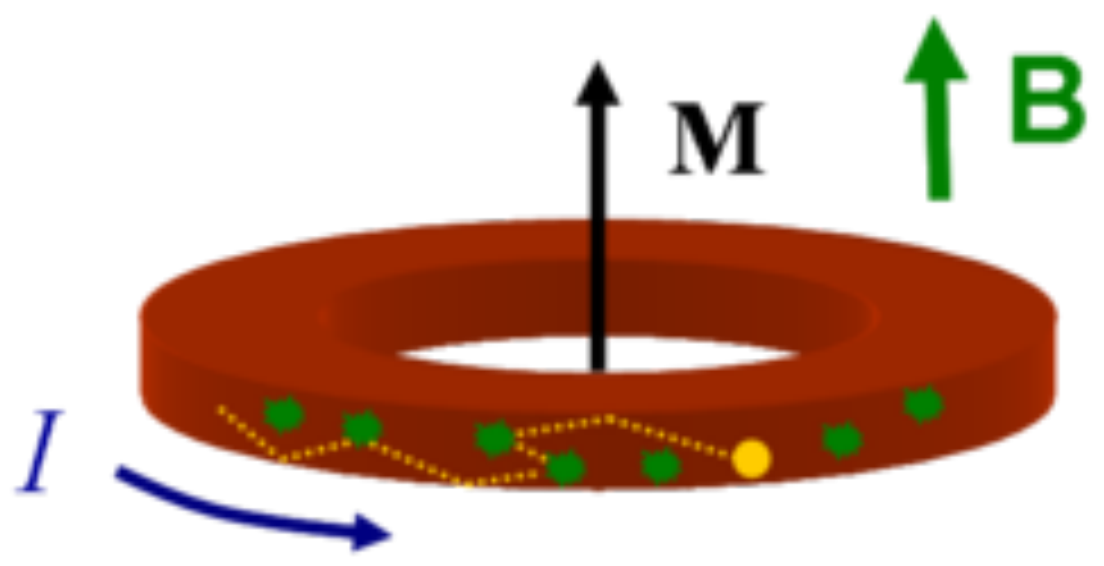
$$i(k_n) = -e\frac{\hbar k_n}{m_*} = -\frac{2\pi e}{m_* l^2}\left(n + \frac{\Phi_{\mathbf{B}}}{h/e}\right)$$ current per level

Experiments

700 isolated rings L.P. Lévy *et al*, Phys. Rev. Lett. **64**, 2074 (1990)

Electronic coherence: permanent currents

$$\psi(x+L) = \mathrm{e}^{2\pi\mathrm{i}\frac{\Phi_\mathbf{B}}{h/e}}\psi(x)$$



M. Büttiker, Y. Imry and R. Landauer, Phys. Lett. A **96**, 365 (1983)

$$\psi(x) = \frac{1}{\sqrt{l}}\mathrm{e}^{\mathrm{i}kx} \qquad k_n = \frac{2\pi}{l}\left(n + \frac{\Phi_B}{h/e}\right)$$

$$i(k_n) = -e\frac{\hbar k_n}{m_*} = -\frac{2\pi e}{m_* l^2}\left(n + \frac{\Phi_\mathbf{B}}{h/e}\right)$$ current per level

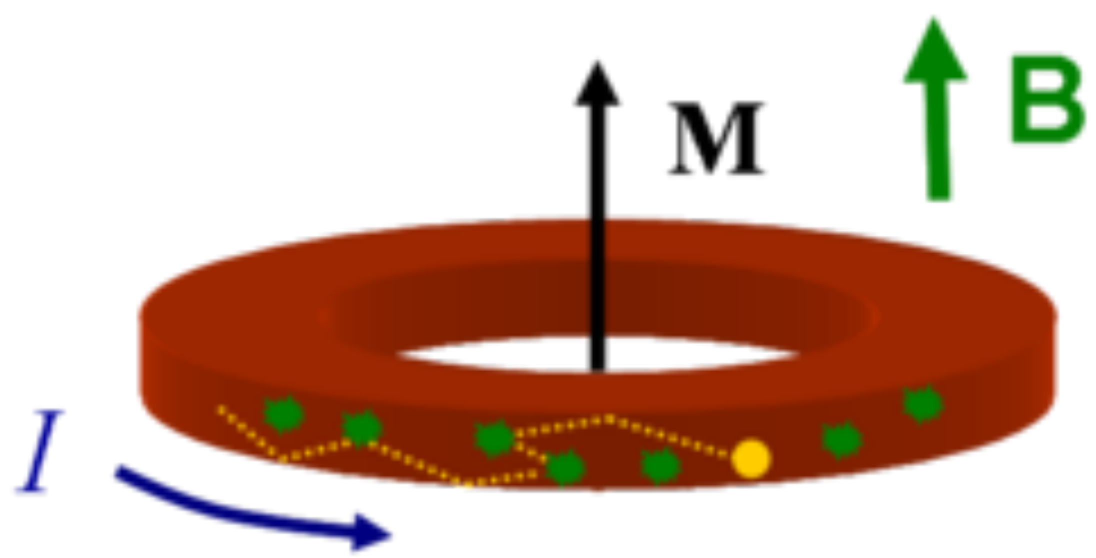
Experiments

700 isolated rings — L.P. Lévy *et al*, Phys. Rev. Lett. **64**, 2074 (1990)

single ring — D. Mailly *et al*, Phys. Rev. Lett. **70**, 2020 (1993)

Electronic coherence: permanent currents

$$\psi(x+L) = \mathrm{e}^{2\pi \mathrm{i} \frac{\Phi_{\mathbf{B}}}{h/e}} \psi(x)$$



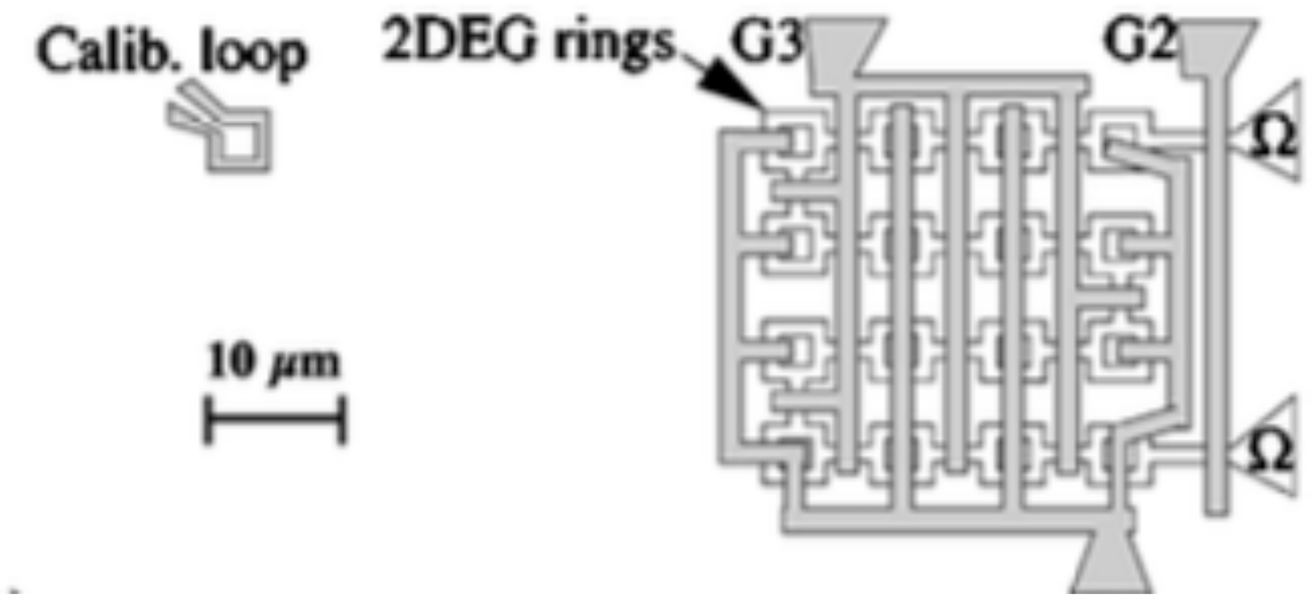
M. Büttiker, Y. Imry and R. Landauer, Phys. Lett. A **96**, 365 (1983)

$$\psi(x) = \frac{1}{\sqrt{l}} \mathrm{e}^{\mathrm{i}kx} \qquad k_n = \frac{2\pi}{l}\left(n + \frac{\Phi_B}{h/e}\right)$$

$$i(k_n) = -e\frac{\hbar k_n}{m_*} = -\frac{2\pi e}{m_* l^2}\left(n + \frac{\Phi_{\mathbf{B}}}{h/e}\right)$$

current per level

Experiments



700 isolated rings — L.P. Lévy *et al*, Phys. Rev. Lett. **64**, 2074 (1990)

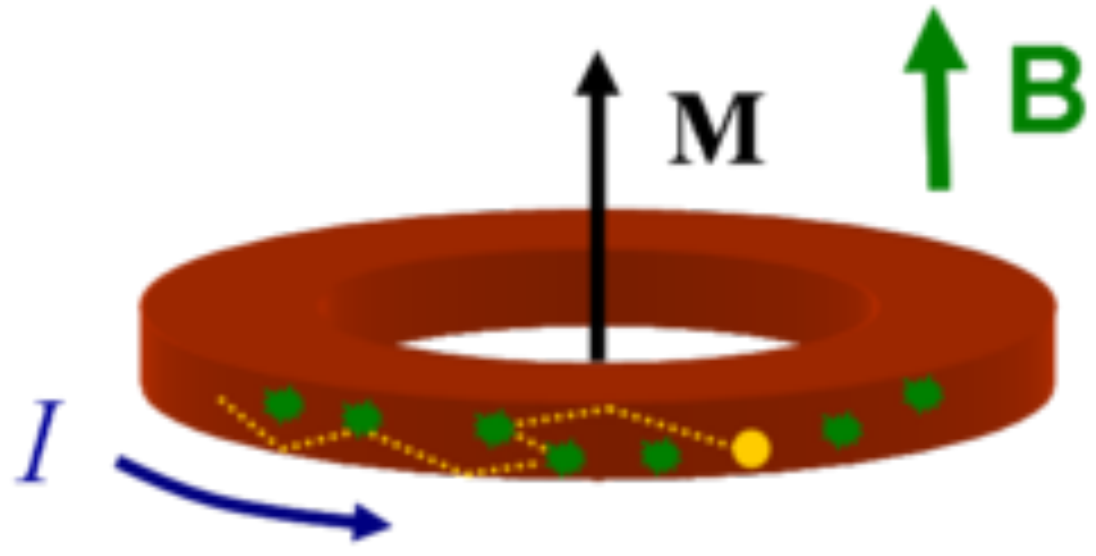
single ring — D. Mailly *et al*, Phys. Rev. Lett. **70**, 2020 (1993)

connected rings — W. Rabaud *et al*, Phys. Rev. Lett. **86**, 3124 (2001)

$l_\phi \sim 20\ \mu\mathrm{m}$

Electronic coherence: permanent currents

$$\psi(x+L) = \mathrm{e}^{2\pi \mathrm{i}\frac{\Phi_{\mathbf{B}}}{h/e}}\psi(x)$$

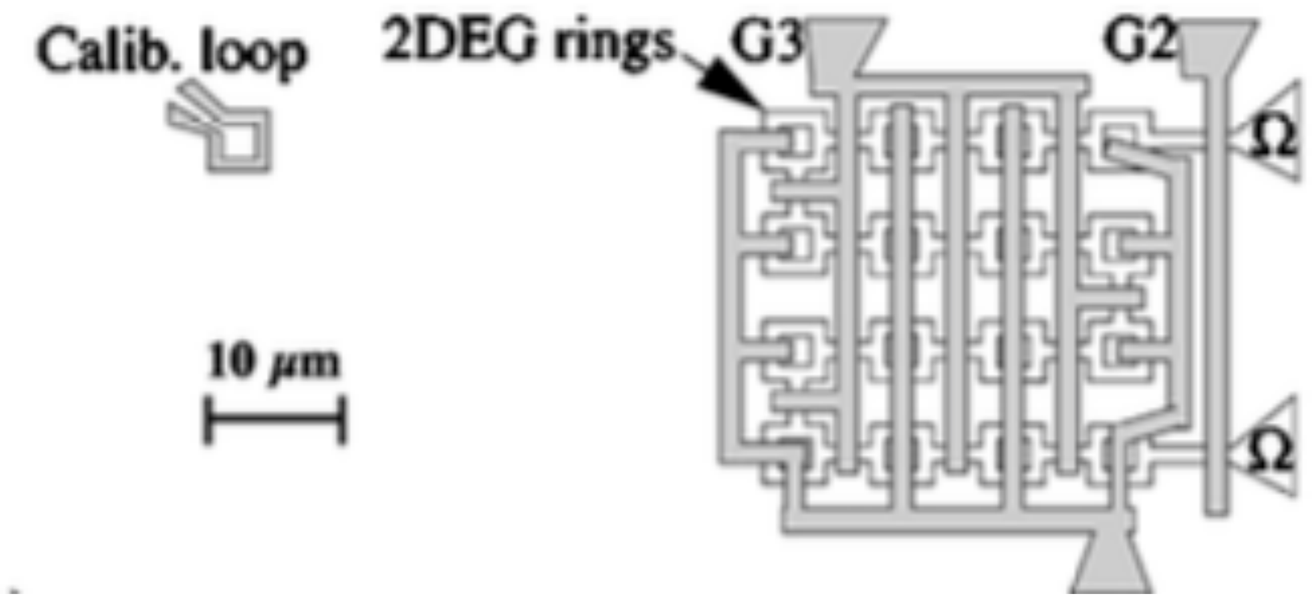


M. Büttiker, Y. Imry and R. Landauer, Phys. Lett. A **96**, 365 (1983)

$$\psi(x) = \frac{1}{\sqrt{l}}\mathrm{e}^{\mathrm{i}kx} \qquad k_n = \frac{2\pi}{l}\left(n + \frac{\Phi_B}{h/e}\right)$$

$$i(k_n) = -e\frac{\hbar k_n}{m_*} = -\frac{2\pi e}{m_* l^2}\left(n + \frac{\Phi_{\mathbf{B}}}{h/e}\right)$$ current per level

Experiments



700 isolated rings — L.P. Lévy *et al*, Phys. Rev. Lett. **64**, 2074 (1990)

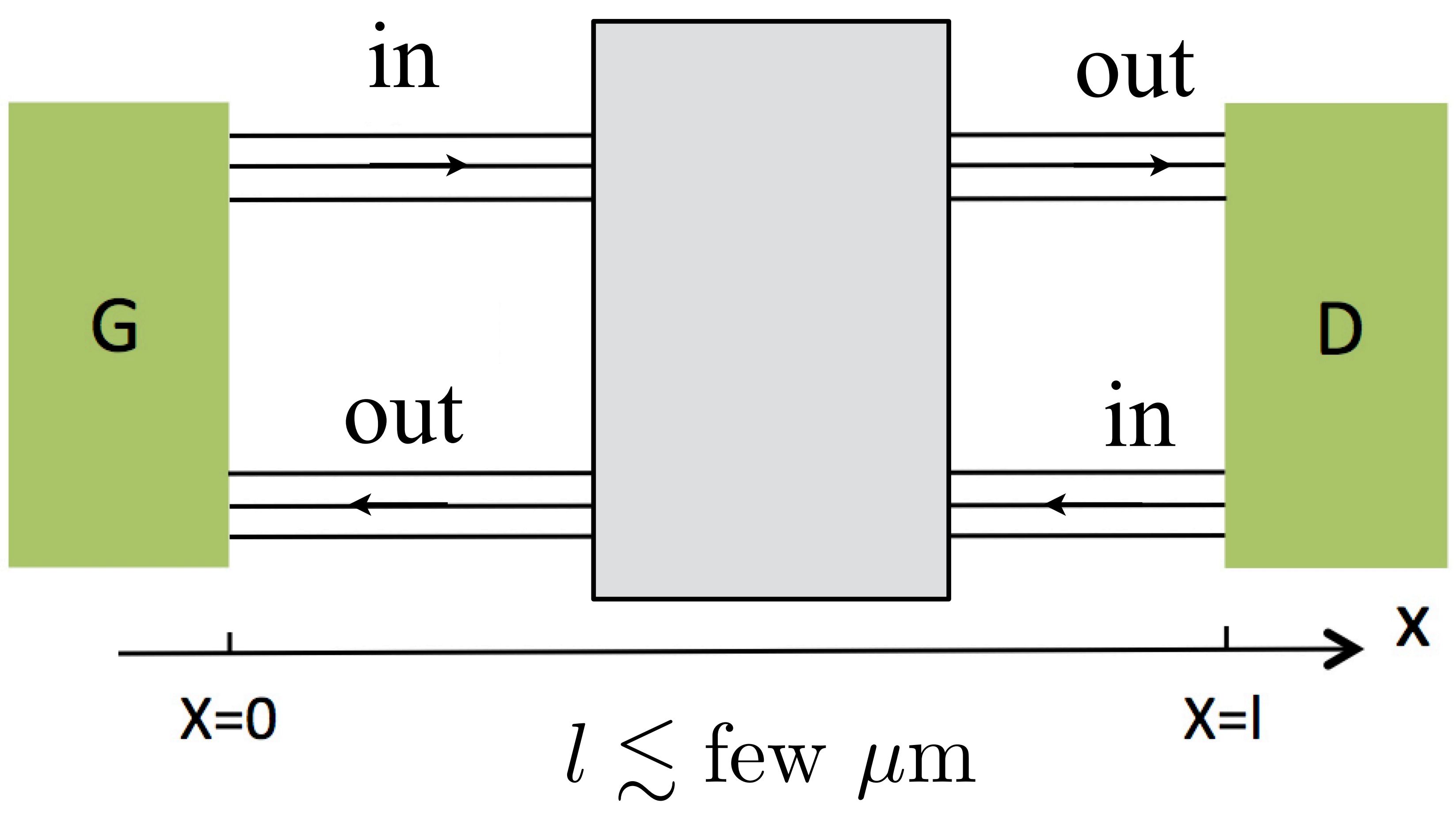
single ring — D. Mailly *et al*, Phys. Rev. Lett. **70**, 2020 (1993)

connected rings $l_\phi \sim 20\ \mu\mathrm{m}$ — W. Rabaud *et al*, Phys. Rev. Lett. **86**, 3124 (2001)

Macroscopic molecular currents *à la Ampère*!

