

Flocks and crowds: active fluids

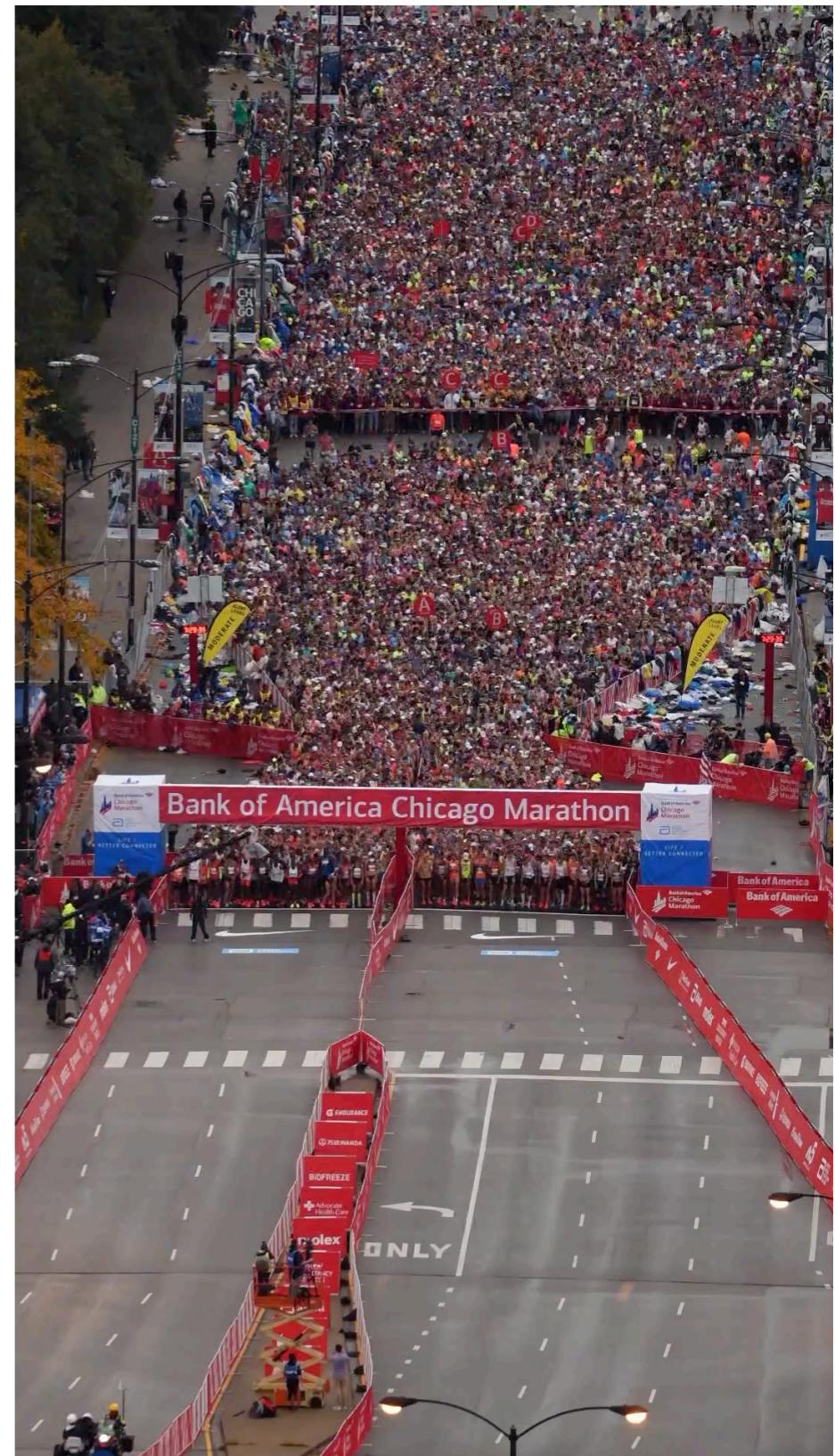
Denis BARTOLO

Laboratoire de Physique, ENS de Lyon

Flock hydraulics



Crowd hydrodynamics



How does active matter flow?

Active Matter

We are active solids



Marin Bartolo
Sardinia 2019

Fish school



Jake Butters and Denis Bartolo

Bird flock

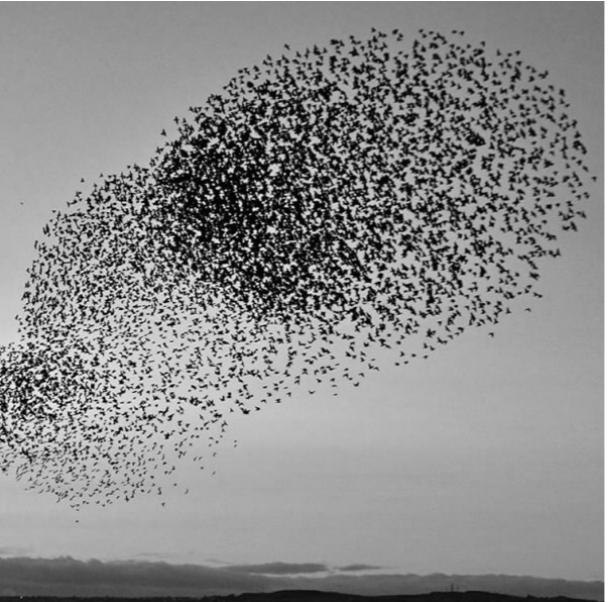


Starling flock, Roma (BBC)