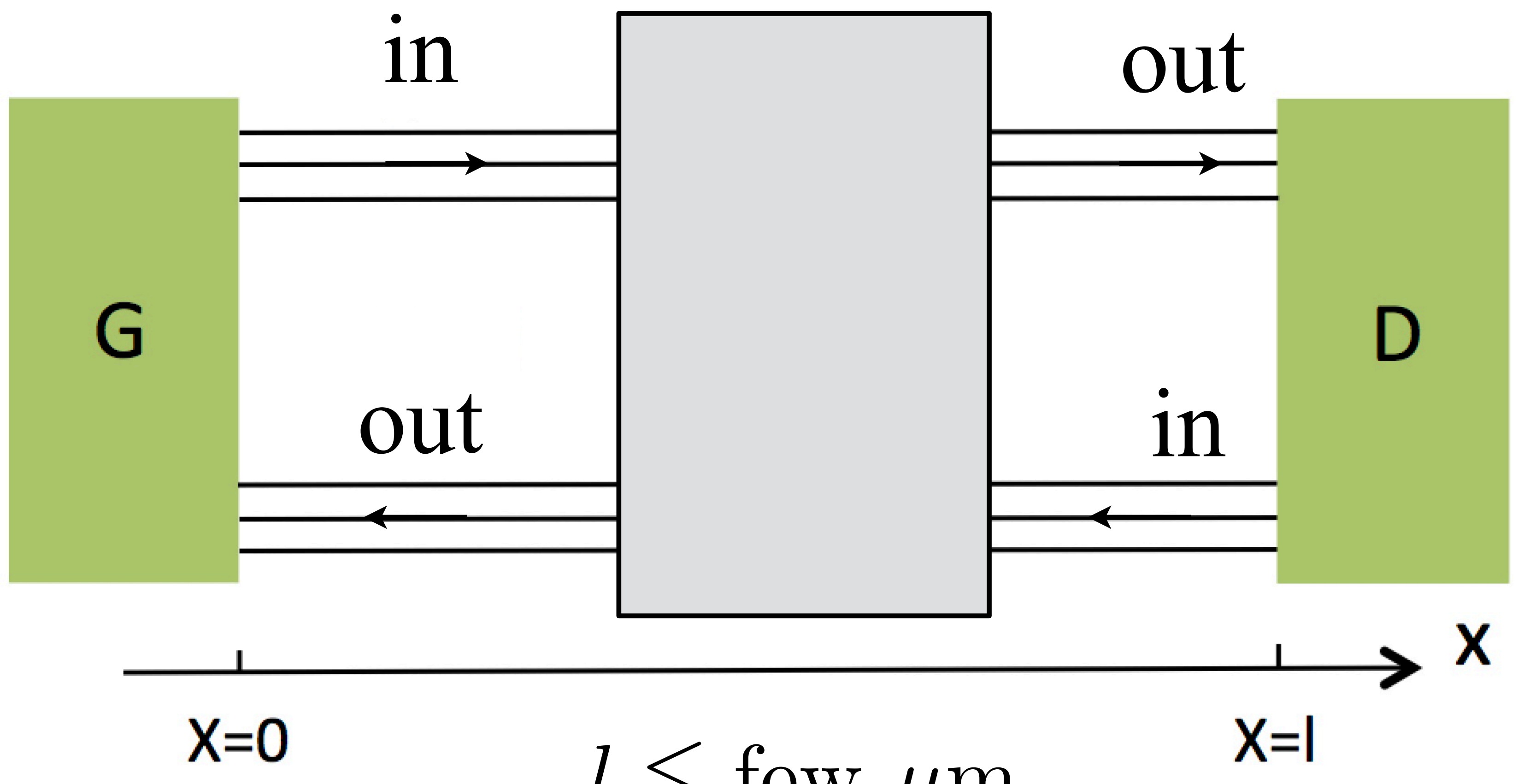
dc transport in quantum conductors: electronic scattering

Quantum conductor = Coherent electronic scatterer



dc transport in quantum conductors: electronic scattering

Quantum conductor = Coherent electronic scatterer



$l \lesssim$ few μm

Reservoirs:

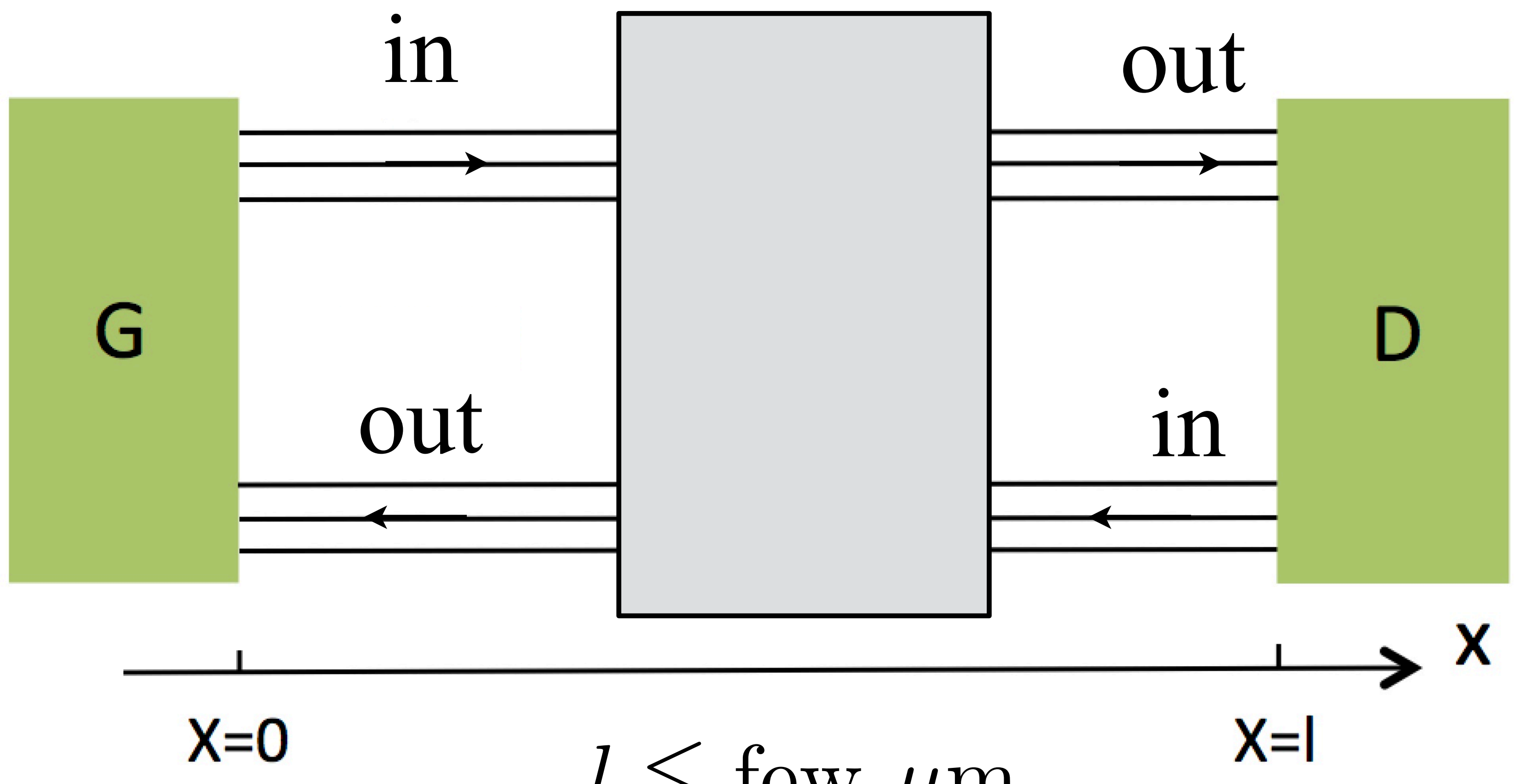
Nothing comes back from reservoirs!

Emit equilibrium streams of electrons $(\mu_D, T_{\rm el,D})$

$$f_D(\omega) = \frac{1}{\mathrm{e}^{(\hbar\omega - \mu_D)/k_B T_{\mathrm{el},D}} + 1}$$

dc transport in quantum conductors: electronic scattering

Quantum conductor = Coherent electronic scatterer



$l \lesssim$ few μm

$$\langle I \rangle = -\frac{e}{h} \int \left(T_{G,D}(E)\, f_G(E) - T_{D,G}(E)\, f_D(E) \right) \mathrm{d}E$$

Transmission probability

Reservoirs:

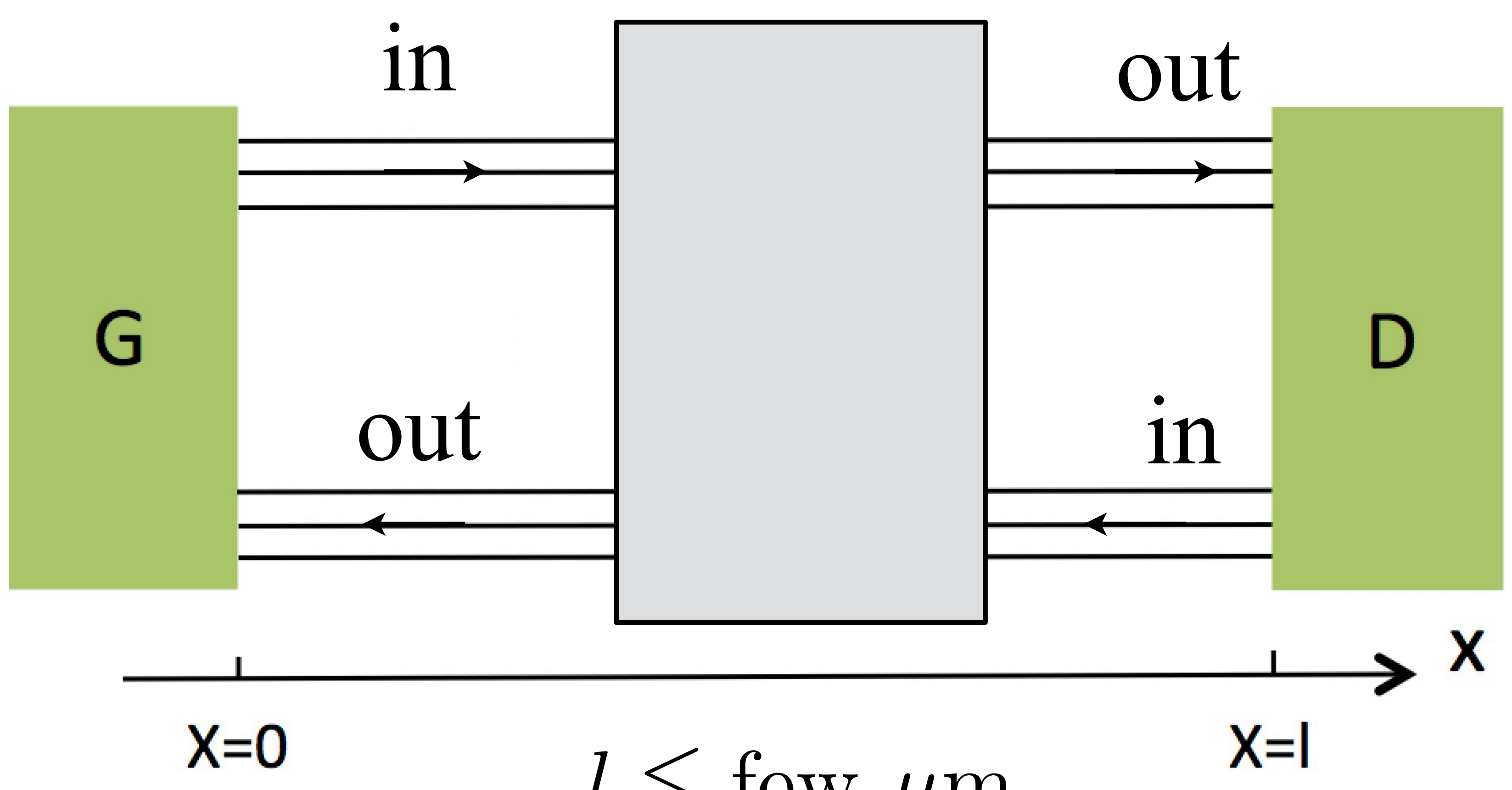
Nothing comes back from reservoirs!

Emit equilibrium streams of electrons $(\mu_D, T_{\mathrm{el,D}})$

$$f_D(\omega) = \frac{1}{\mathrm{e}^{(\hbar\omega - \mu_D)/k_B T_{\mathrm{el},D}} + 1}$$

dc transport in quantum conductors: electronic scattering

Quantum conductor = Coherent electronic scatterer



$l \lesssim$ few μm

$$\langle I \rangle = -\frac{e}{h} \int \left(T_{G,D}(E)\, f_G(E) - T_{D,G}(E)\, f_D(E) \right) \mathrm{d}E$$

Transmission probability

Reservoirs:

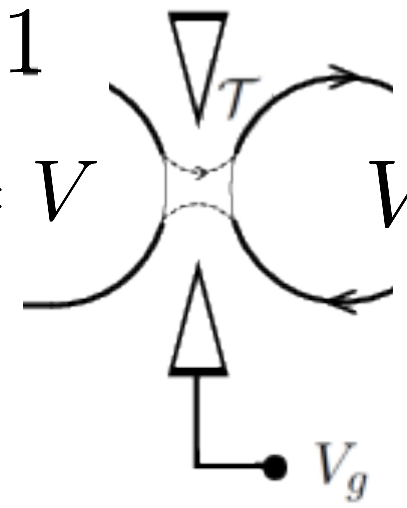
Nothing comes back from reservoirs!

Emit equilibrium streams of electrons $(\mu_D, T_{\mathrm{el,D}})$

$$f_D(\omega) = \frac{1}{\mathrm{e}^{(\hbar\omega - \mu_D)/k_B T_{\mathrm{el},D}} + 1}$$

$\nu = 1$

$V_D = V$

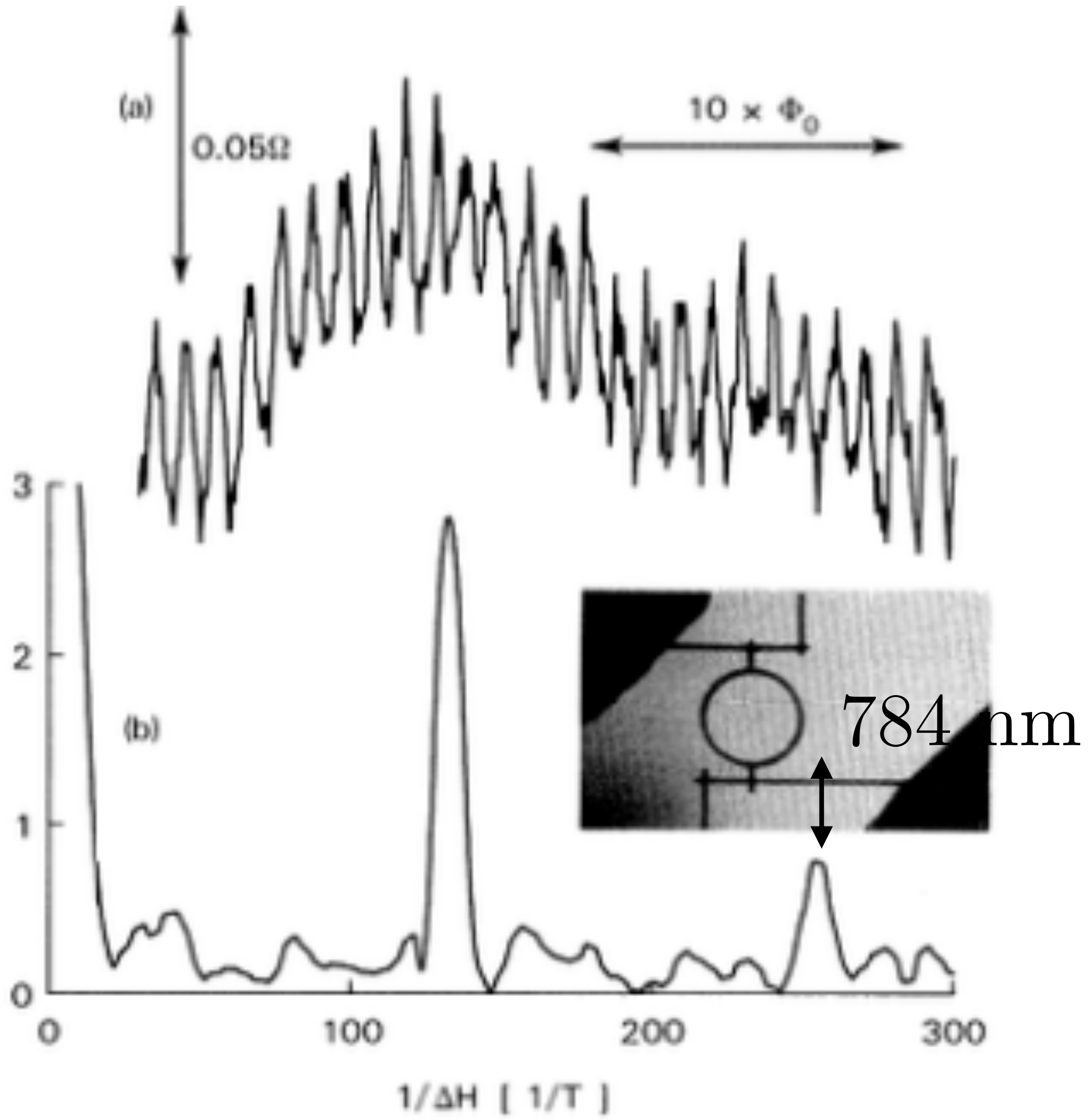


$V_G = 0$

$$I = \frac{e^2}{h} \mathcal{T} V$$

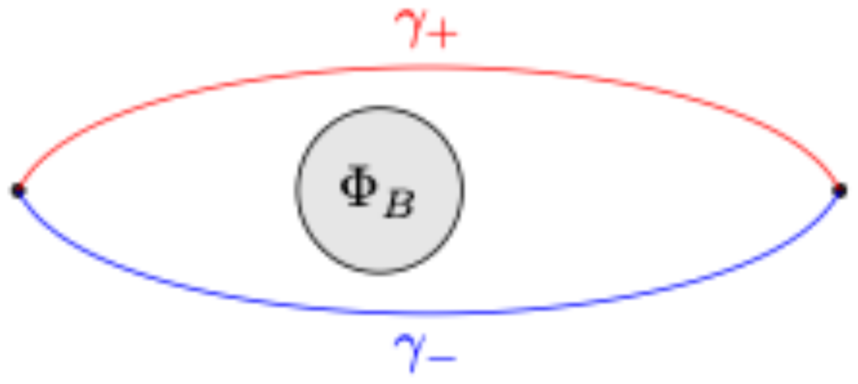
Electronic coherence: breakdown of impedance composition laws

Conductances do not add in parallel !