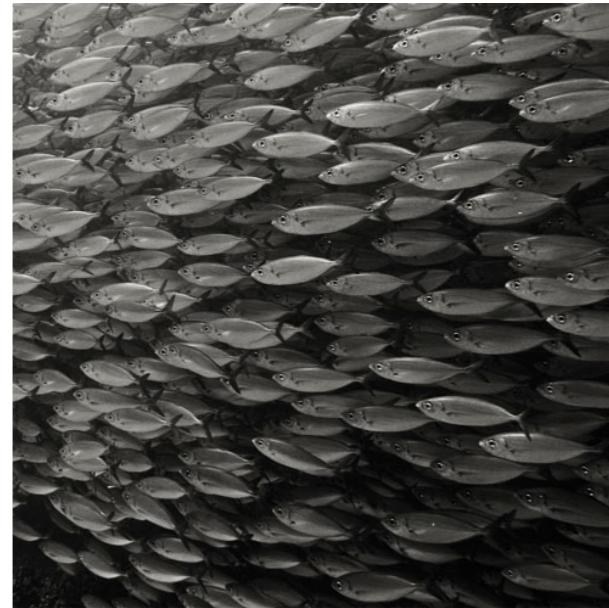
Locust swarm



Active liquids



F. Nureldine, AFP



Steve Dunleavy



National geographic



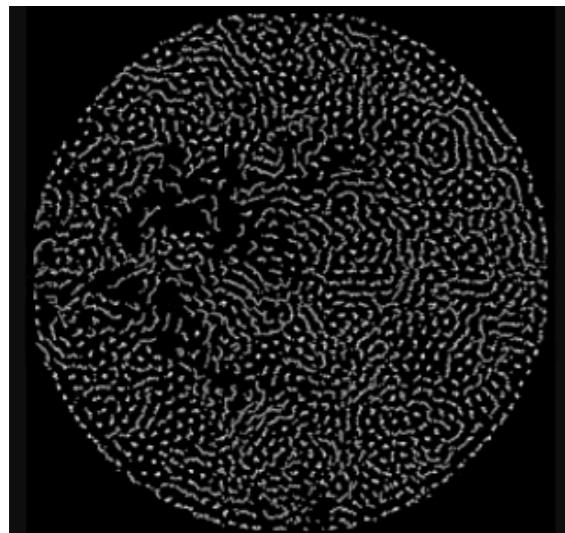
National geographic

Flocks, schools and herds
as spontaneously flowing liquids

Synthetic active matter

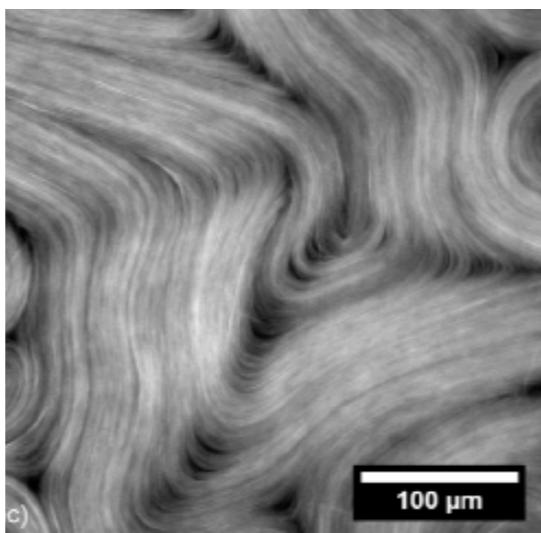
Synthetic active matter

Active emulsions



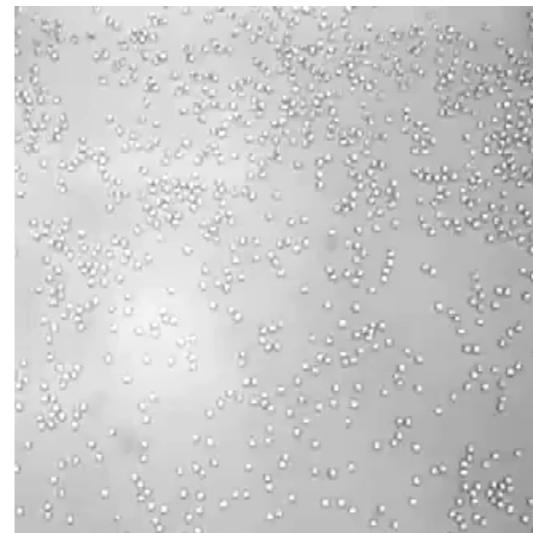
Thutupalli et al *NJP* (2011)

Active nematics



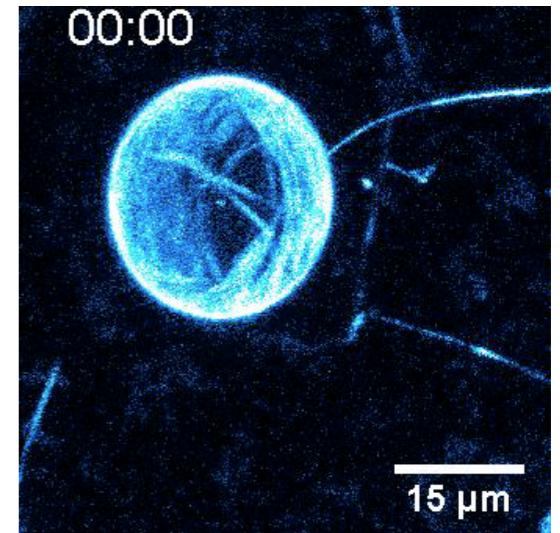
Sanchez et al *Nature* (2012)

Active colloids



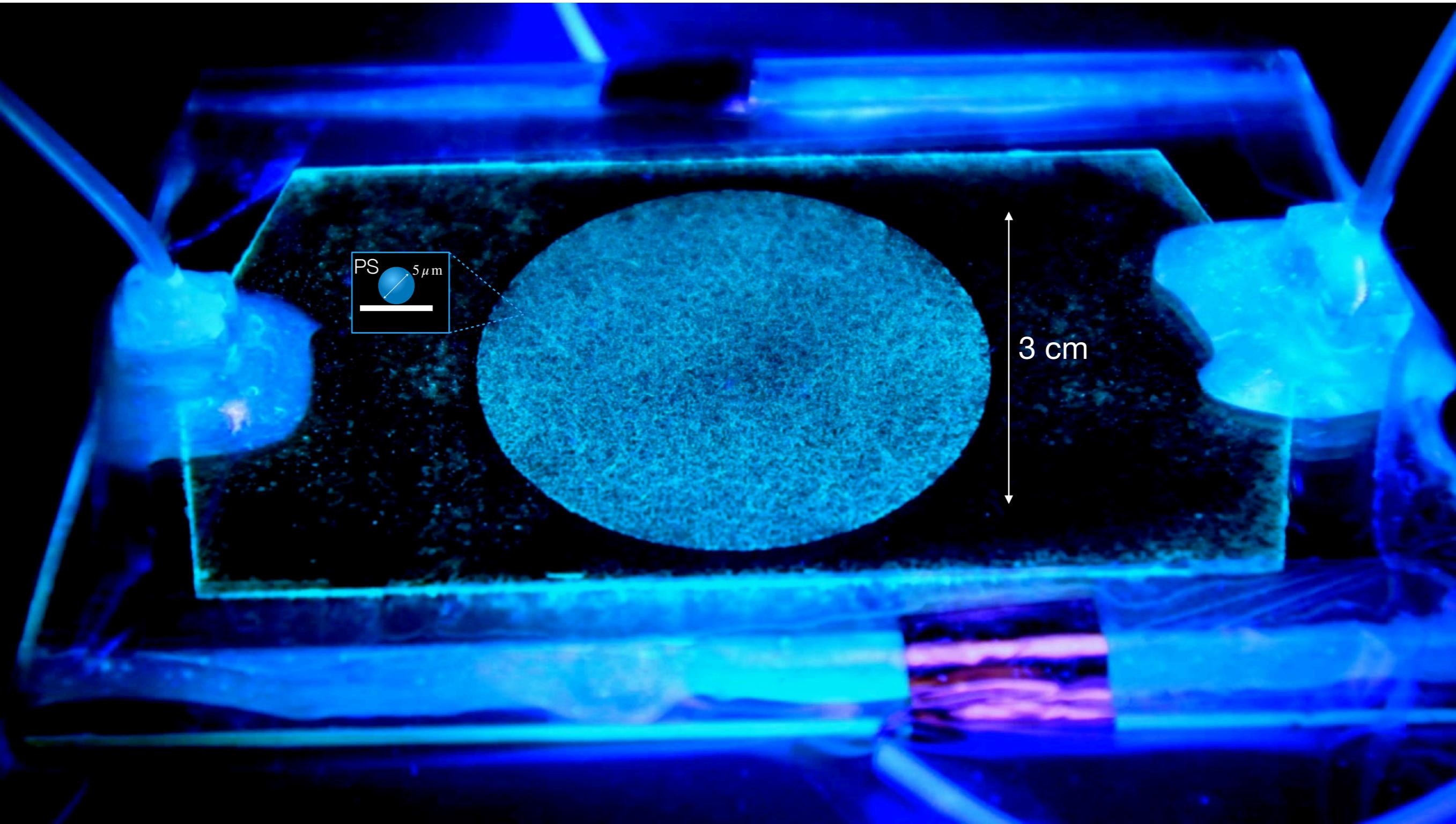
Palacci et al *Science* (2013)