thickness 41 nm

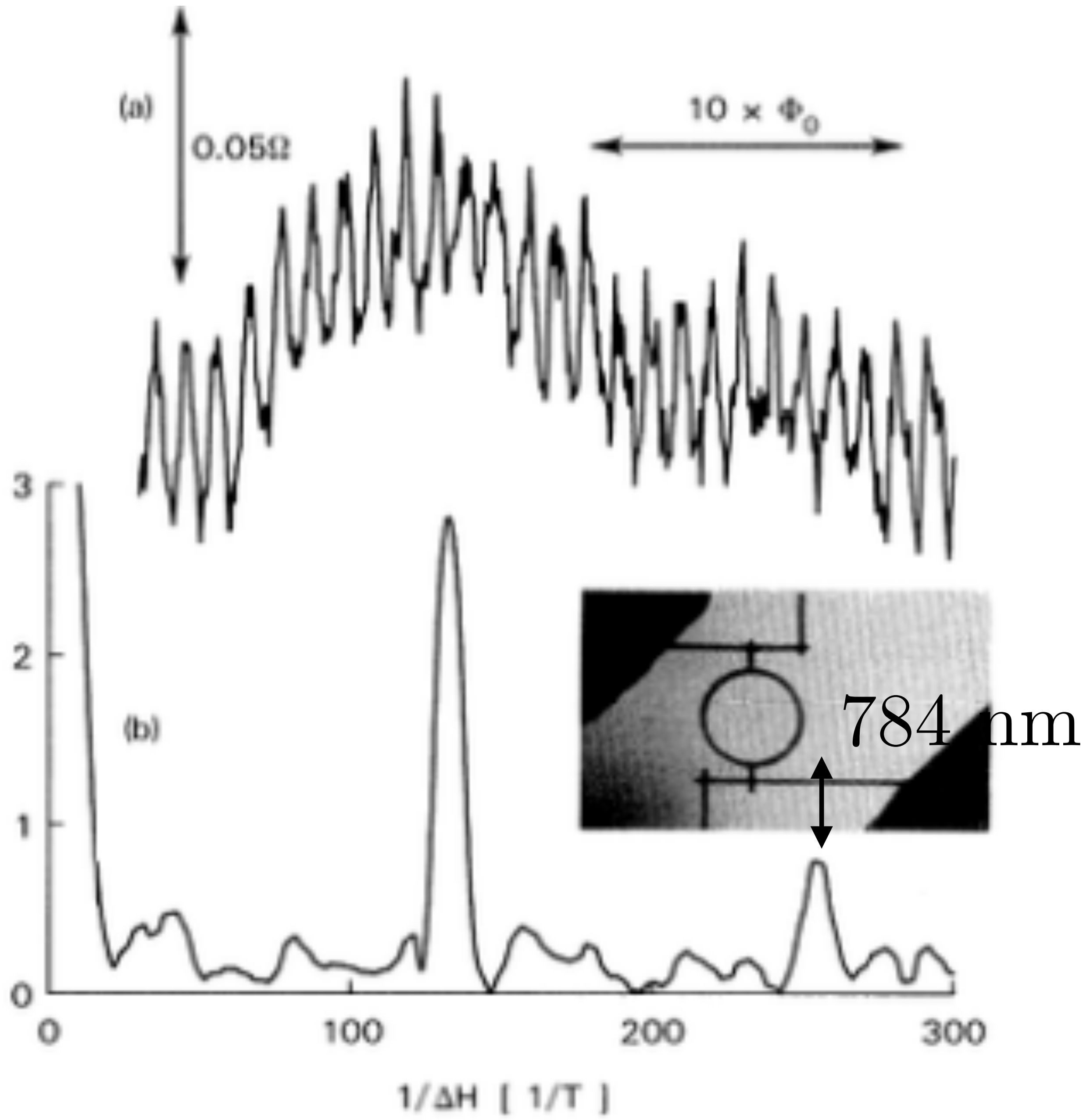
Periodicity: $\Phi \mapsto \Phi + \dfrac{h}{e}$



R.A. Webb *et al*, Phys. Rev. Lett. **54**, 2696 (1985)

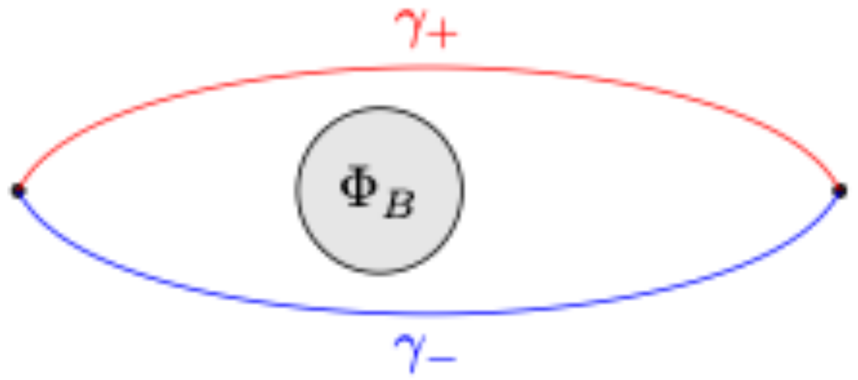
Electronic coherence: breakdown of impedance composition laws

Conductances do not add in parallel !



thickness 41 nm

Periodicity: $\Phi \mapsto \Phi + \dfrac{h}{e}$

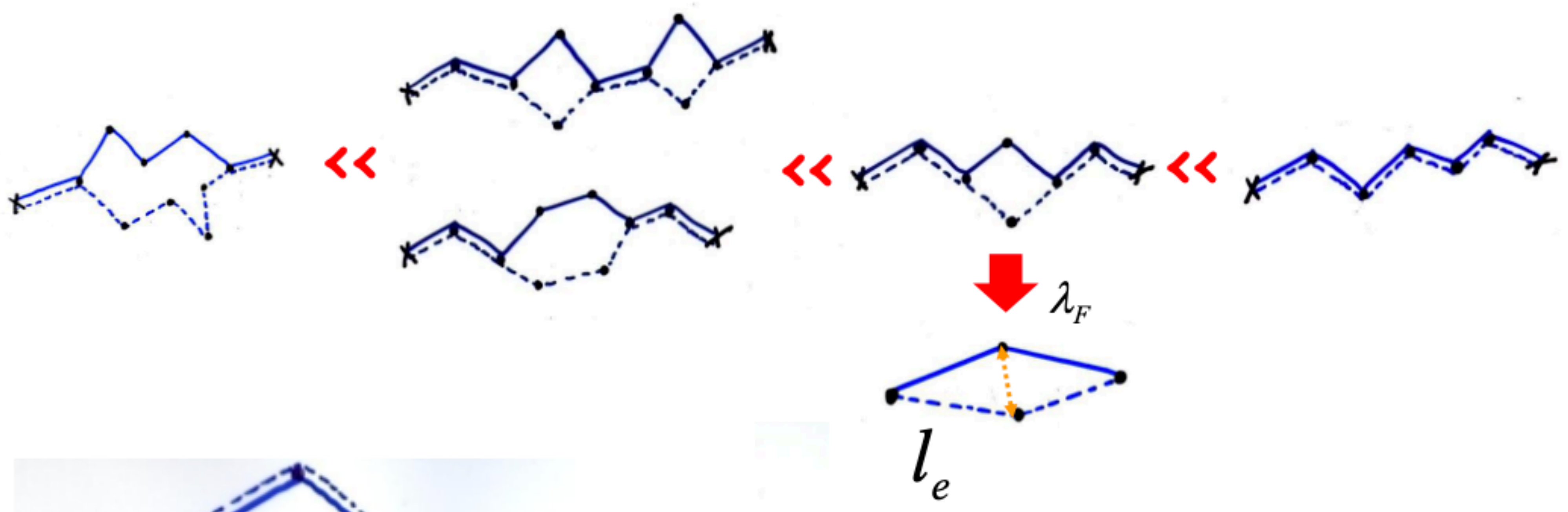


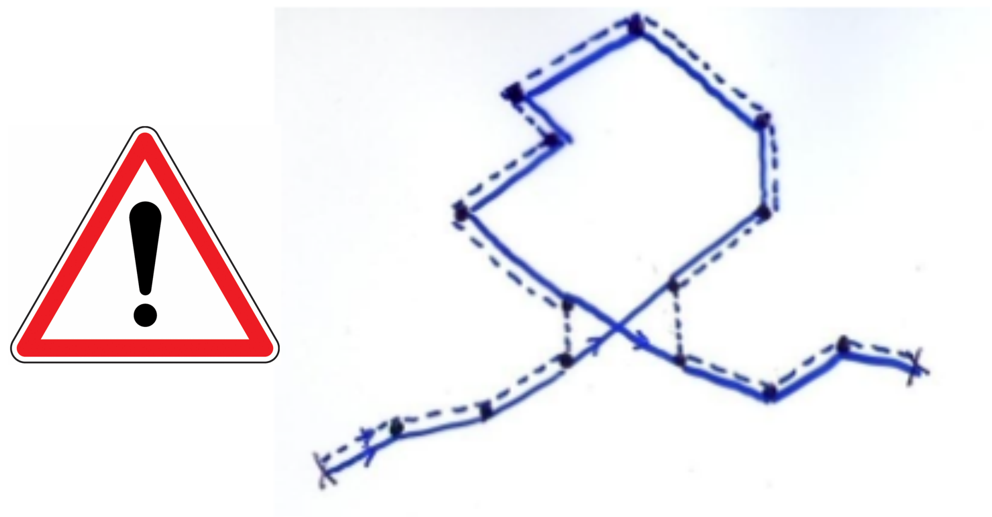
R.A. Webb *et al*, Phys. Rev. Lett. **54**, 2696 (1985)

Older experience: $\Phi \mapsto \Phi + \dfrac{h}{2e}$

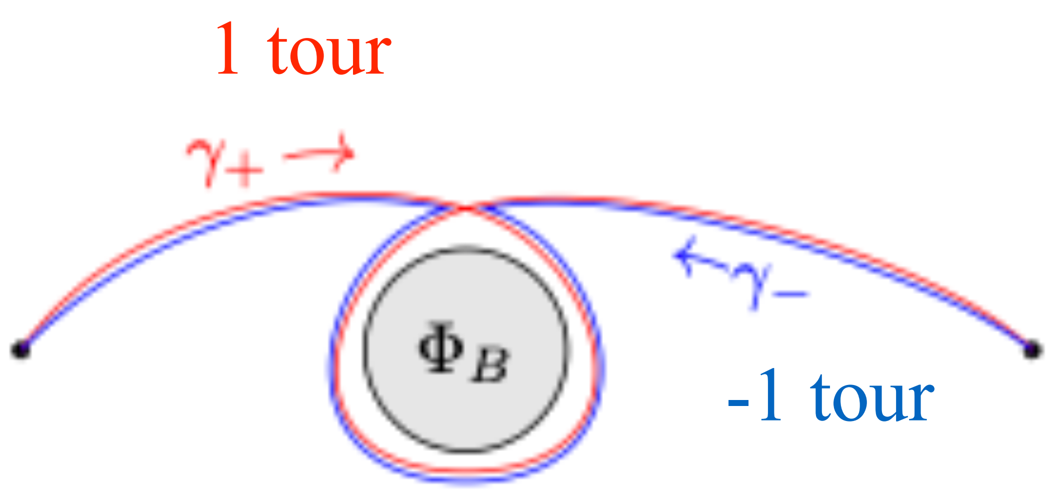
D. Yu. and Yu. V. Sharvin, JETP Lett. **34** (1981)

Classical contributions

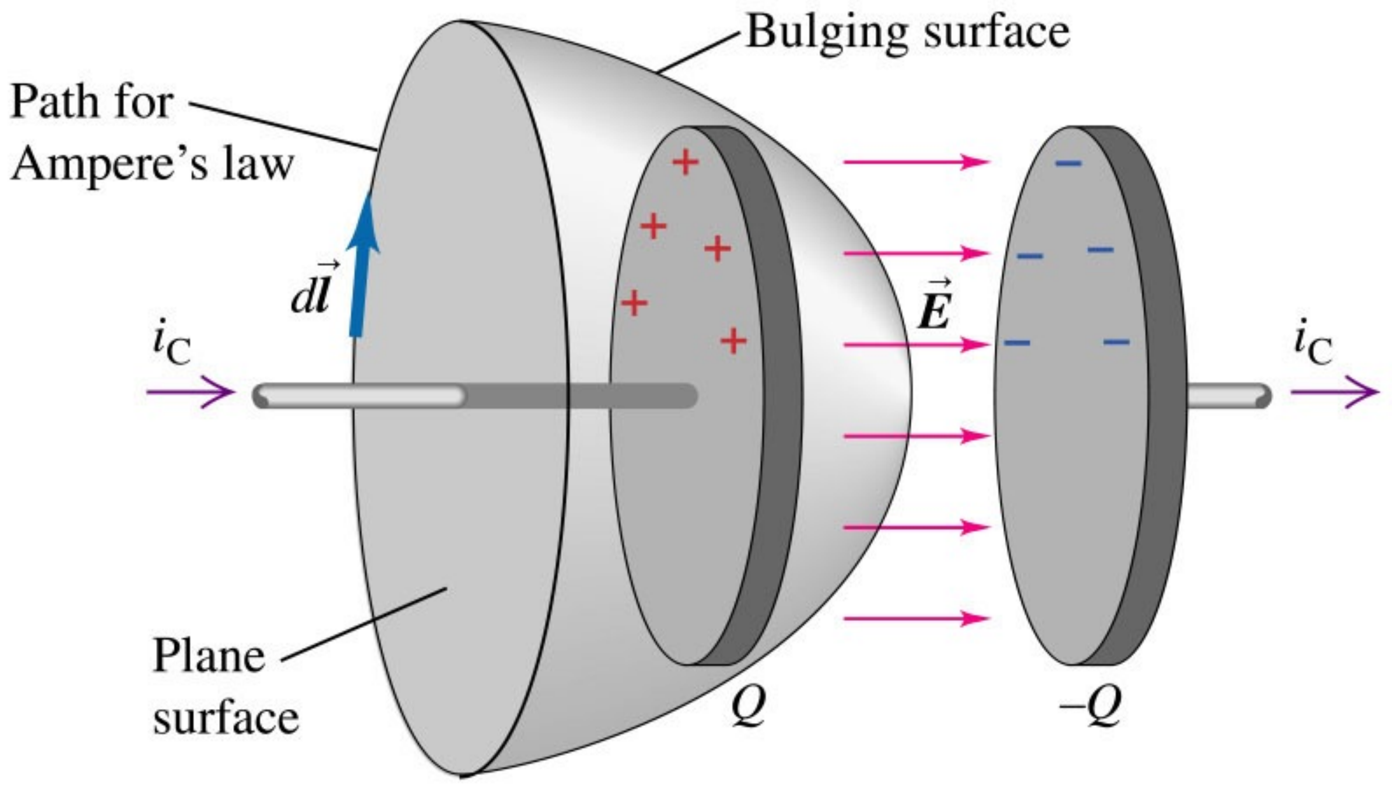




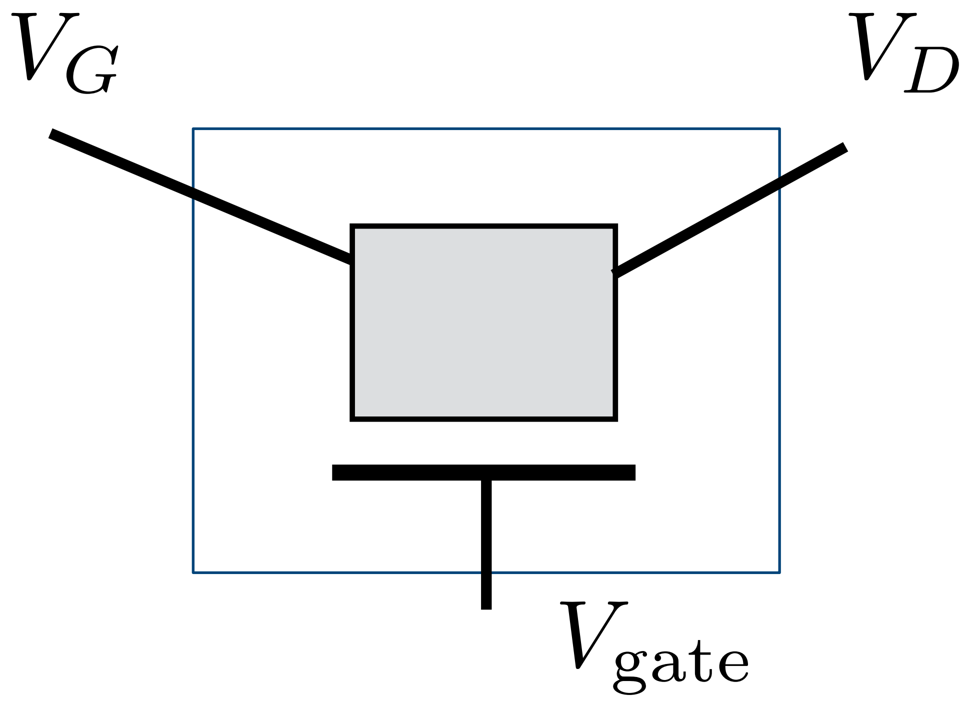
Quantum corrections:
Weak for $R \ll R_K$



AC transport and Coulomb interactions



$$c^2\,\overrightarrow{\mathrm{rot}}(\mathbf{B}) = \frac{\mathbf{j}}{\varepsilon_0} + \frac{\partial \mathbf{E}}{\partial t}$$



$$\sum_\alpha I_{\alpha,\mathrm{in}} = 0$$ current conservation

$$\forall \alpha,\ V_\alpha \mapsto V_\alpha + V$$ changes nothing — gauge invariance

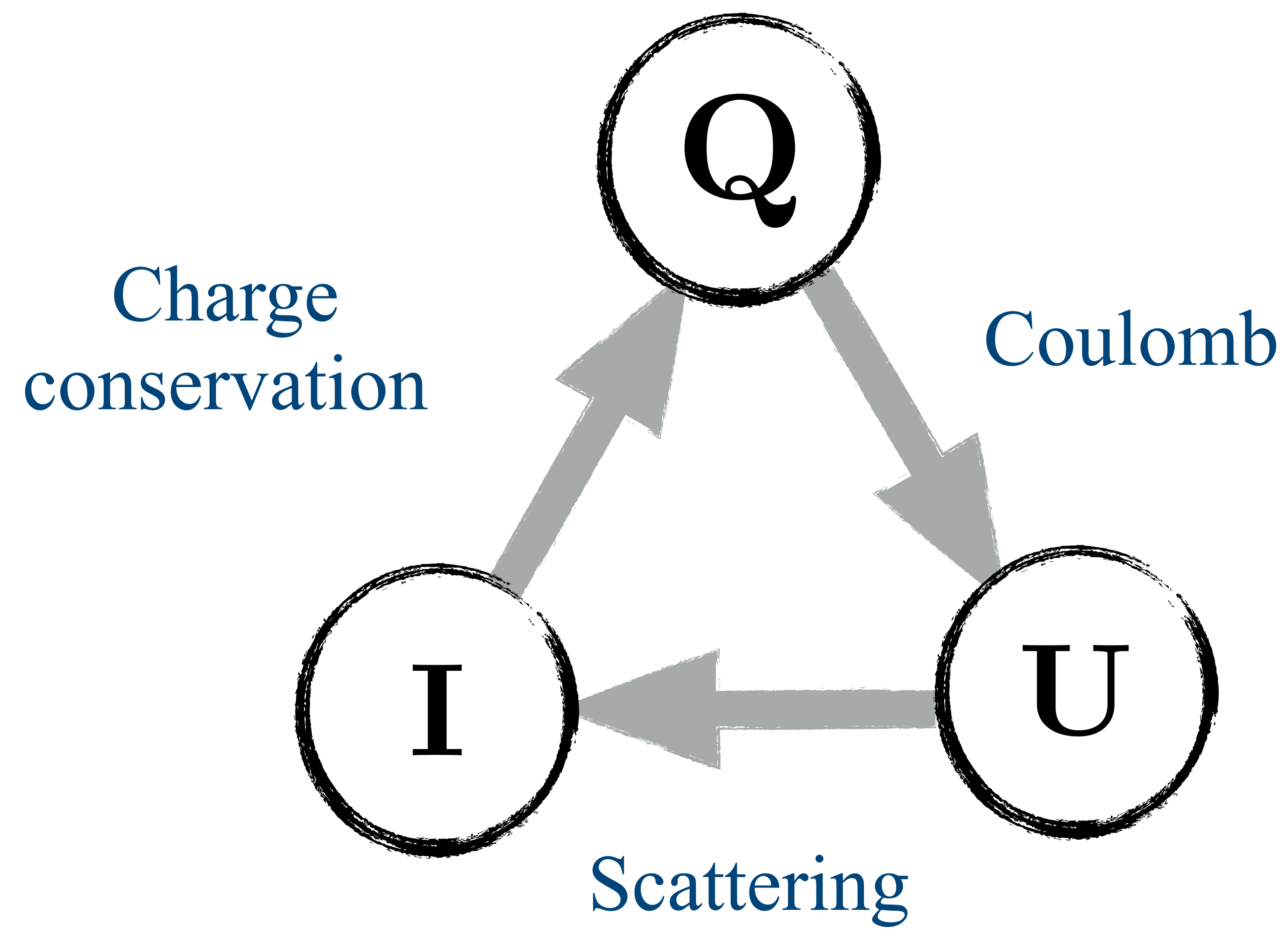
Gauge invariance and charge conservation require Coulomb interactions

M. Büttiker, J. Phys.: Cond. Matt. **5**, 9361 (1993)

Classical self-consistant potential for electronic scattering

AC quantum transport and Coulomb interactions

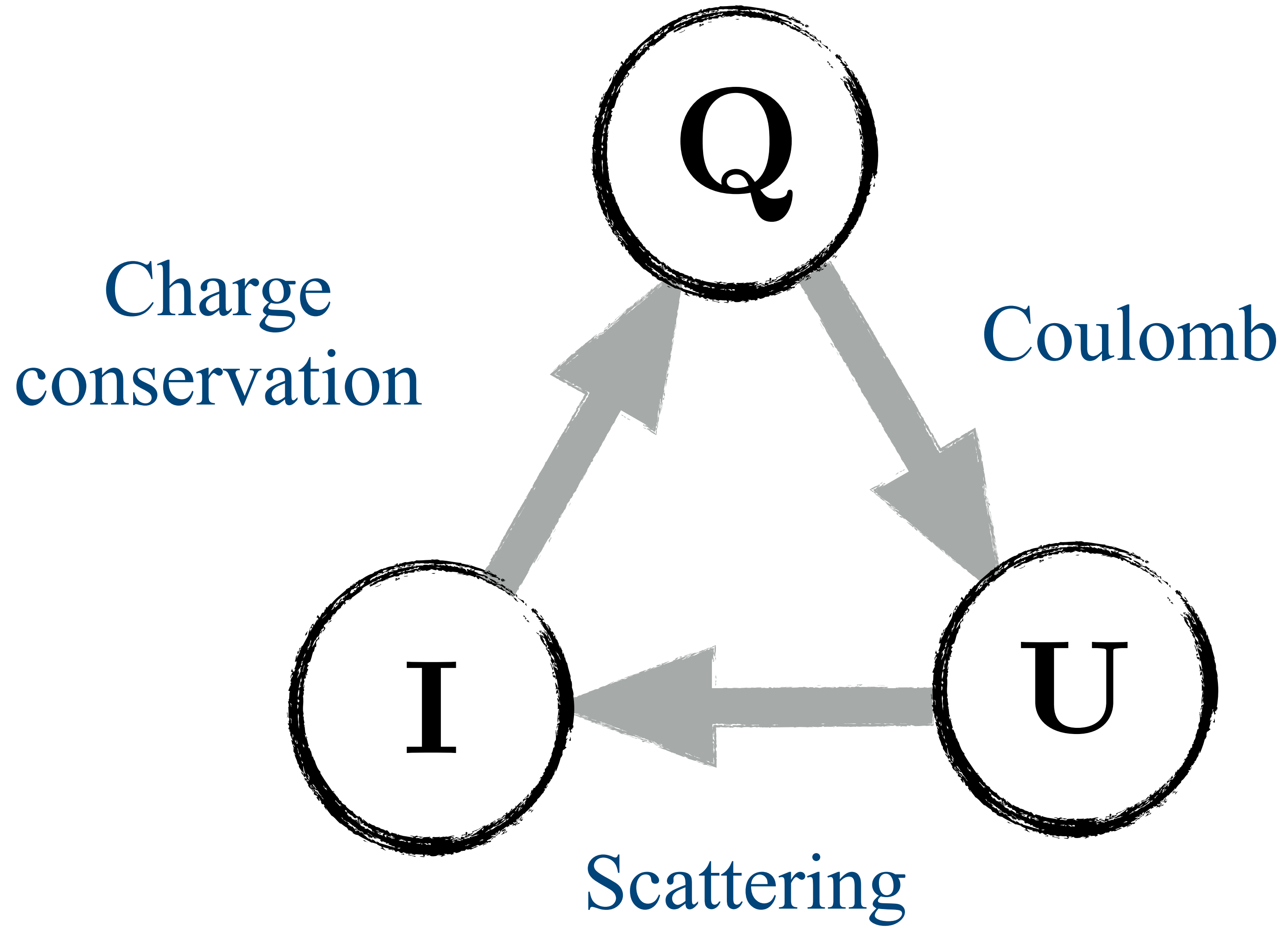
Classical self-consistant potential for electronic scattering



$$S(\omega; [U])$$

AC quantum transport and Coulomb interactions

Classical self-consistant potential for electronic scattering



$$S(\omega; [U])$$

Charges from potentials (Coulomb) $$Q_\alpha = \sum_\beta C^{(\mathrm{geom})}_{\alpha\beta} U_\beta$$

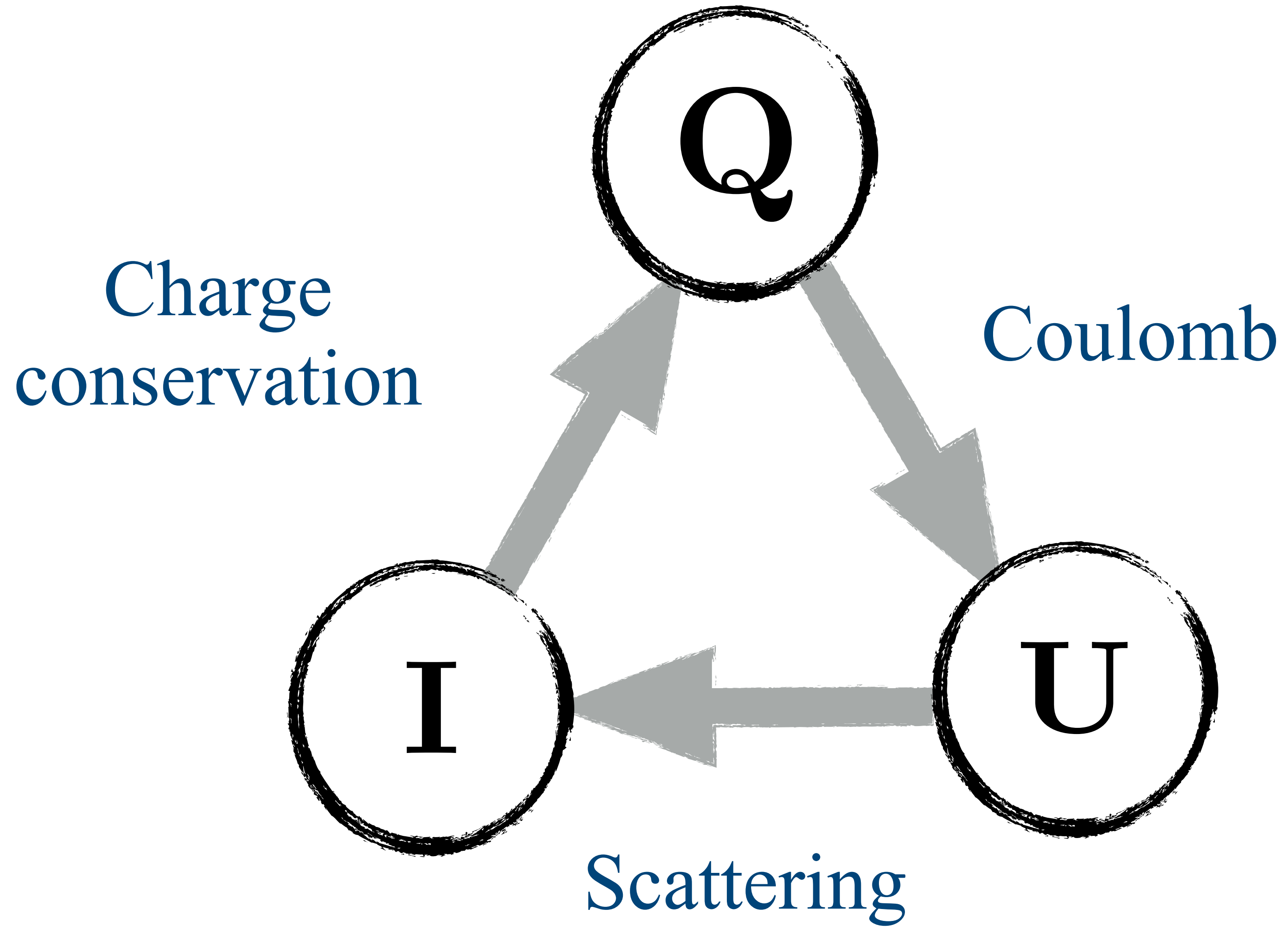
Charges from currents $$-\mathrm{i}\omega\, Q_\alpha(\omega) = \sum_{j\mapsto\alpha} I_j(\omega)$$

Charges from electronic scattering $$I_j(\omega) = \mathcal{I}_j(\omega; [S, V_j])$$

A Prêtre *et al*, Phys. Rev. B **54**, 8130 (1996)

AC quantum transport and Coulomb interactions

Classical self-consistant potential for electronic scattering



$S(\omega; [U])$

Charges from potentials (Coulomb) $$Q_\alpha = \sum_\beta C^{(\mathrm{geom})}_{\alpha\beta} U_\beta$$

Charges from currents $$-\mathrm{i}\omega\, Q_\alpha(\omega) = \sum_{j\mapsto\alpha} I_j(\omega)$$

Charges from electronic scattering $$I_j(\omega) = \mathcal{I}_j(\omega; [S, V_j])$$

A Prêtre *et al*, Phys. Rev. B **54**, 8130 (1996)

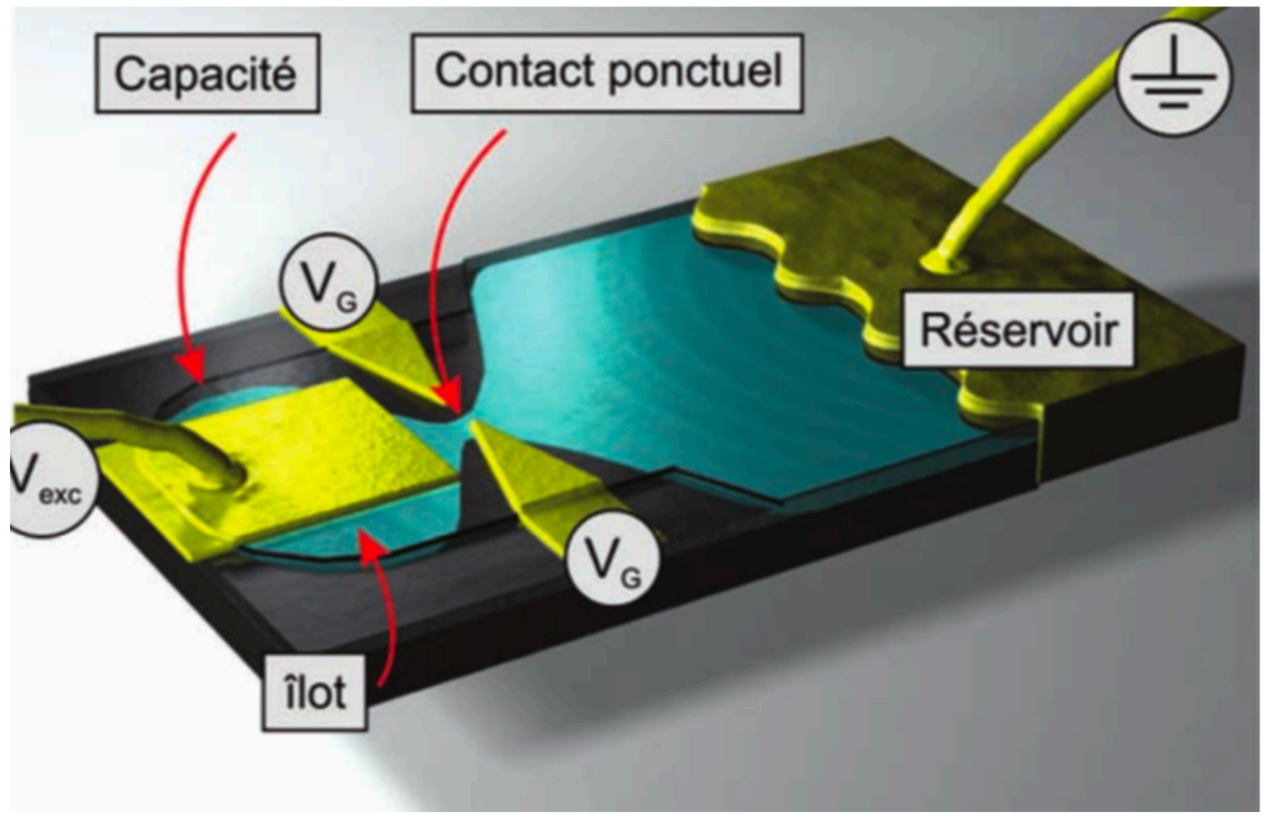
Non-linear dc response

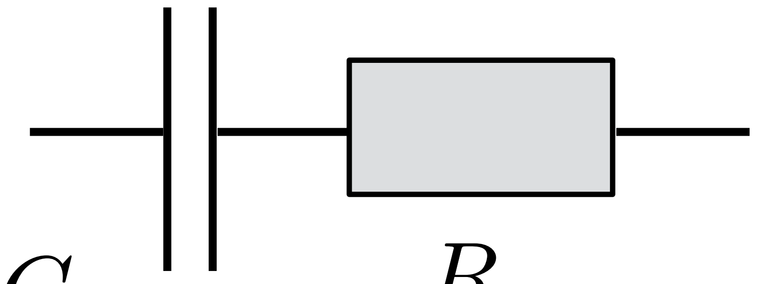
T. Christen and M. Büttiker, Europhys. Lett. **35**, 523 (1996)

AC quantum transport: the mesoscopic RC circuit

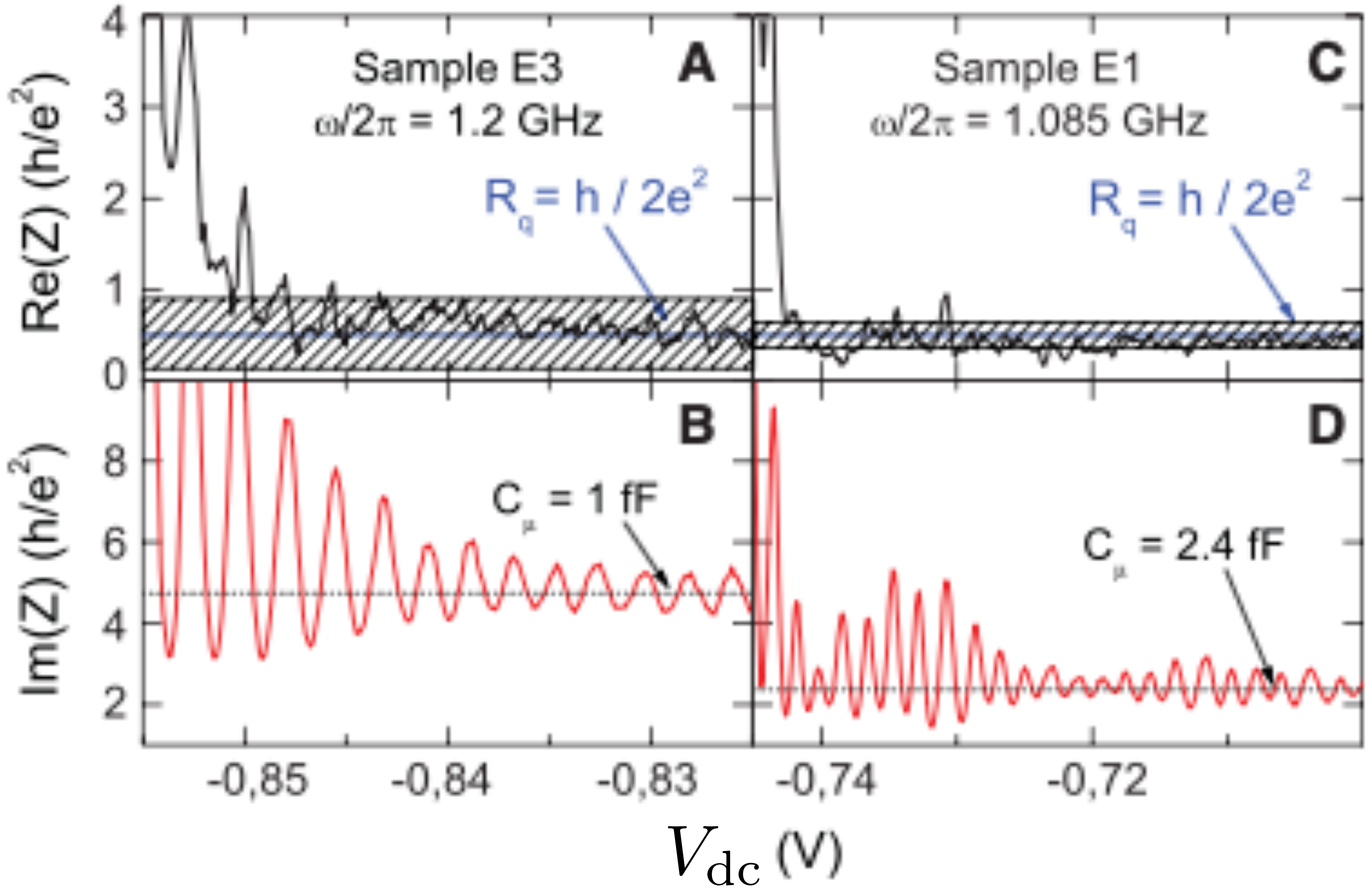
GHz frequency measurement of the impedance