Active membranes

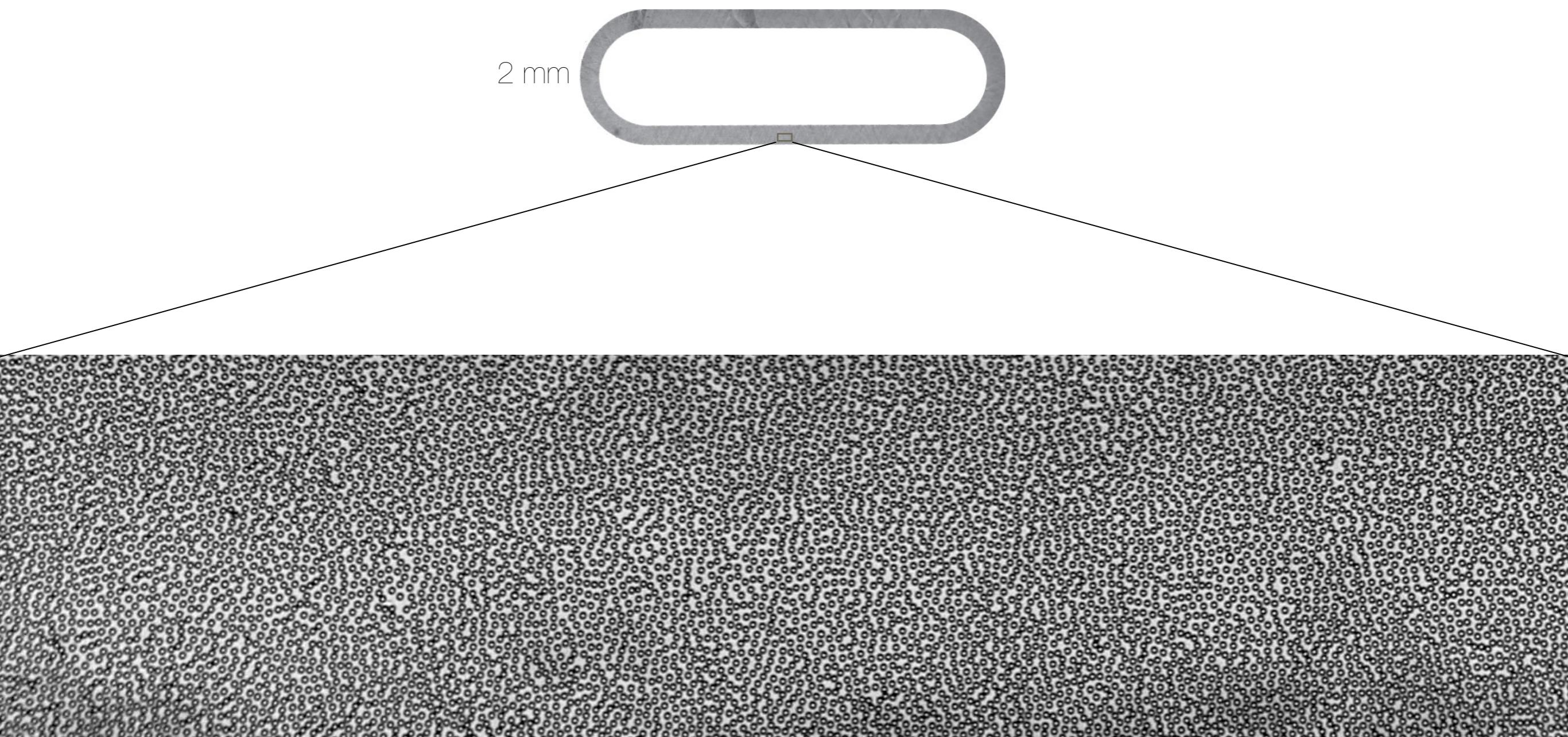


Keber et al *Science* (2014)

Synthetic flocks



Flocking fluids: laminar flows

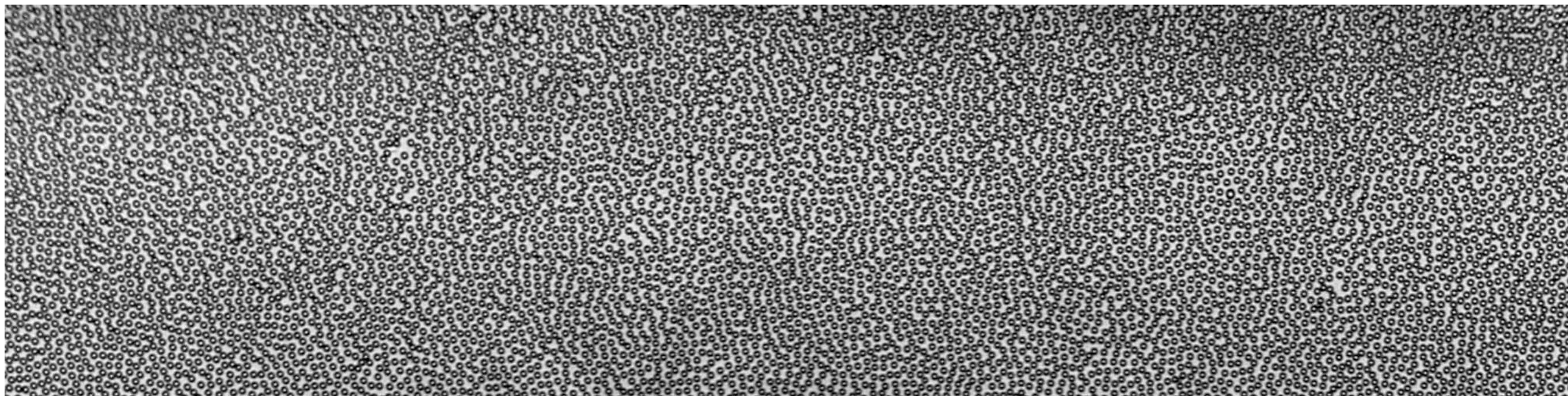


Colloidal rollers

$\phi \sim 3 \times 10^{-1}$

Engineering flocking fluids

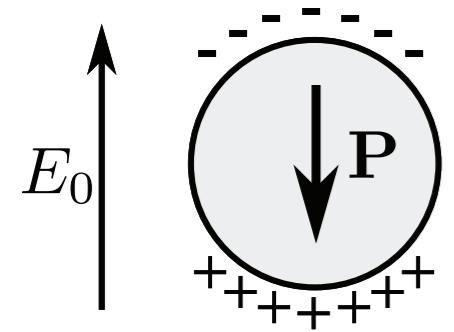
$$\phi \sim 3 \times 10^{-1}$$



Self-propelled units



Colloidal rollers



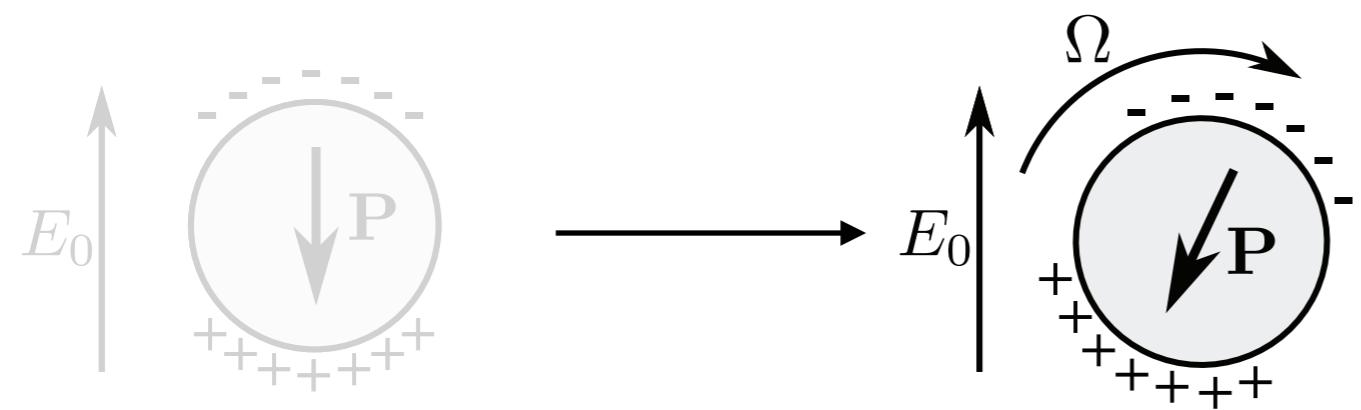
Insulating bead

Conducting fluid

DC **E** field