J. Gabelli *et al*, Science **313**, 499 (2006)





C_μ R_q



$$\frac{1}{C_\mu} = \frac{1}{C_g} + \frac{1}{C_q(V_{\mathrm{dc}})}$$

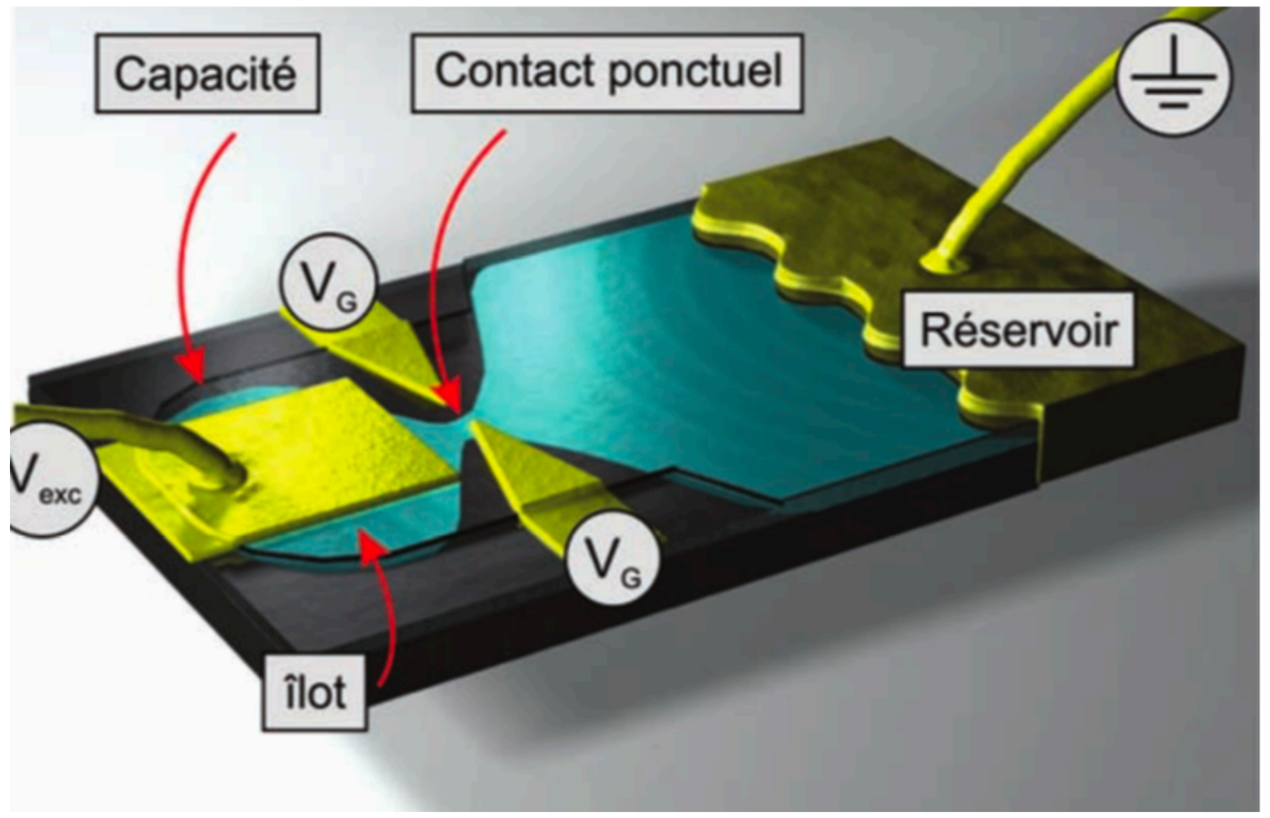
$$R_q = \frac{R_K}{2} \neq \frac{R_K}{\mathcal{T}}$$

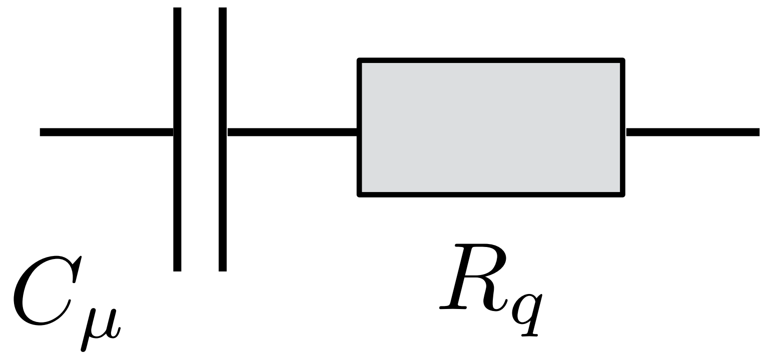
Theory: M. Büttiker *et al*, Phys. Lett. A 180, 364 (1993)

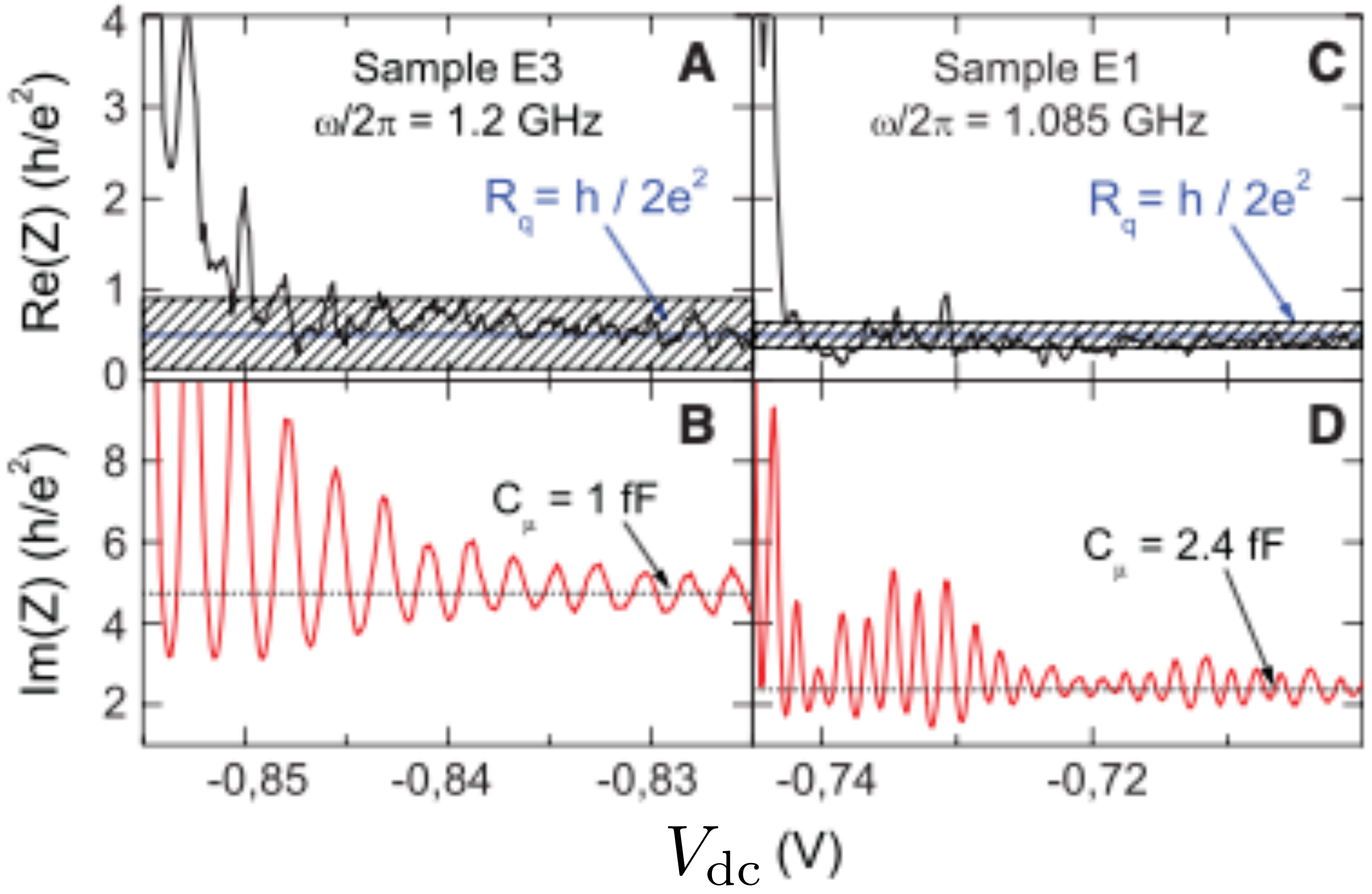
AC quantum transport: the mesoscopic RC circuit

GHz frequency measurement of the impedance

J. Gabelli *et al*, Science **313**, 499 (2006)







$$\frac{1}{C_\mu} = \frac{1}{C_g} + \frac{1}{C_q(V_{dc})}$$

$$R_q = \frac{R_K}{2} \neq \frac{R_K}{\mathcal{T}}$$

Violation of Kirchhoff's Laws for a Coherent *RC* Circuit

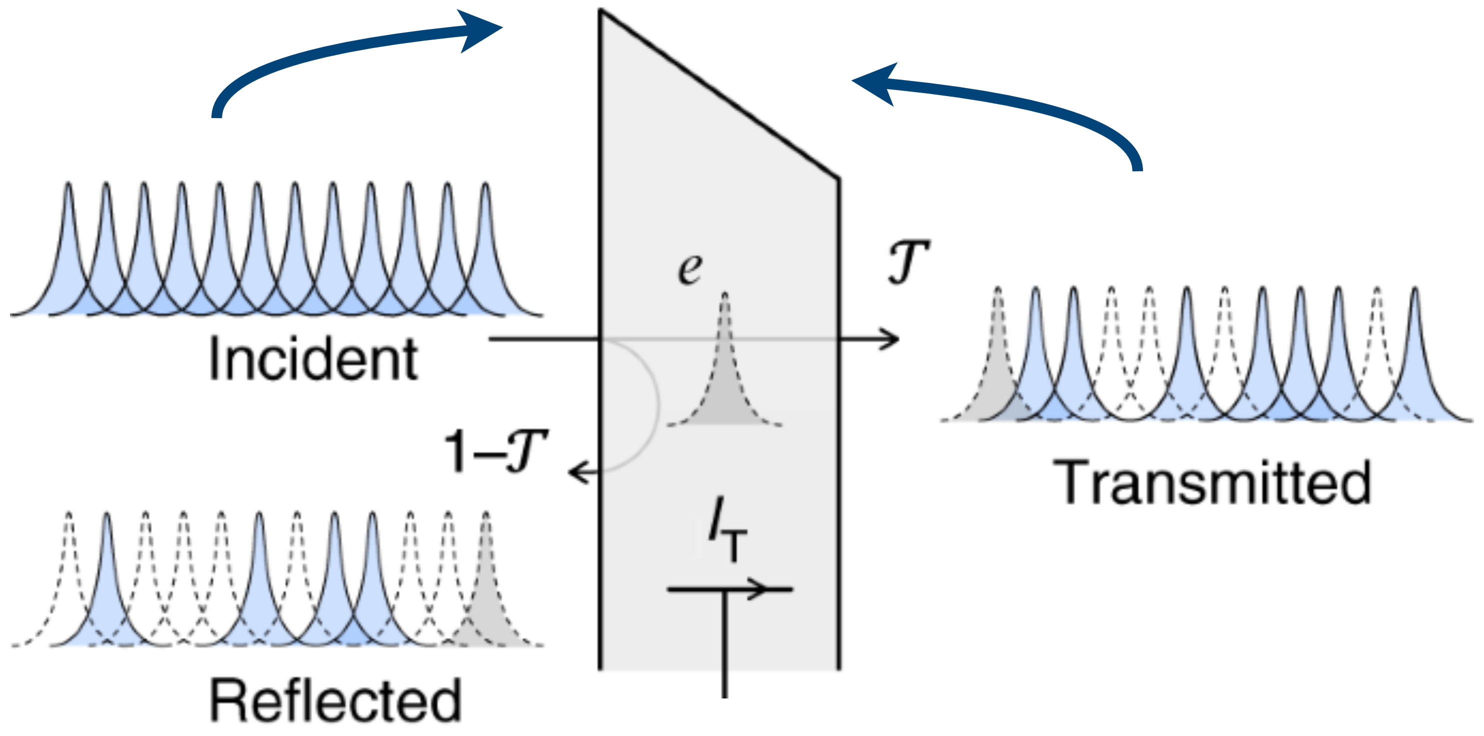
J. Gabelli,[1] G. Fève,[1] J.-M. Berroir,[1] B. Plaçais,[1] A. Cavanna,[2] B. Etienne,[2] Y. Jin,[2] D. C. Glattli[1,3*]

Theory: M. Büttiker *et al*, Phys. Lett. A 180, 364 (1993)

Electronic transport in the quantum domain

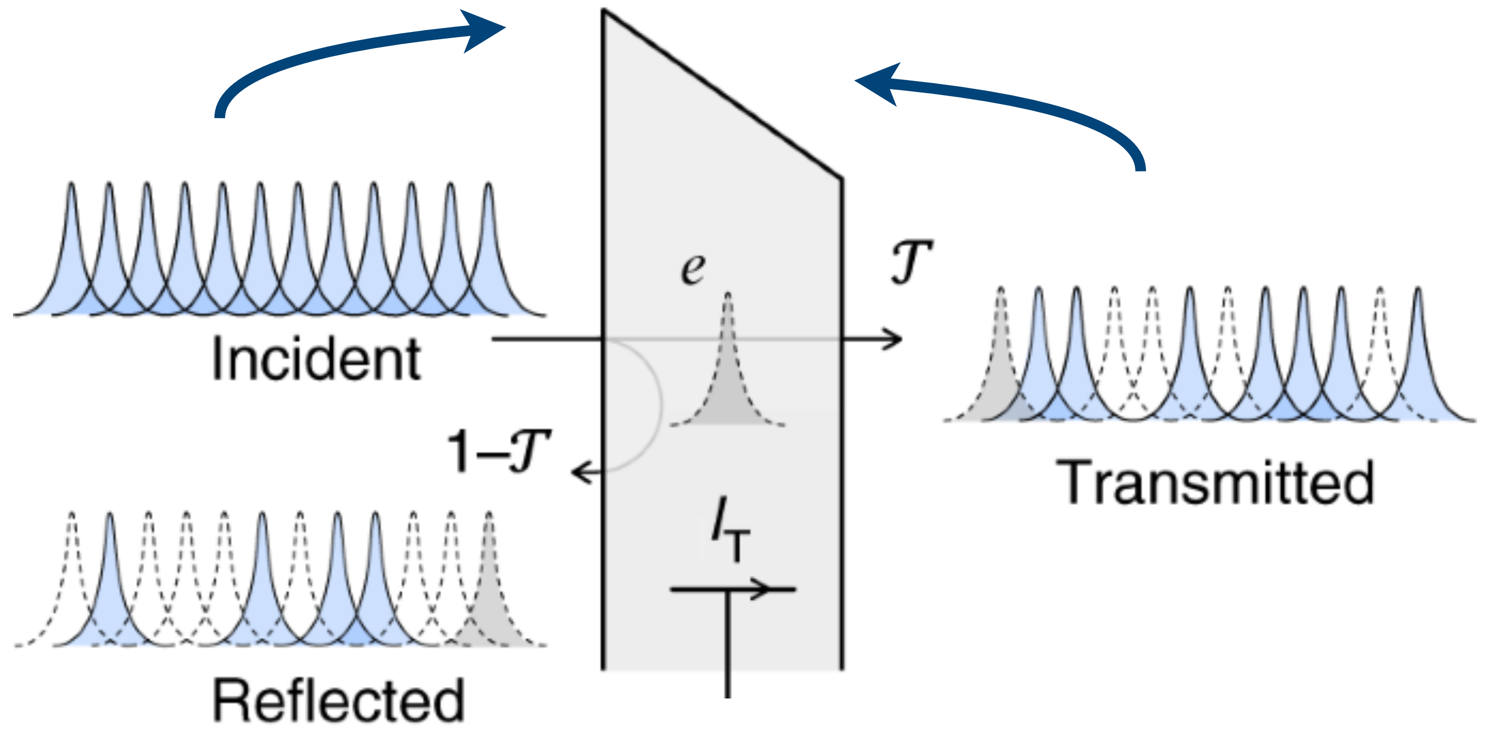
Büttikerian paradigm: conductors as electronic scatterers and waveguides

Self consistent approach



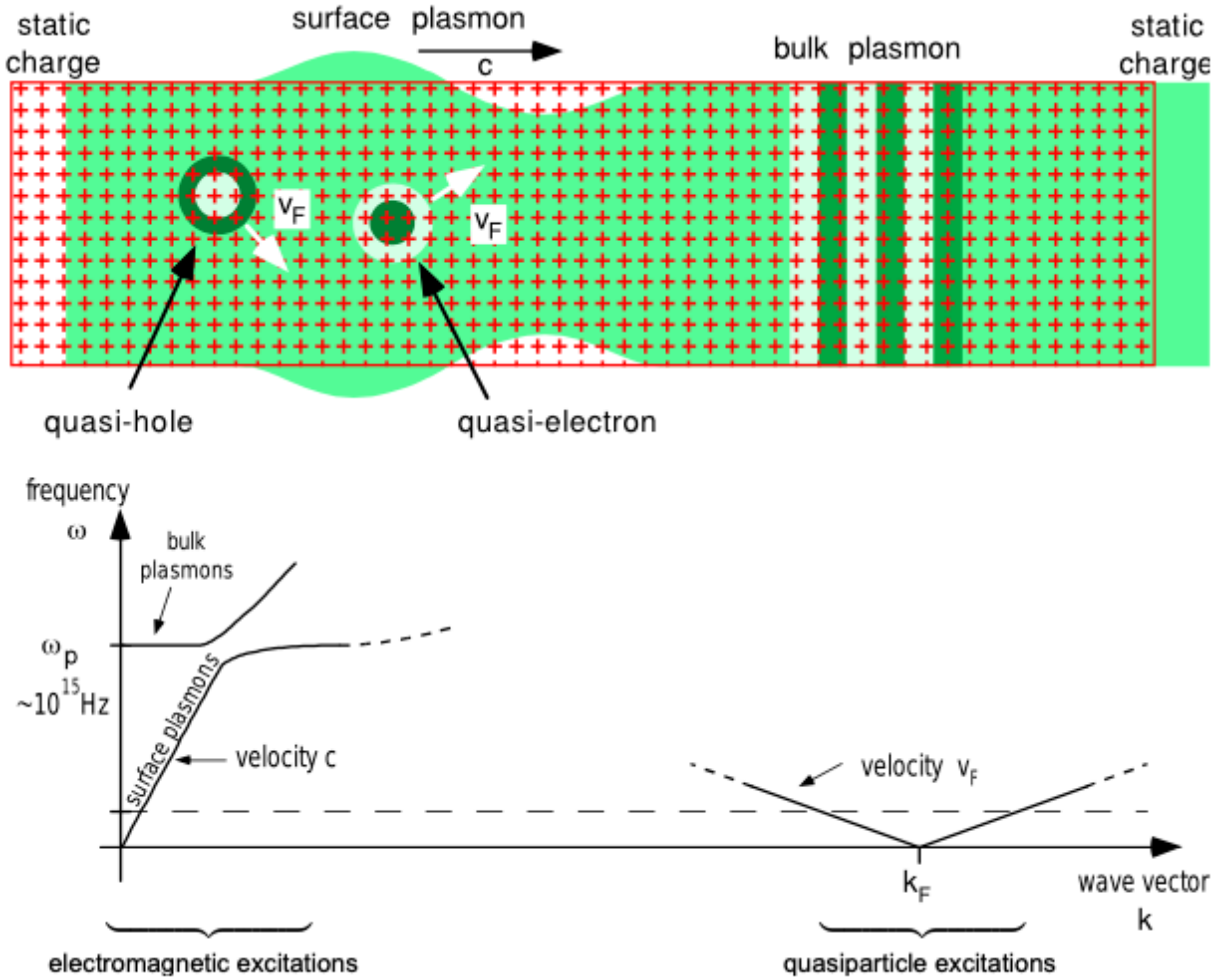
Büttikerian paradigm: conductors as electronic scatterers and waveguides