Colloidal rollers

Spontaneous rotation



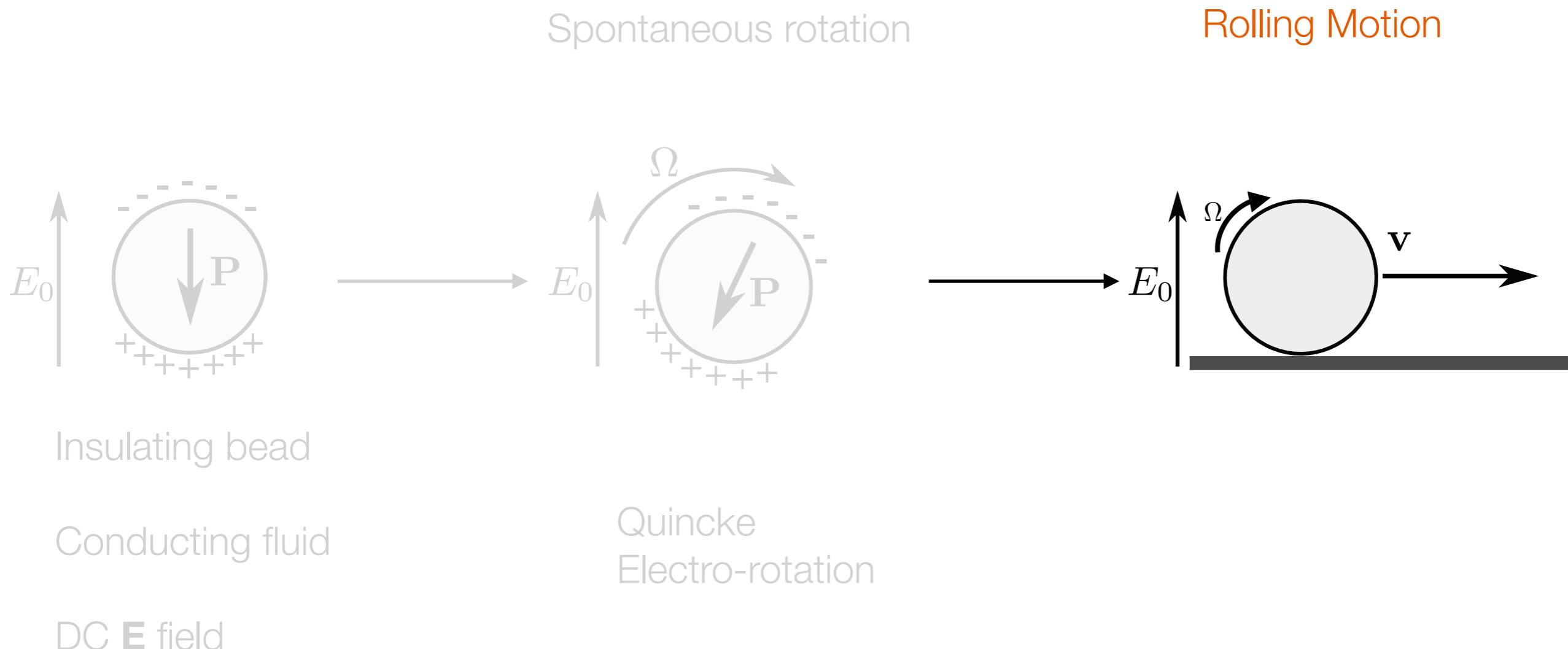
Insulating bead

Conducting fluid

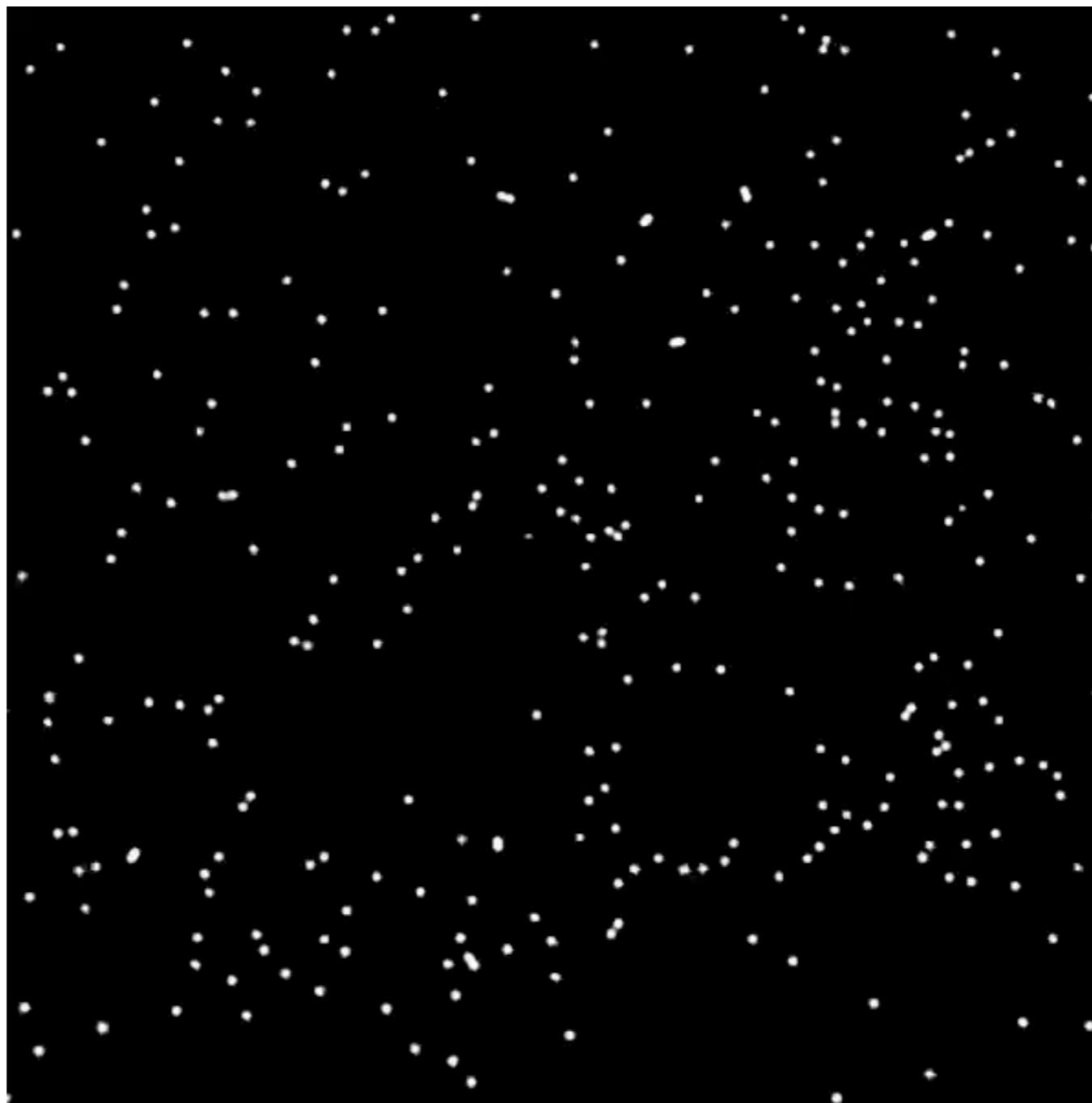
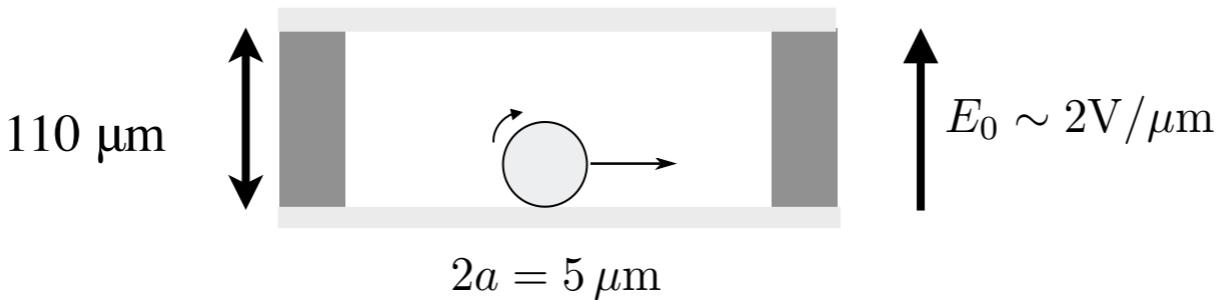
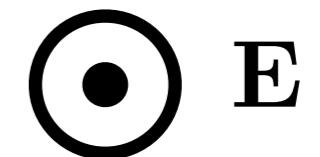
DC **E** field

Quincke
Electro-rotation

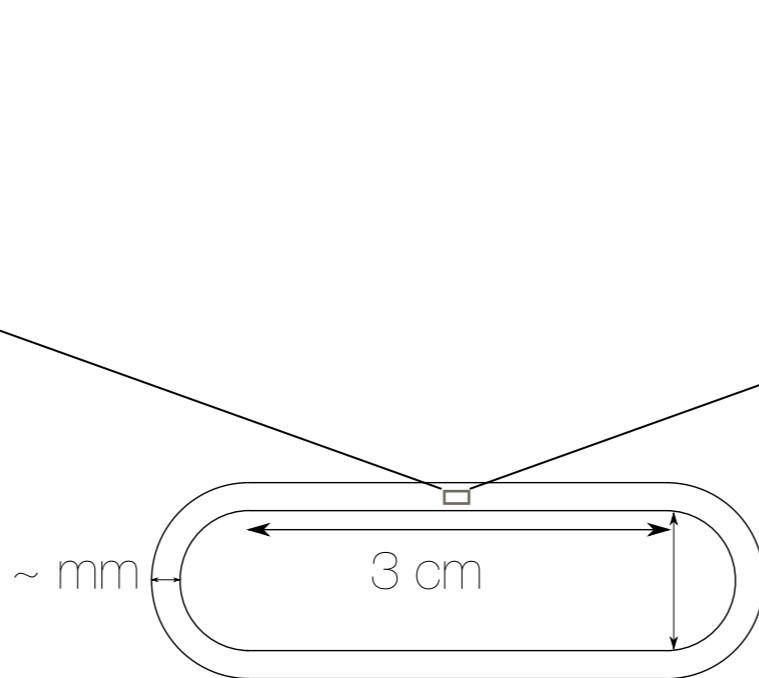
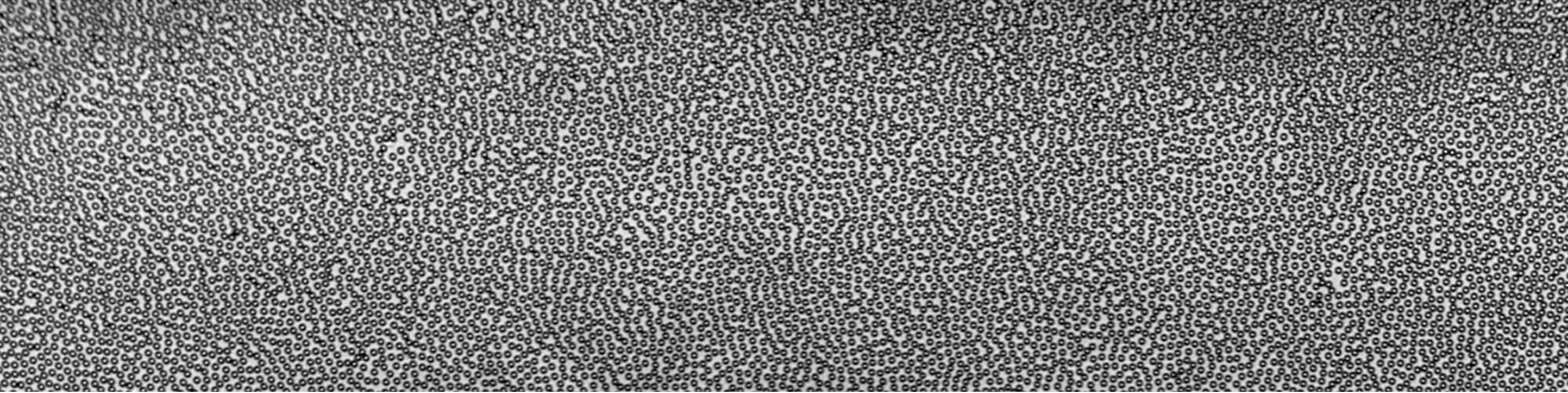
Colloidal rollers



Quincke rollers

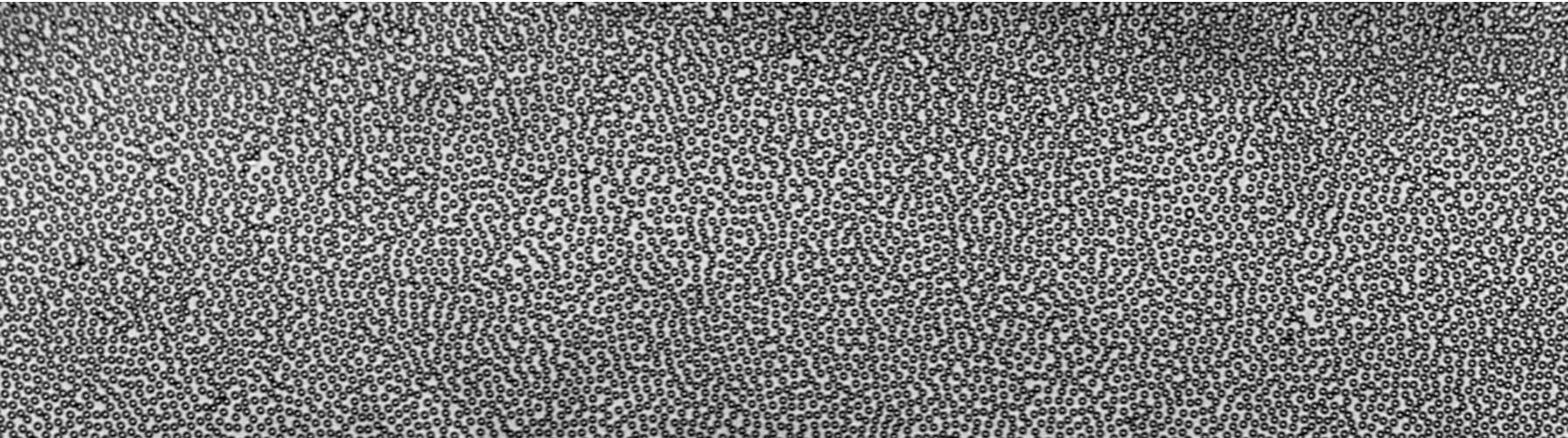
 Ω  \mathbf{E} $\phi \sim 10^{-4}$

Flocking transition



$$\phi \sim 3 \times 10^{-1}$$

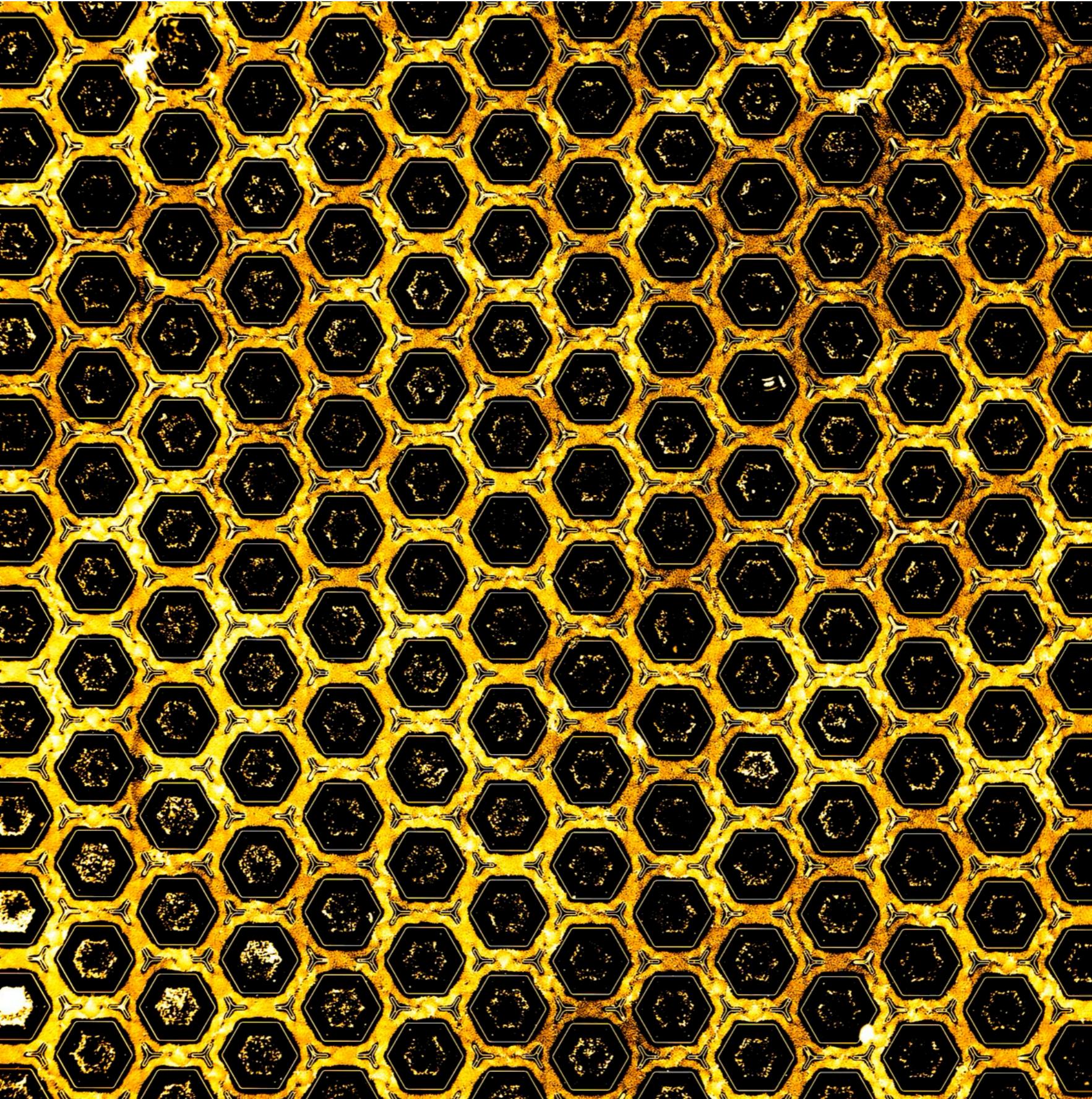
Flocking fluids