Self consistent approach



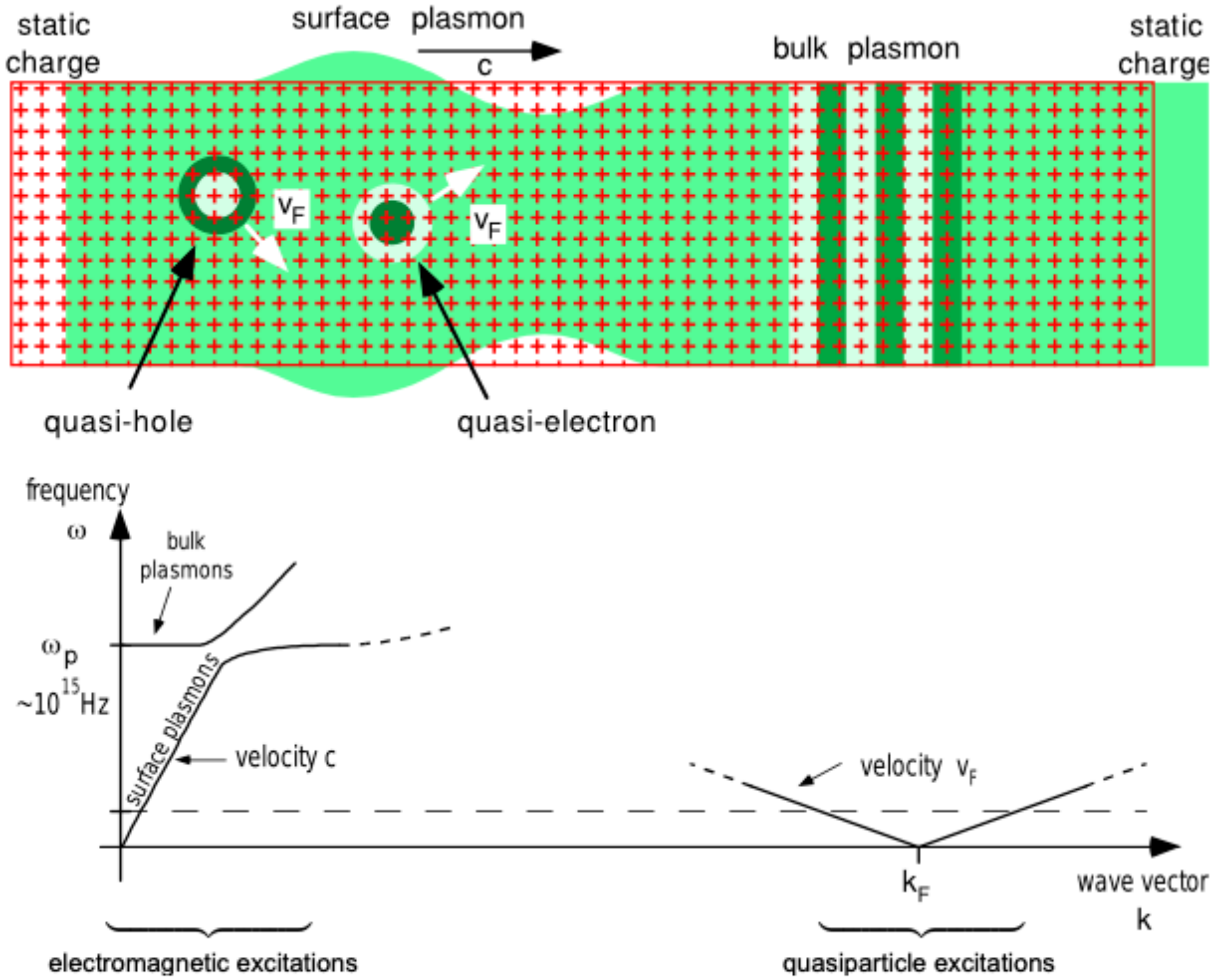
Question: what hides beyond this « classical scatterer » picture ?

Excitations of the electronic fluid in a metal



Ph. Joyez, *Introduction to quantum circuits* (Univ. Denis Diderot, Master DQ)

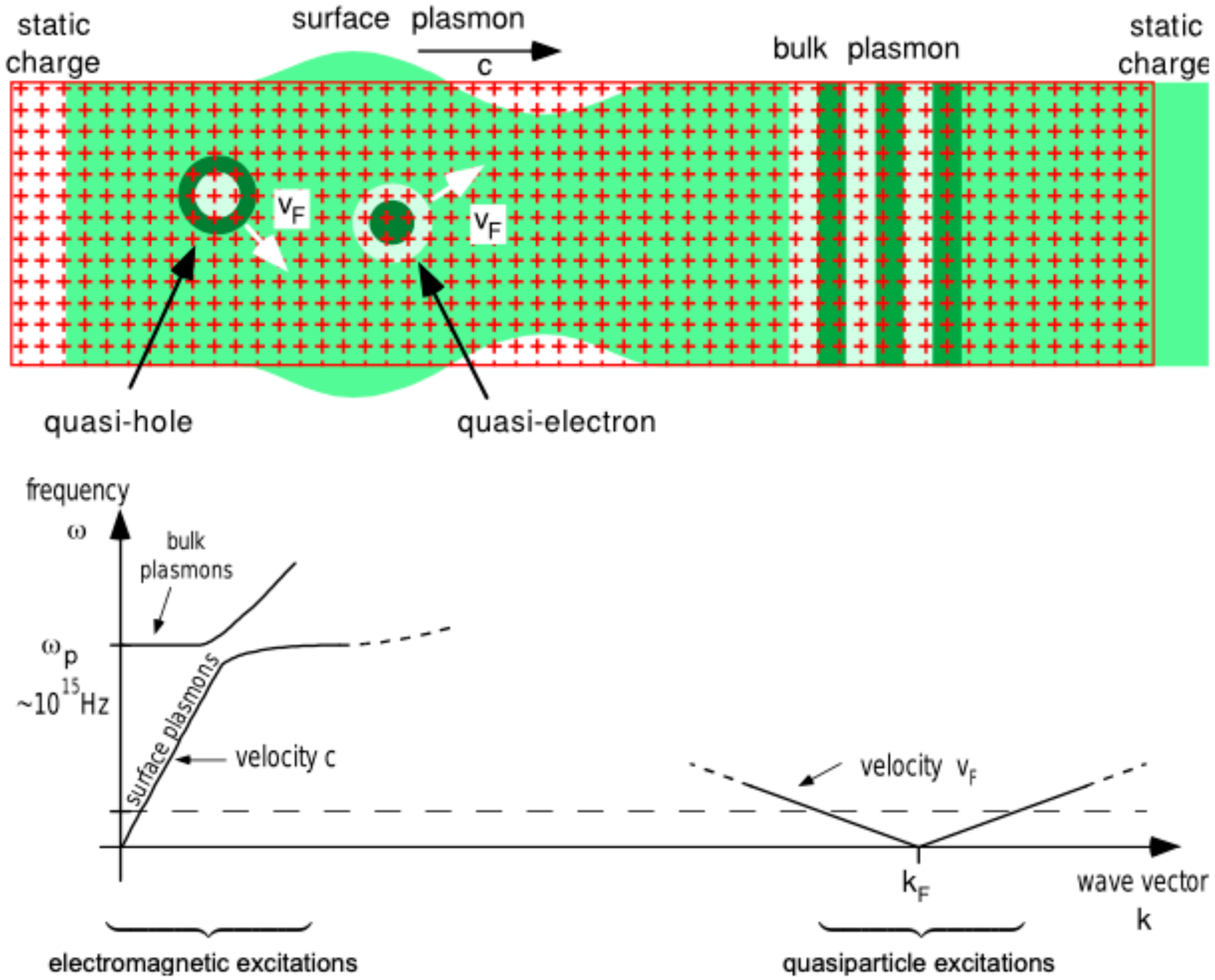
Excitations of the electronic fluid in a metal



Static charges: role in dc transport.

Ph. Joyez, *Introduction to quantum circuits* (Univ. Denis Diderot, Master DQ)

Excitations of the electronic fluid in a metal



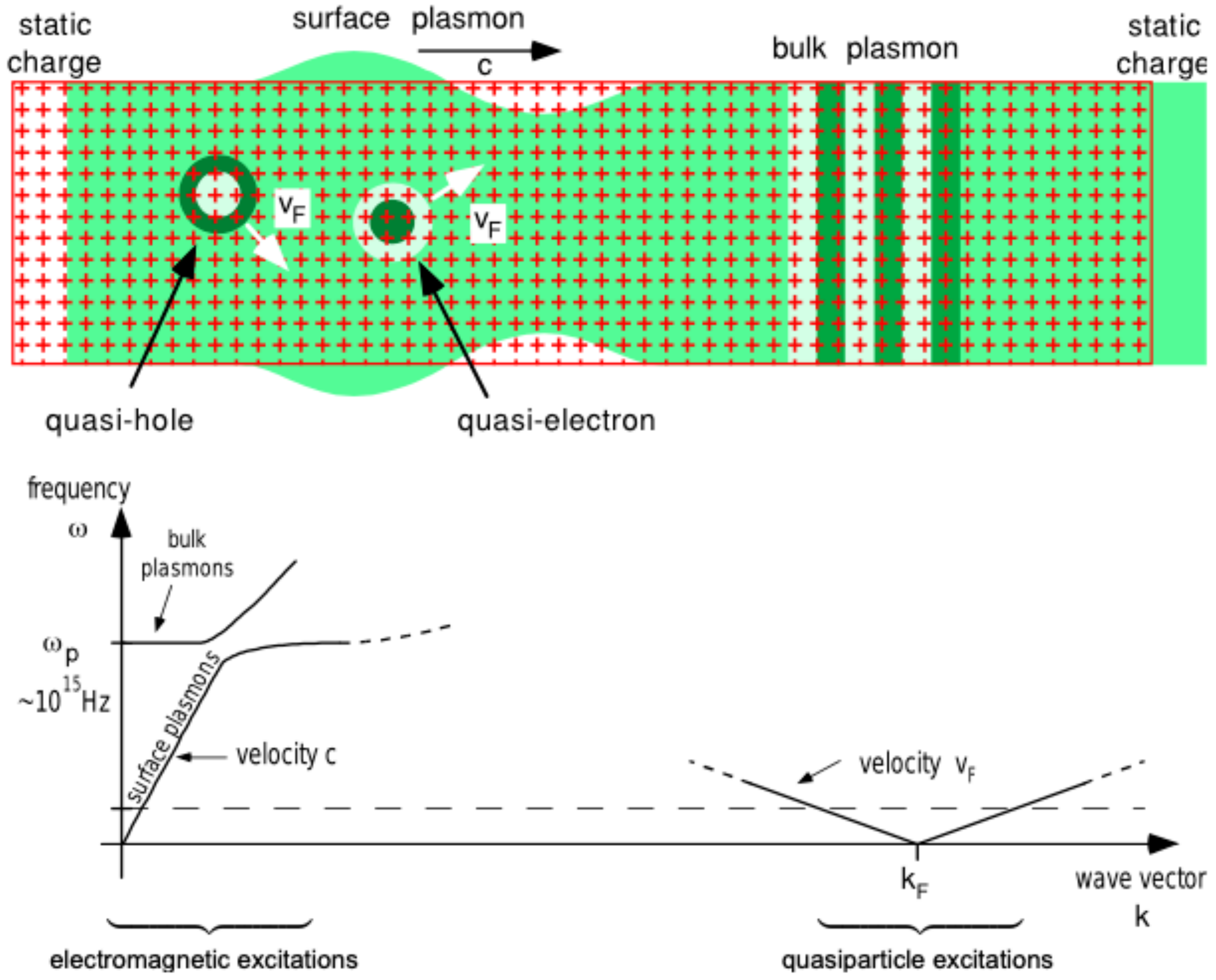
Static charges: role in dc transport.

Low energy plasmons (surface charge density waves): role in ac transport and transient regimes. *Hybridize with the EM field*. Coulomb couplings in mesoscopic systems.

Ph. Joyez, *Introduction to quantum circuits* (Univ. Denis Diderot, Master DQ)

Quantum currents and photons

Excitations of the electronic fluid in a metal



Static charges: role in dc transport.

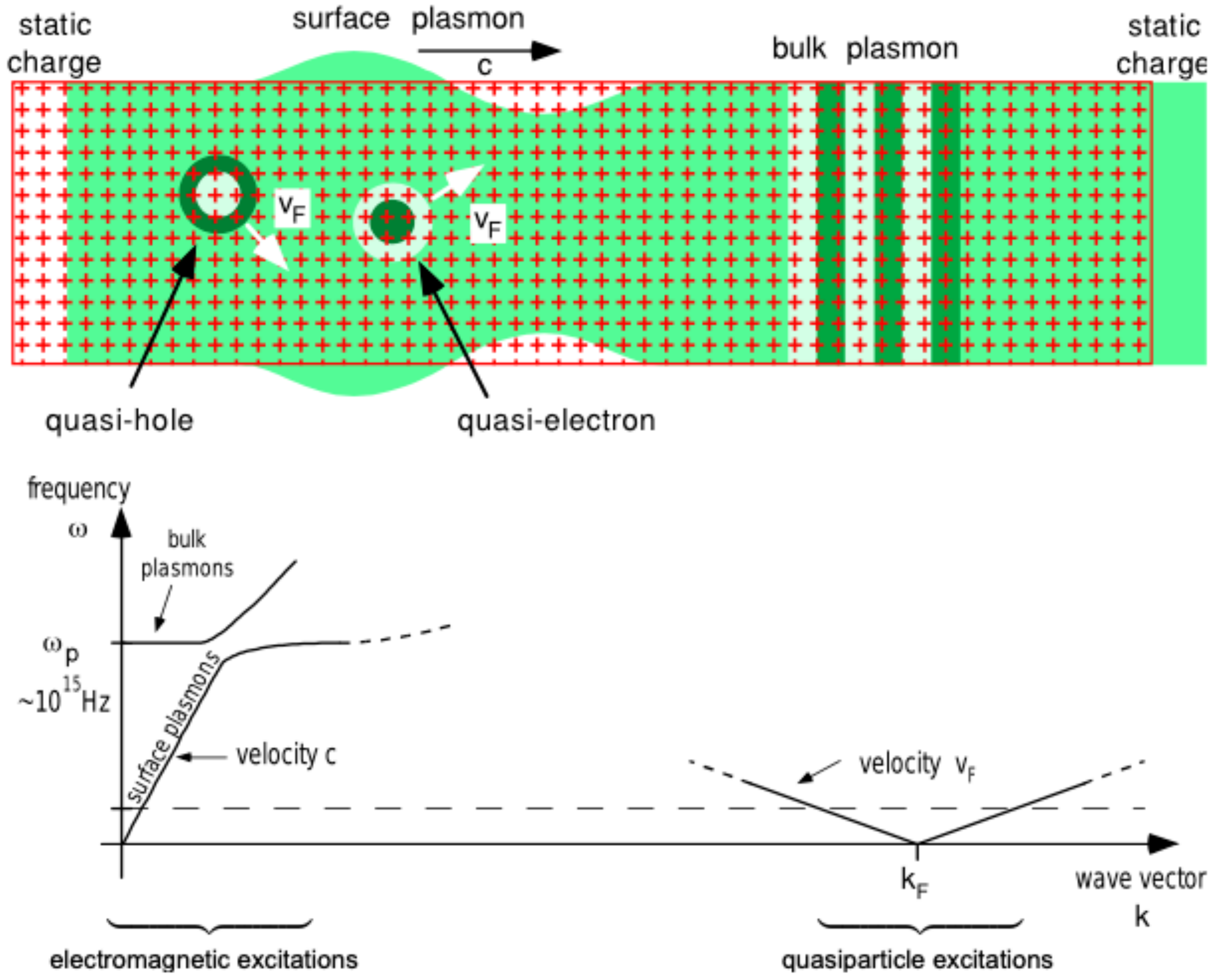
Low energy plasmons (surface charge density waves): role in ac transport and transient regimes. *Hybridize with the EM field*. Coulomb couplings in mesoscopic systems.

Bulk plasmons: optical frequencies (*too high for nano electronics*)

Ph. Joyez, *Introduction to quantum circuits* (Univ. Denis Diderot, Master DQ)

Quantum currents and photons

Excitations of the electronic fluid in a metal



Static charges: role in dc transport.

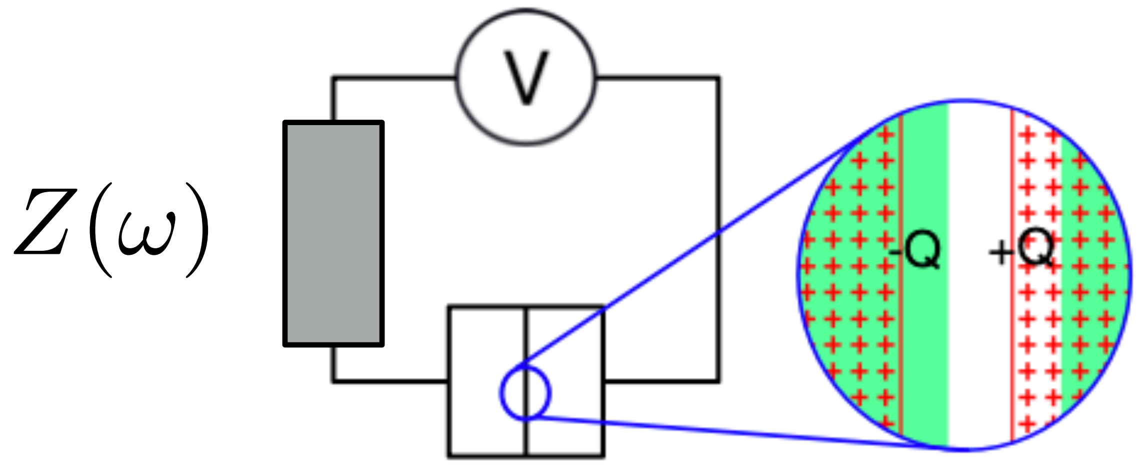
Low energy plasmons (surface charge density waves): role in ac transport and transient regimes. *Hybridize with the EM field*. Coulomb couplings in mesoscopic systems.