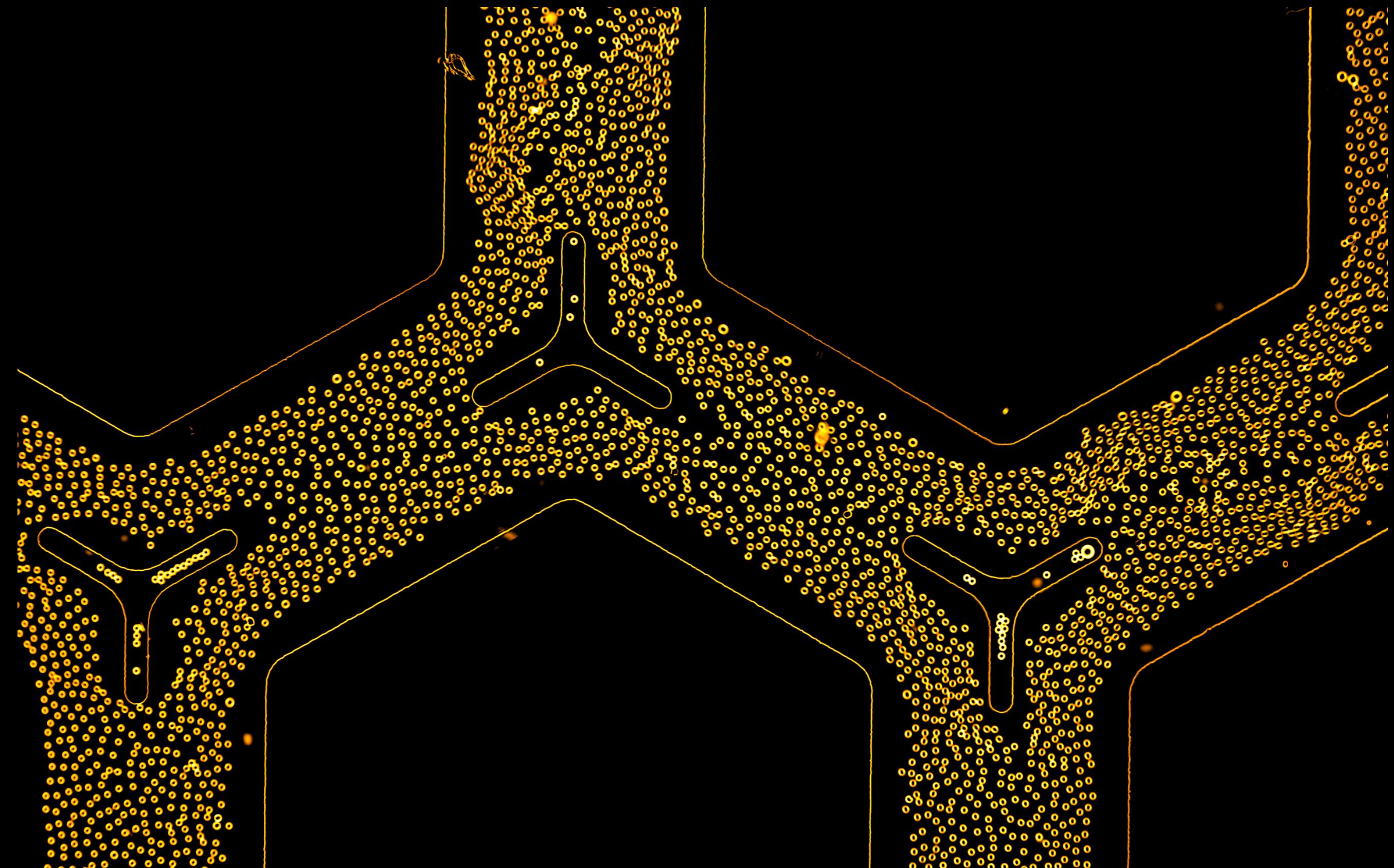
Spontaneous laminar flows
in channels and pipes

$$\phi \sim 3 \times 10^{-1}$$

Active flows in pipe networks?

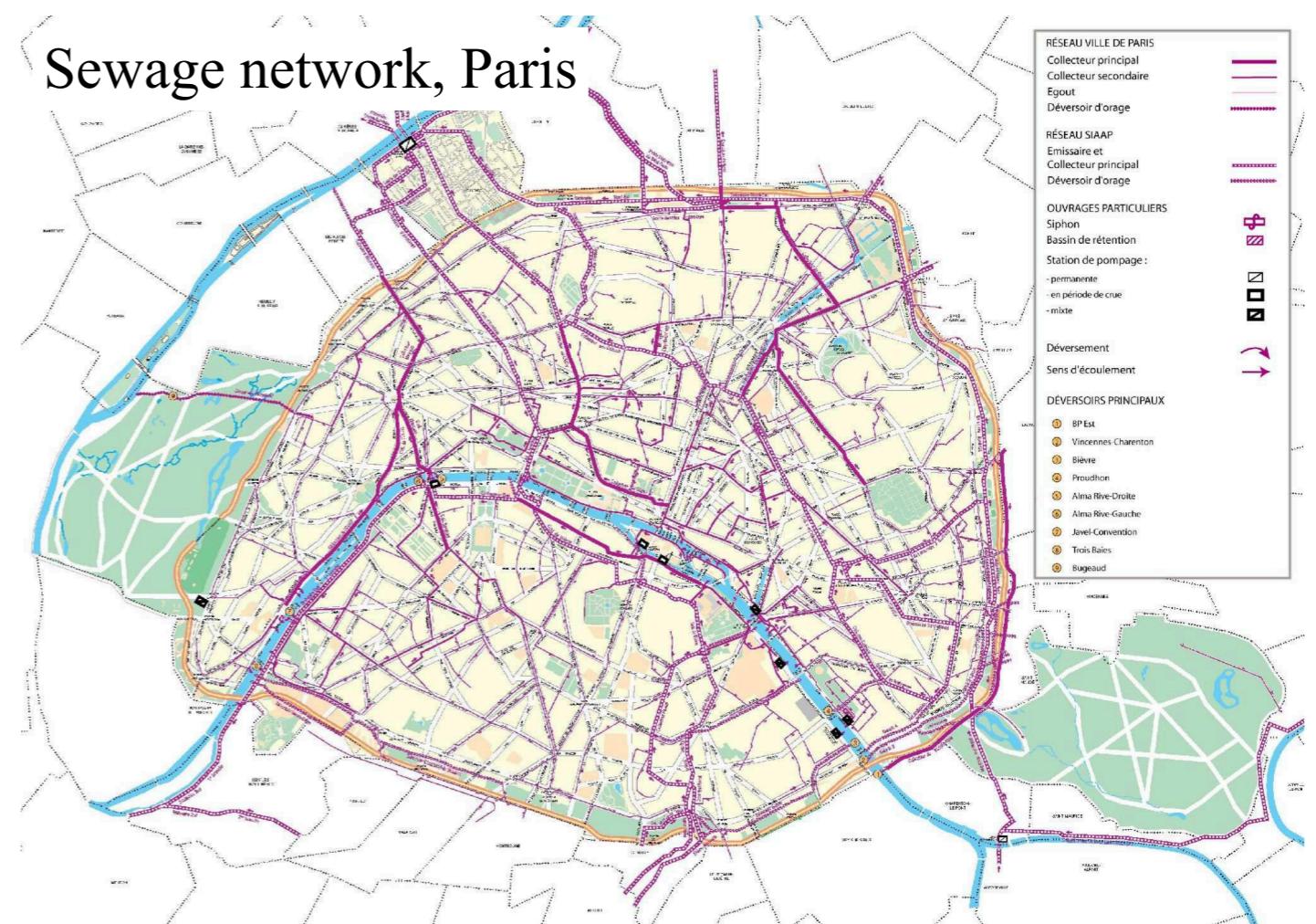
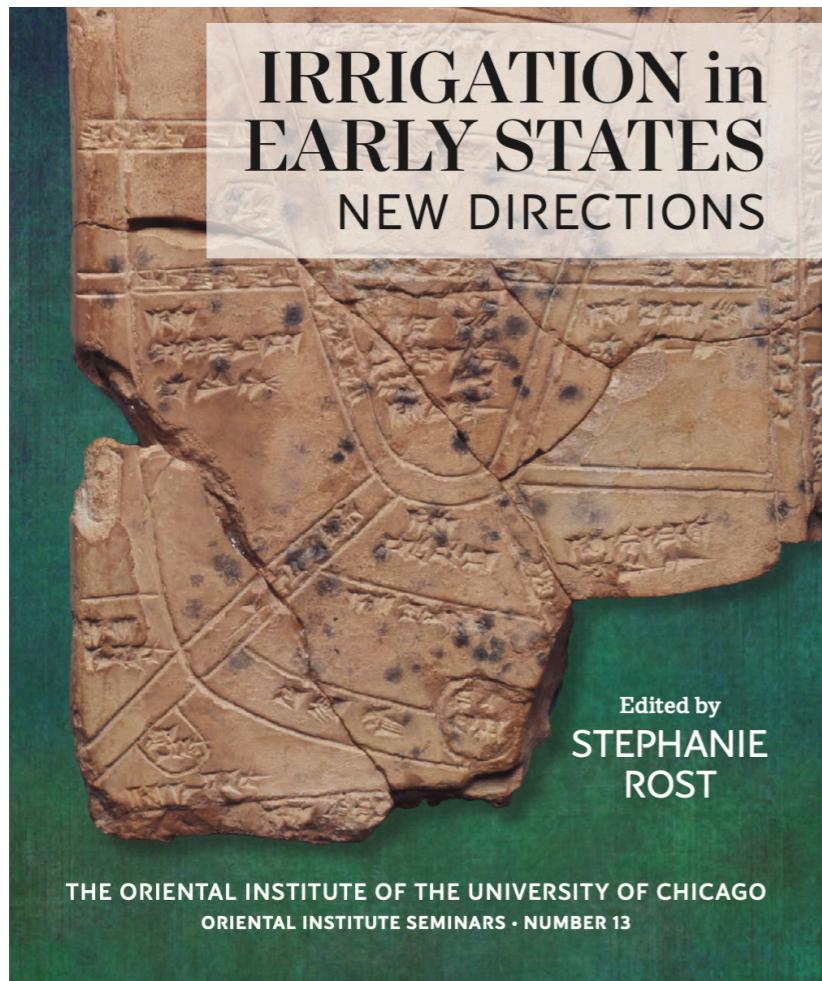