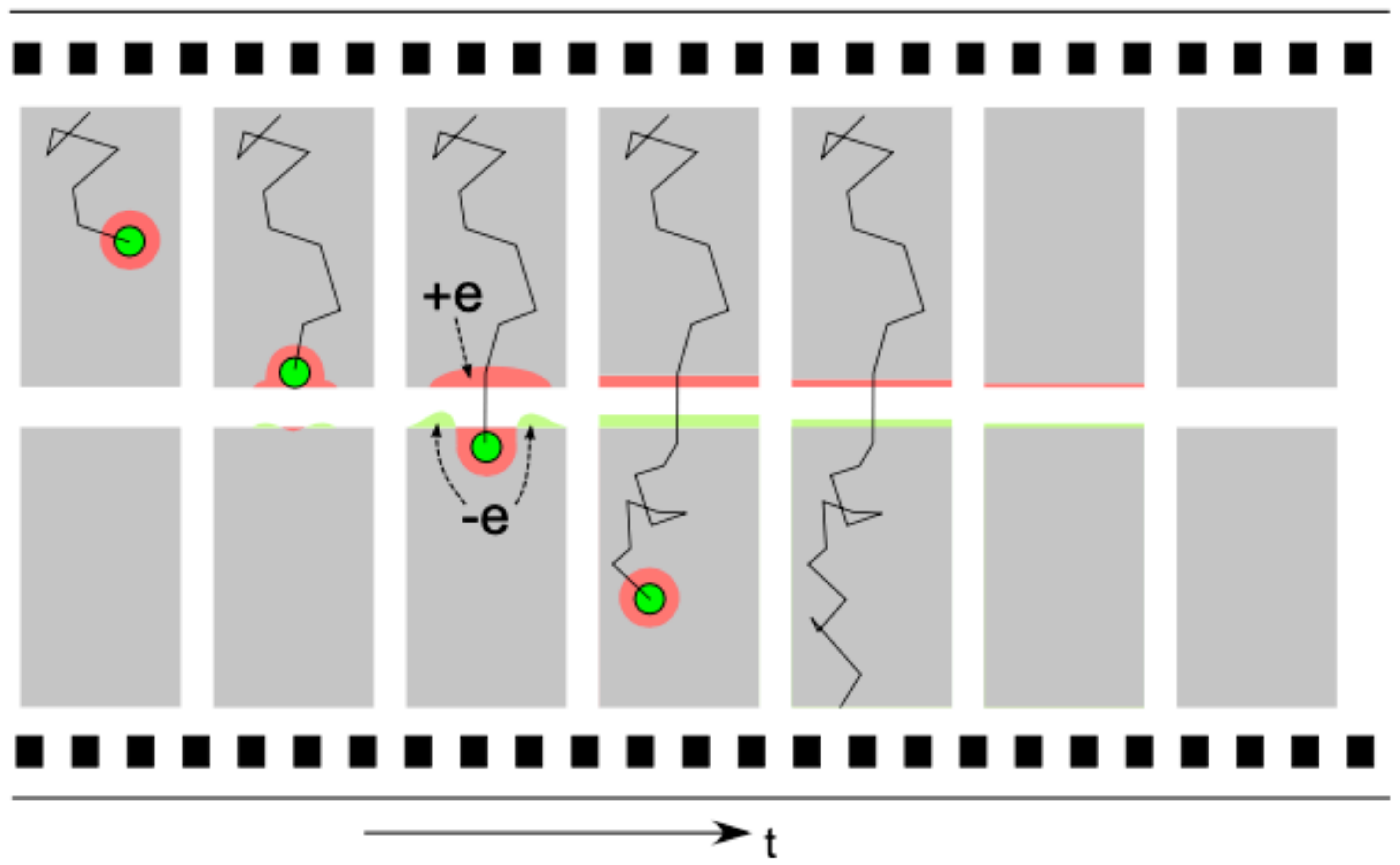
Bulk plasmons: optical frequencies (*too high for nano electronics*)

Quasi-particles: only feel a Coulomb screened interactions (*electron scattering approach*)

Ph. Joyez, *Introduction to quantum circuits* (Univ. Denis Diderot, Master DQ)

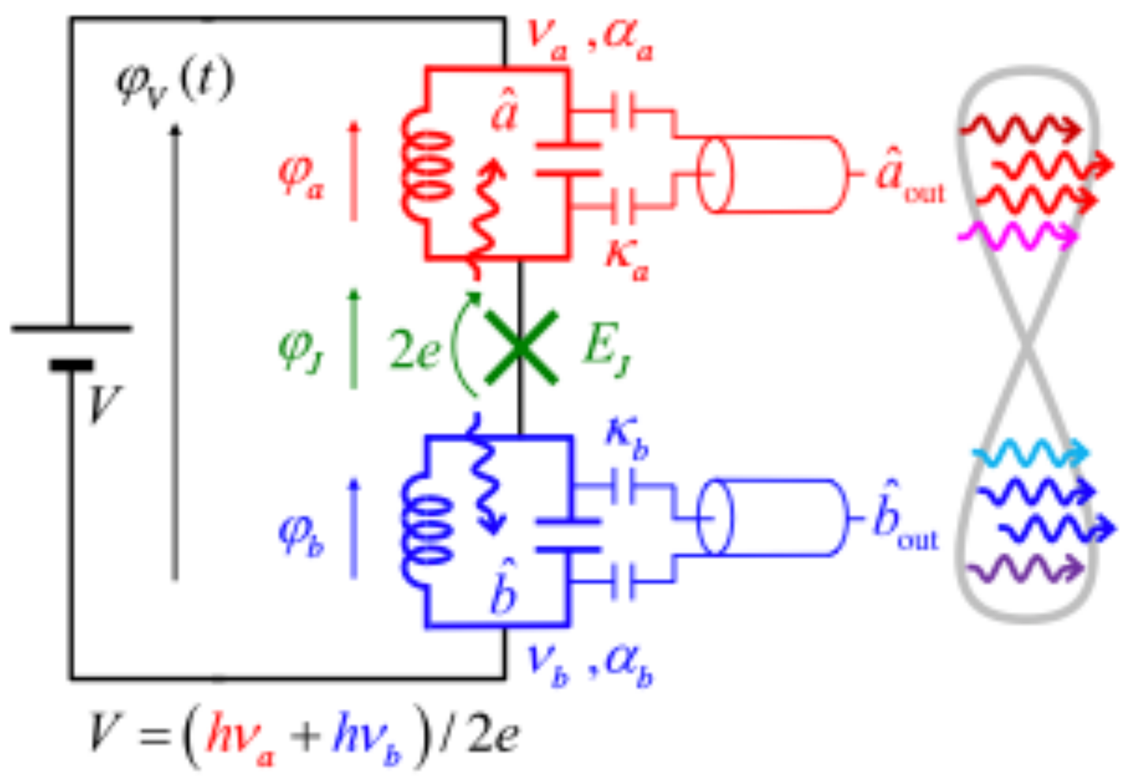
Dynamical Coulomb blockade





See F. Pierre's talk (9/4/2025)

Can be used to generate quantum microwave radiation



A. Peugeot *et al*, Phys. Rev. X **11**, 031008 (2021)

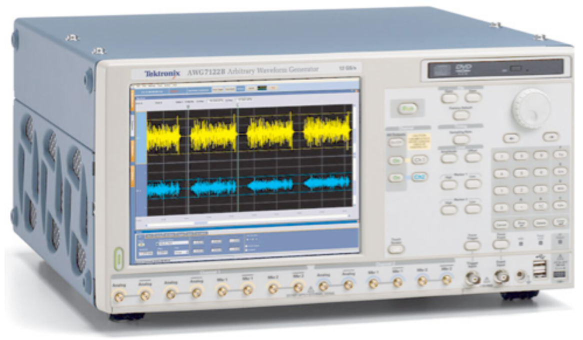
Electron quantum optics

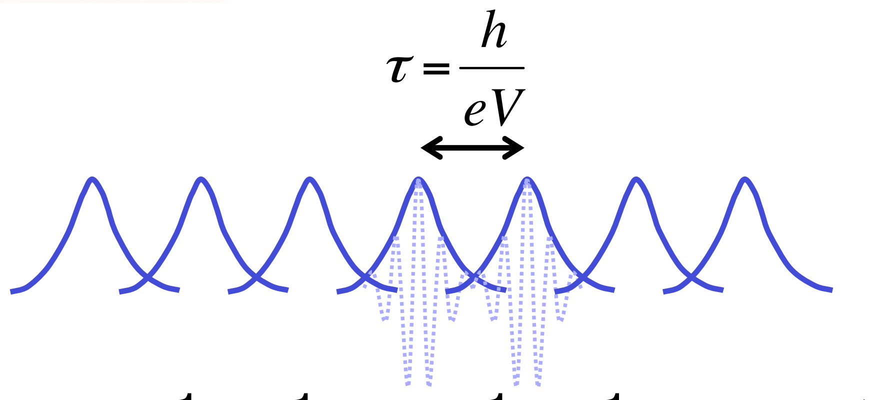
Electron quantum optics

Coherent nano-electronics:
many electrons sources

dc source

ac source





Many overlapping electrons!

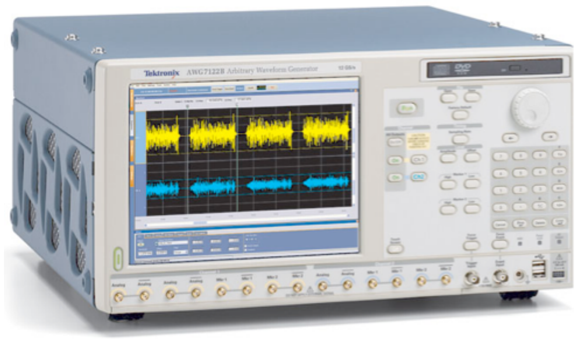
Electron quantum optics

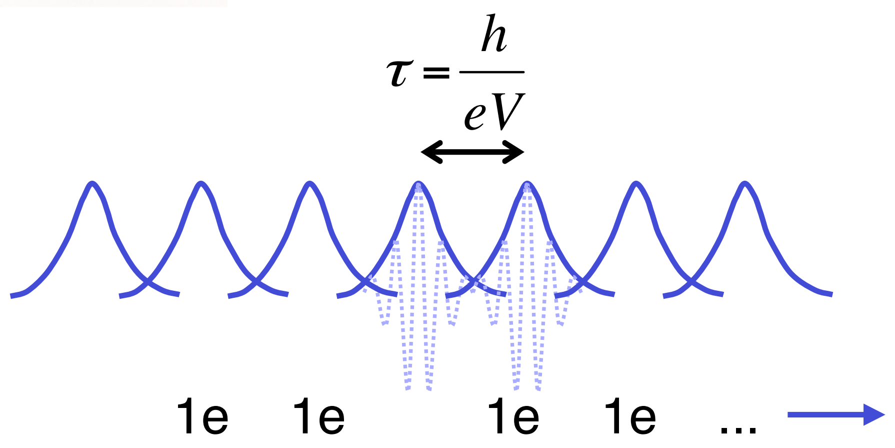
Coherent nano-electronics:
many electrons sources

Electron quantum optics:
single or few electrons sources

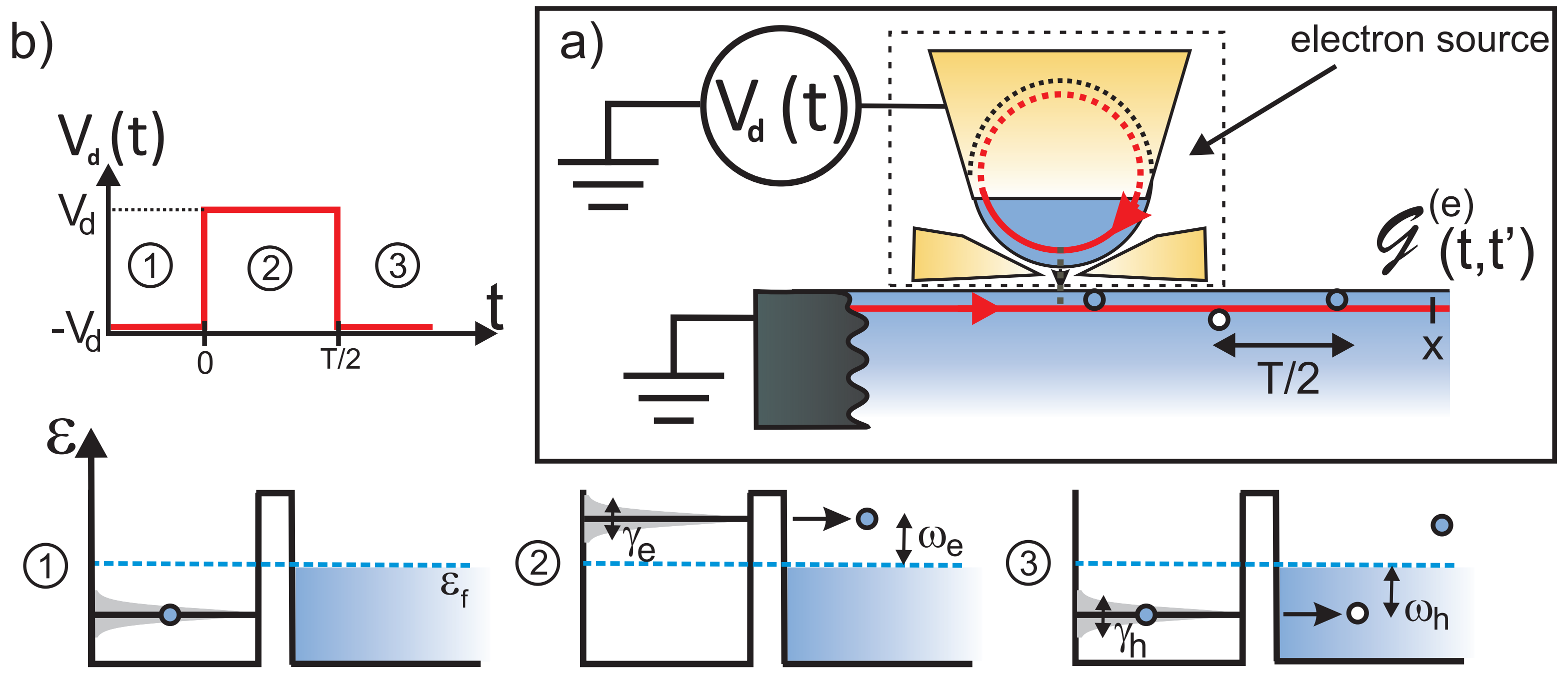
dc source

ac source



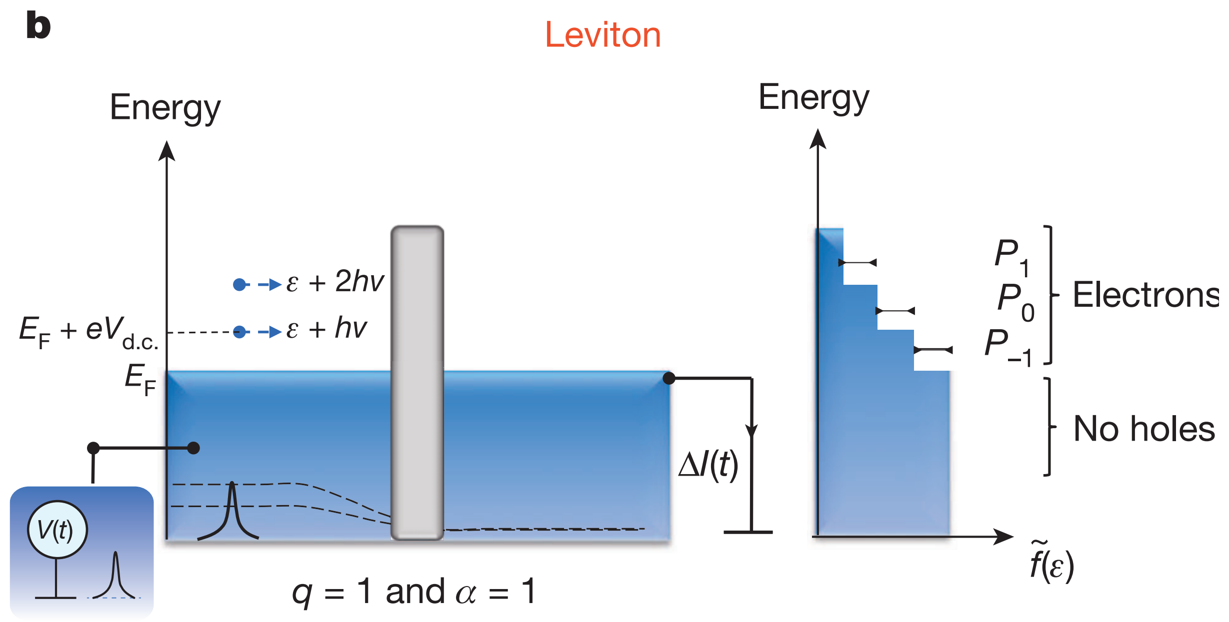


Landau excitation source:



G. Fève *et al*, Science **316**, 1169 (2007)

Leviton source:



J. Dubois *et al*, Nature **502**, 659 (2013)

Many overlapping electrons!

Well separated electronic excitations

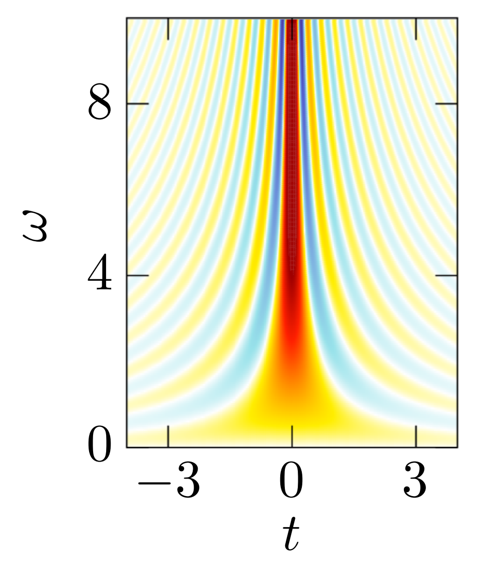
E. Bocquillon *et al*, Ann. Phys. (Berlin) **526**, 1-30 (2014)

See G. Fève's talk (16/04/2025)

Quantum electrons, currents and photons

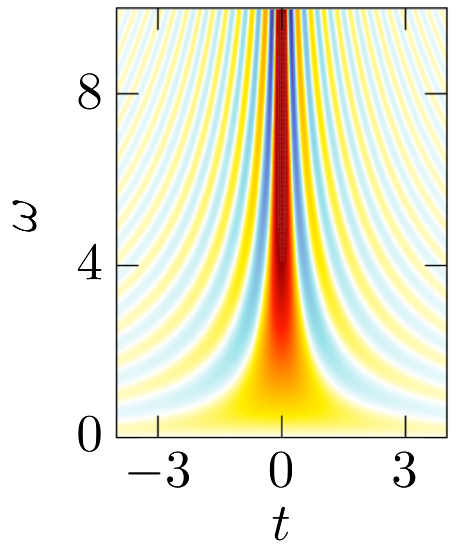
Quantum electrons, currents and photons

$\psi^\dagger(0)|F\rangle$

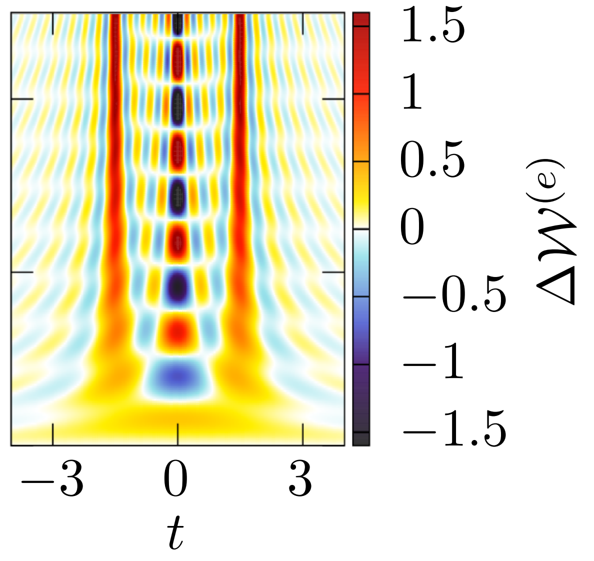


Quantum electrons, currents and photons

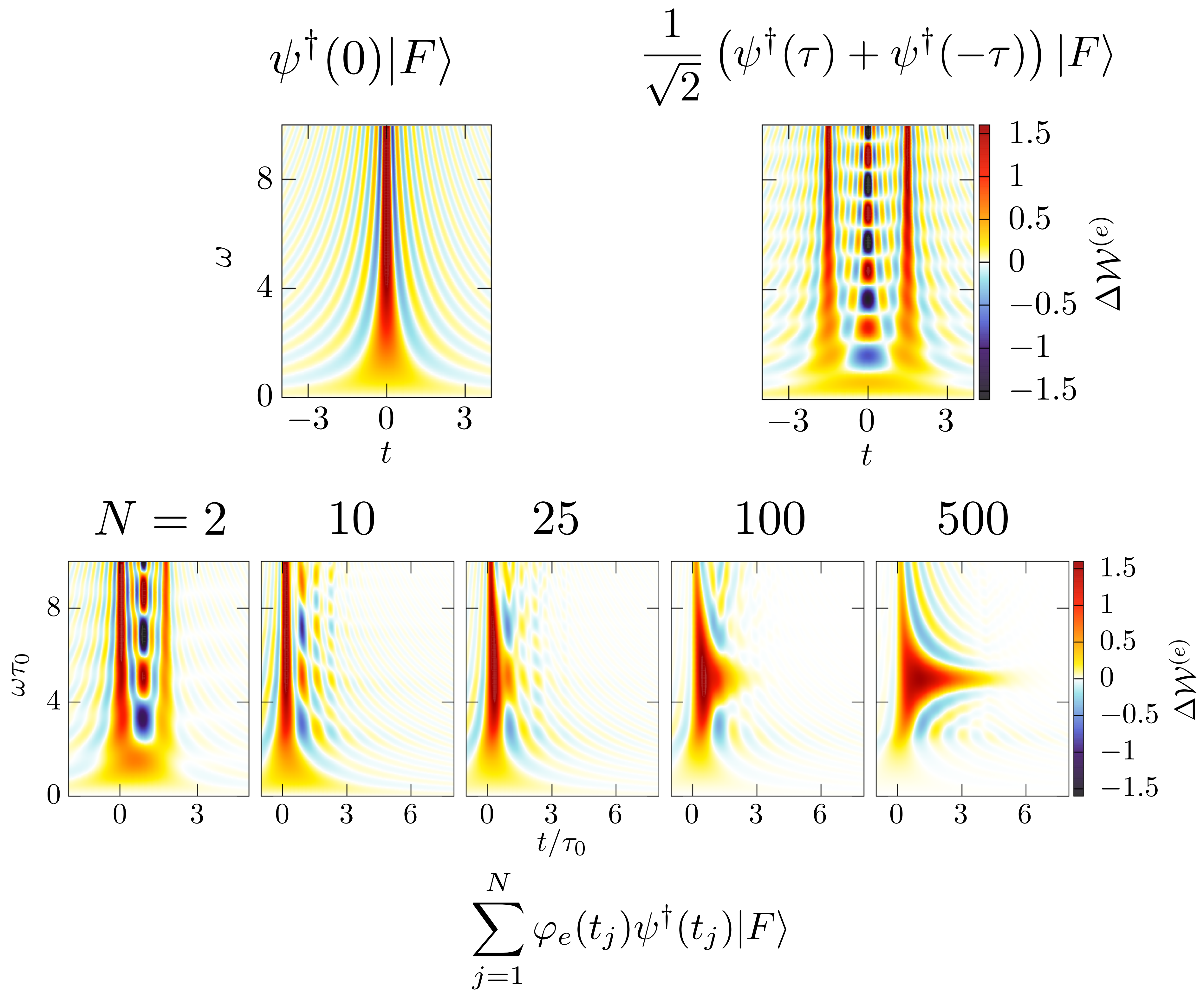
$$\psi^\dagger(0)|F\rangle$$



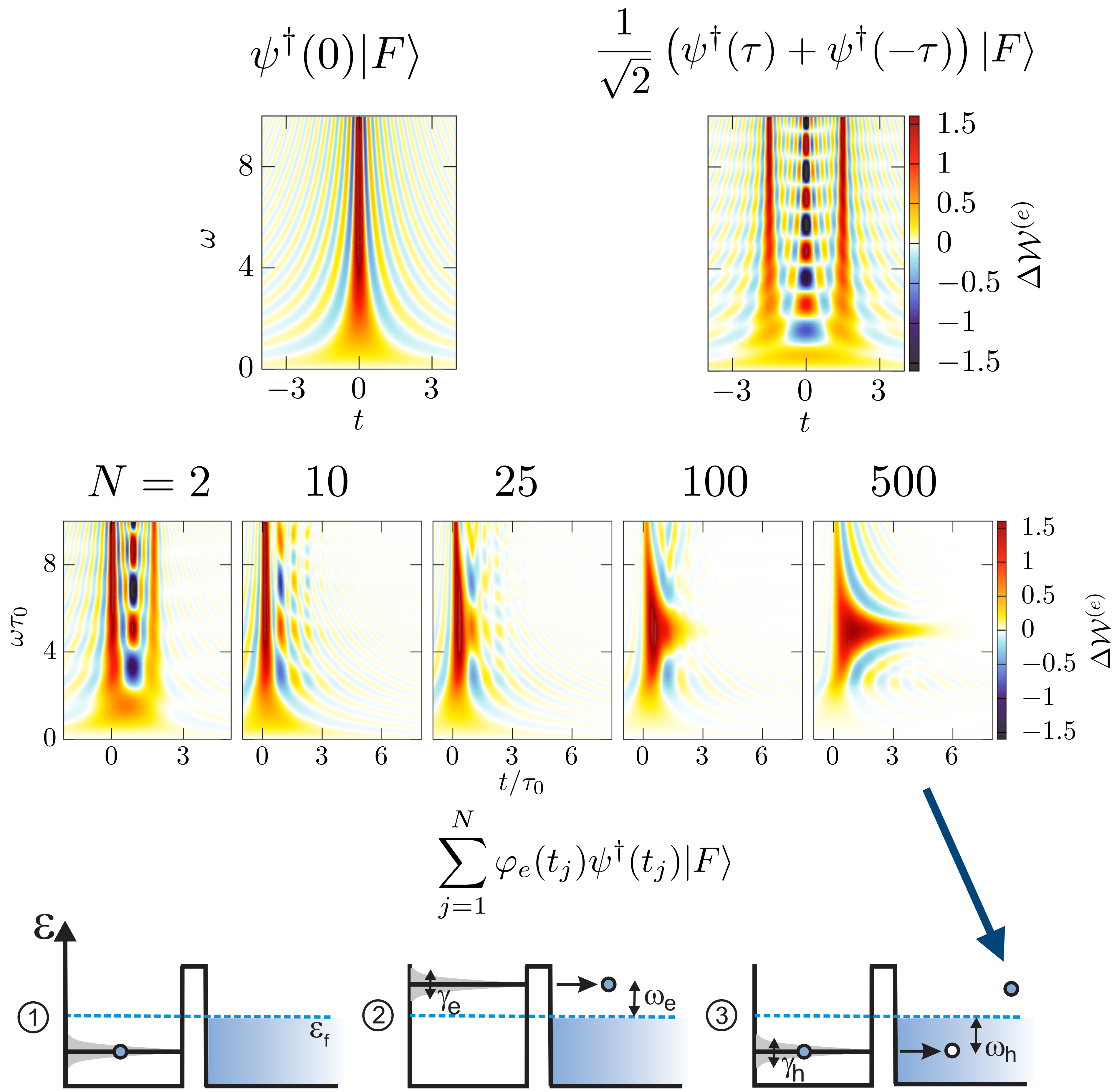
$$\frac{1}{\sqrt{2}}\left(\psi^\dagger(\tau)+\psi^\dagger(-\tau)\right)|F\rangle$$



Quantum electrons, currents and photons

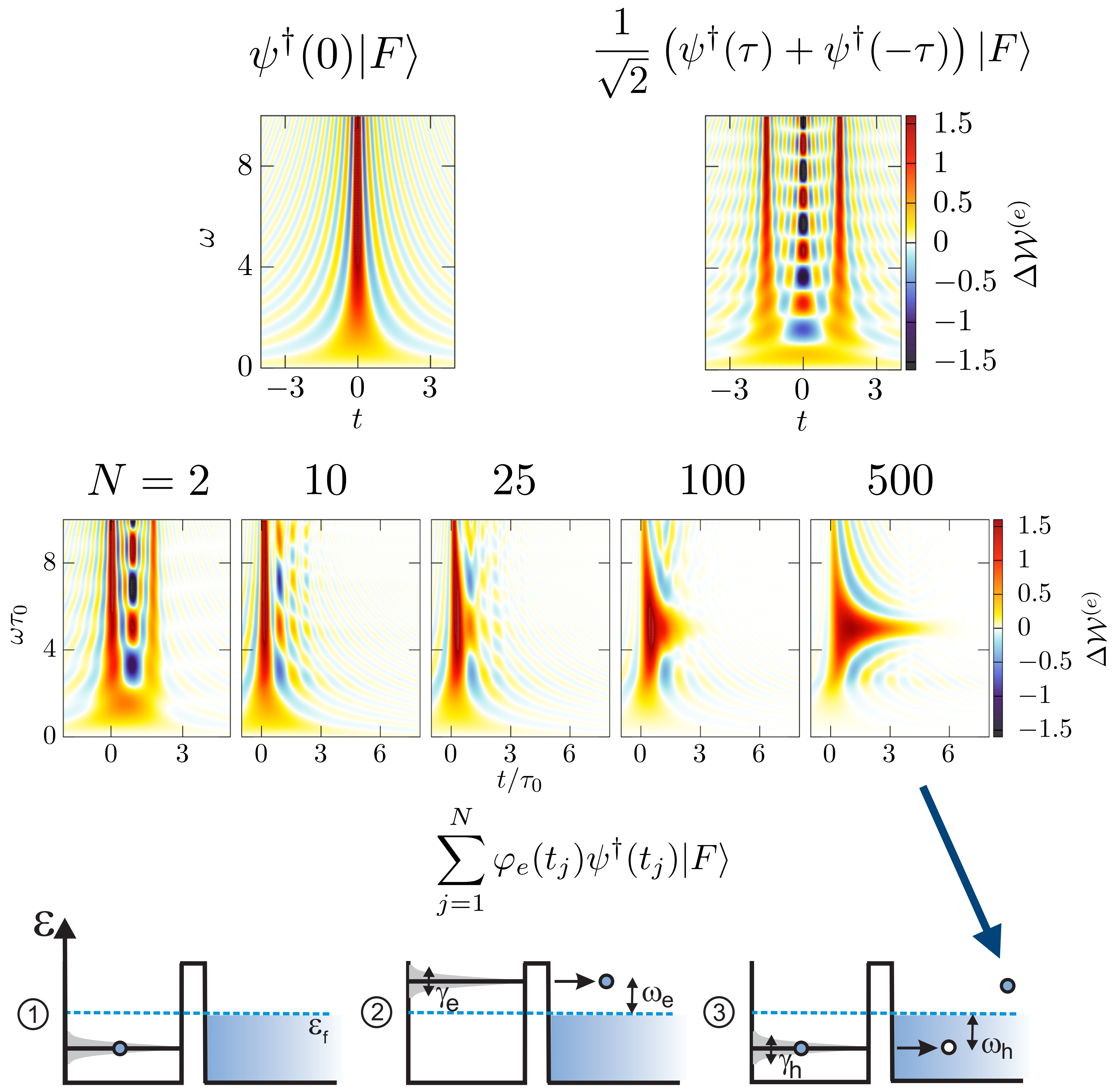


Quantum electrons, currents and photons



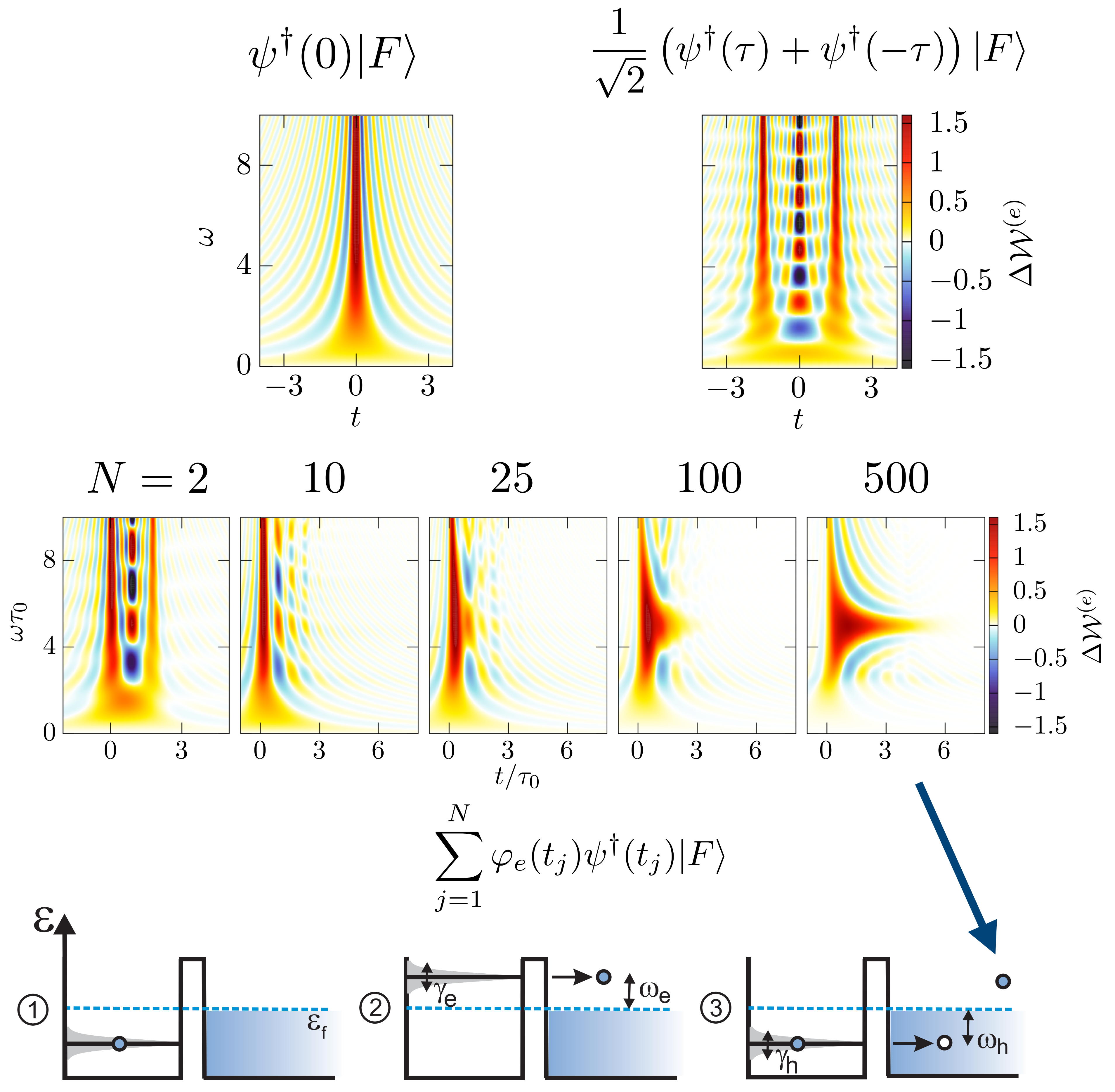
Quantum electrons, currents and photons

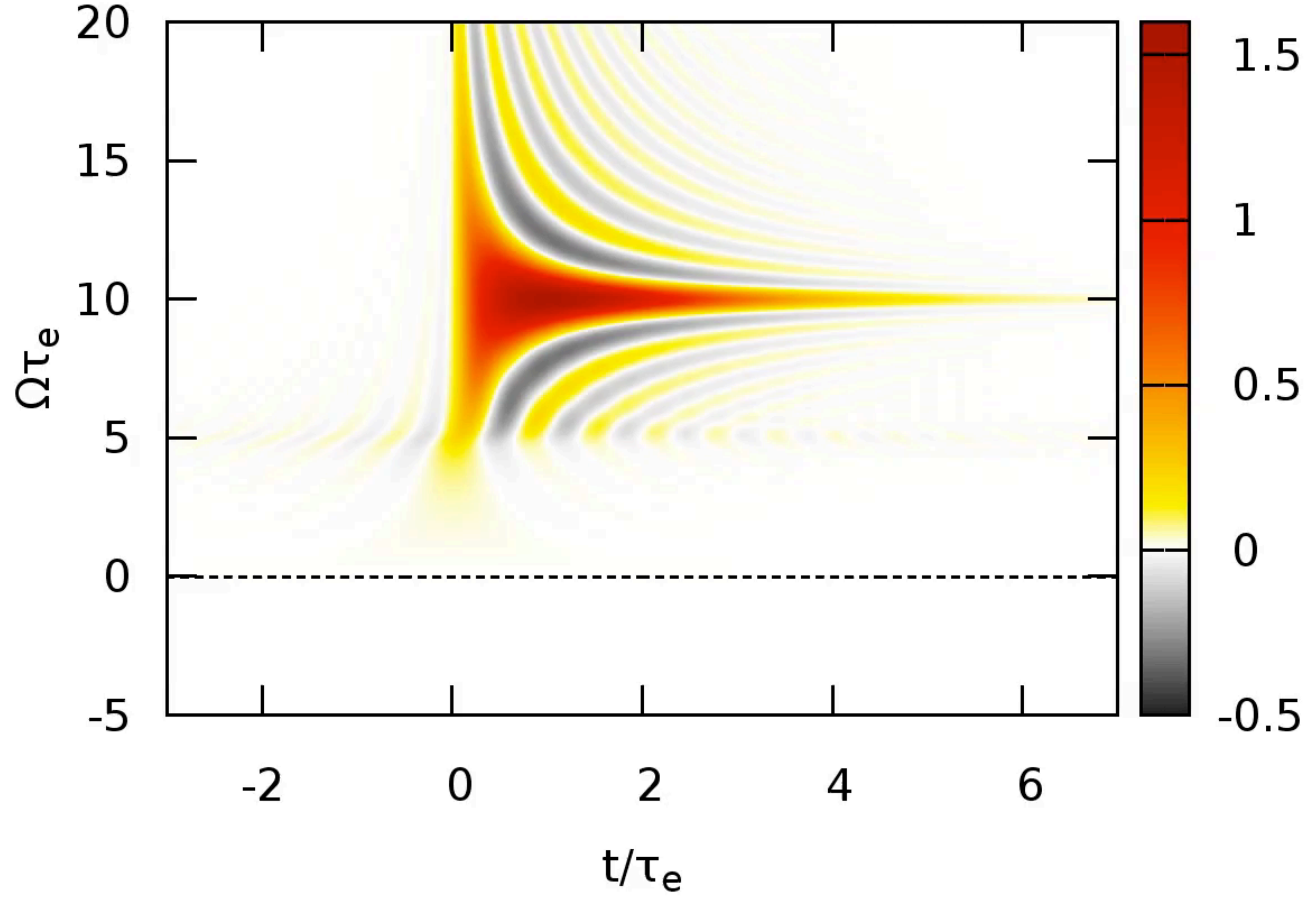
Single electron excitations are generically quantum plasmonic states !



Quantum electrons, currents and photons

Single electron excitations are generically quantum plasmonic states !





Key message on fully quantum electrical circuits

Fully quantum electricity is QED on a chip !

Outline

- From Ampère to electrical circuits
- Quantum physics within electricity
- Towards quantum coherent electronics
- Presentation of the PCP 2025 cycle

Depuis Ampère jusqu'à l'électronique quantique

Hughes Pothier (CEA Saclay/SPEC) 19 mars 2025 (16h30, Amphi Dirac)

La supra-conductivité, de sa découverte aux circuits quantiques

Lucian Prejbeanu (CEA Grenoble/Spintec) 2 avril 2025 (16h30, Amphi Ampère)

Innovations spintroniques pour un numérique frugal et agile

Depuis Ampère jusqu'à l'électronique quantique

Frédéric Pierre (C2N Palaiseau)

9 avril 2025 (16h30, Amphi Ampère)

Circuits quantiques composites: des nouvelles lois du transport à la simulation quantique

Gwendal Fève (LPENS Paris)

16 avril 2025 (16h30, Amphi Ampère)

L'optique quantique électronique: de l'électron unique aux anyons

Anne Lhuillier (Lund University)

30 avril 2025 (16h30, Amphi Ampère)

Mouvement des électrons dans les atomes à l'échelle attoseconde

Merci de votre attention…