Active Hydraulics?



Hydraulics

Conveyance of liquids through pipes and channels



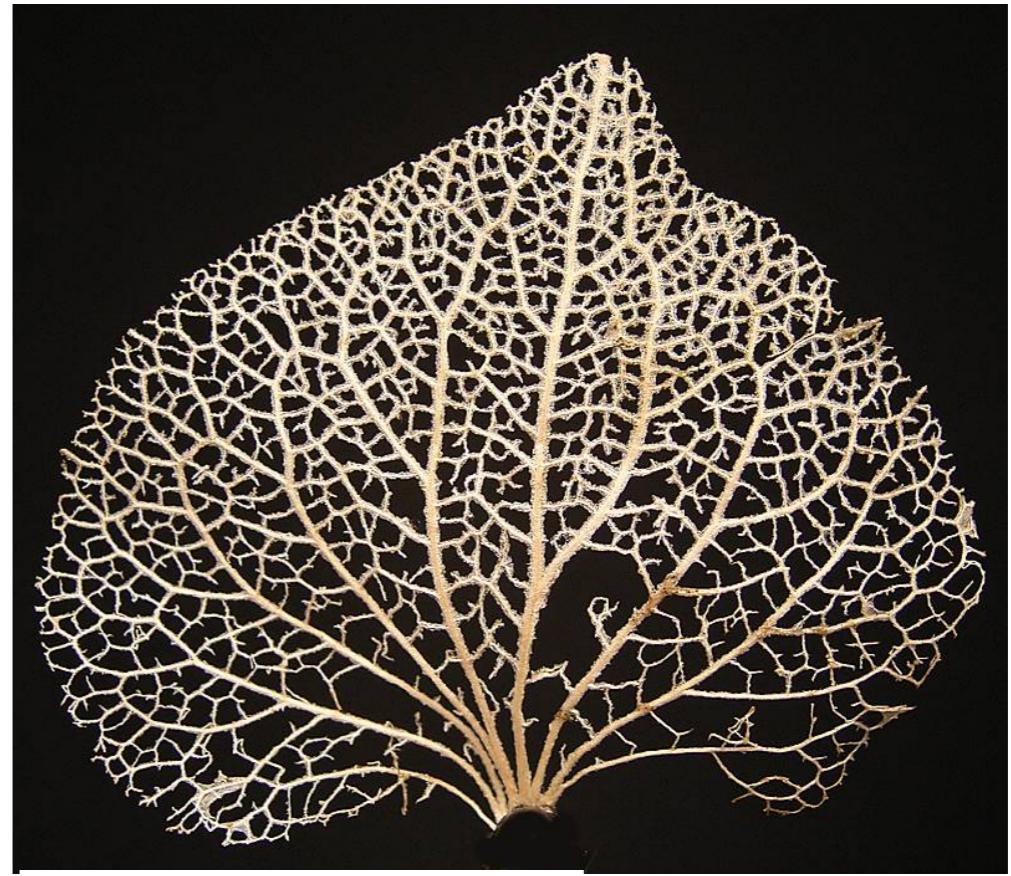
-6000

2022

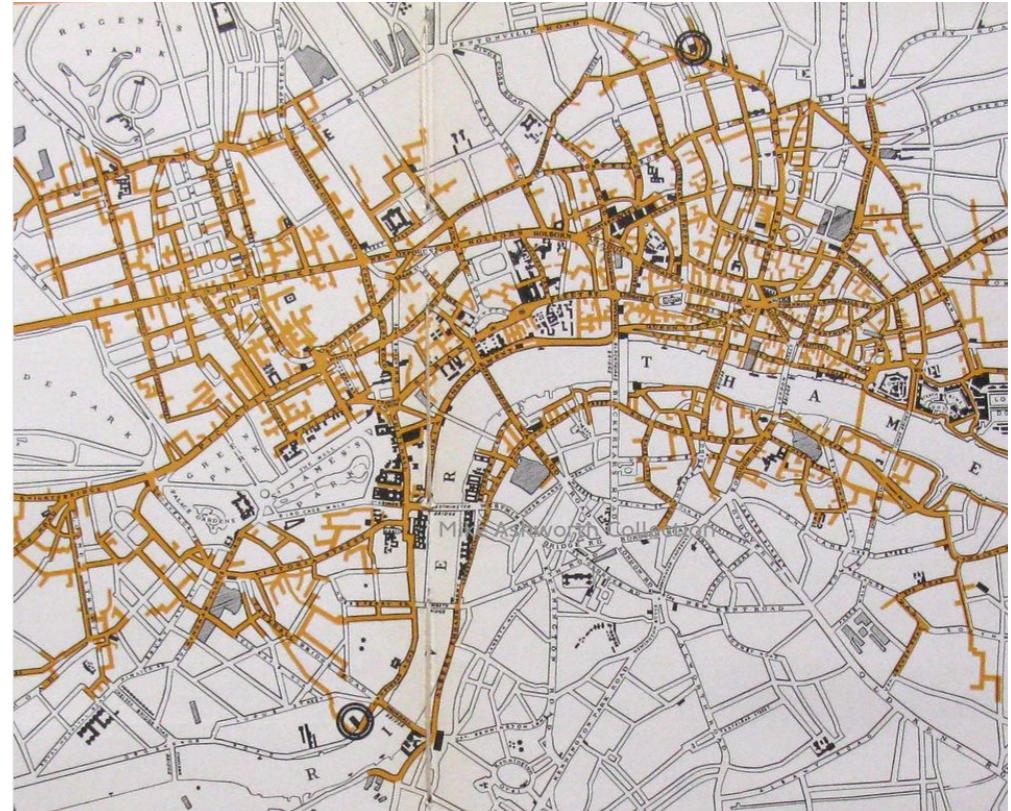
Time [y]

Hydraulics

Linear problem



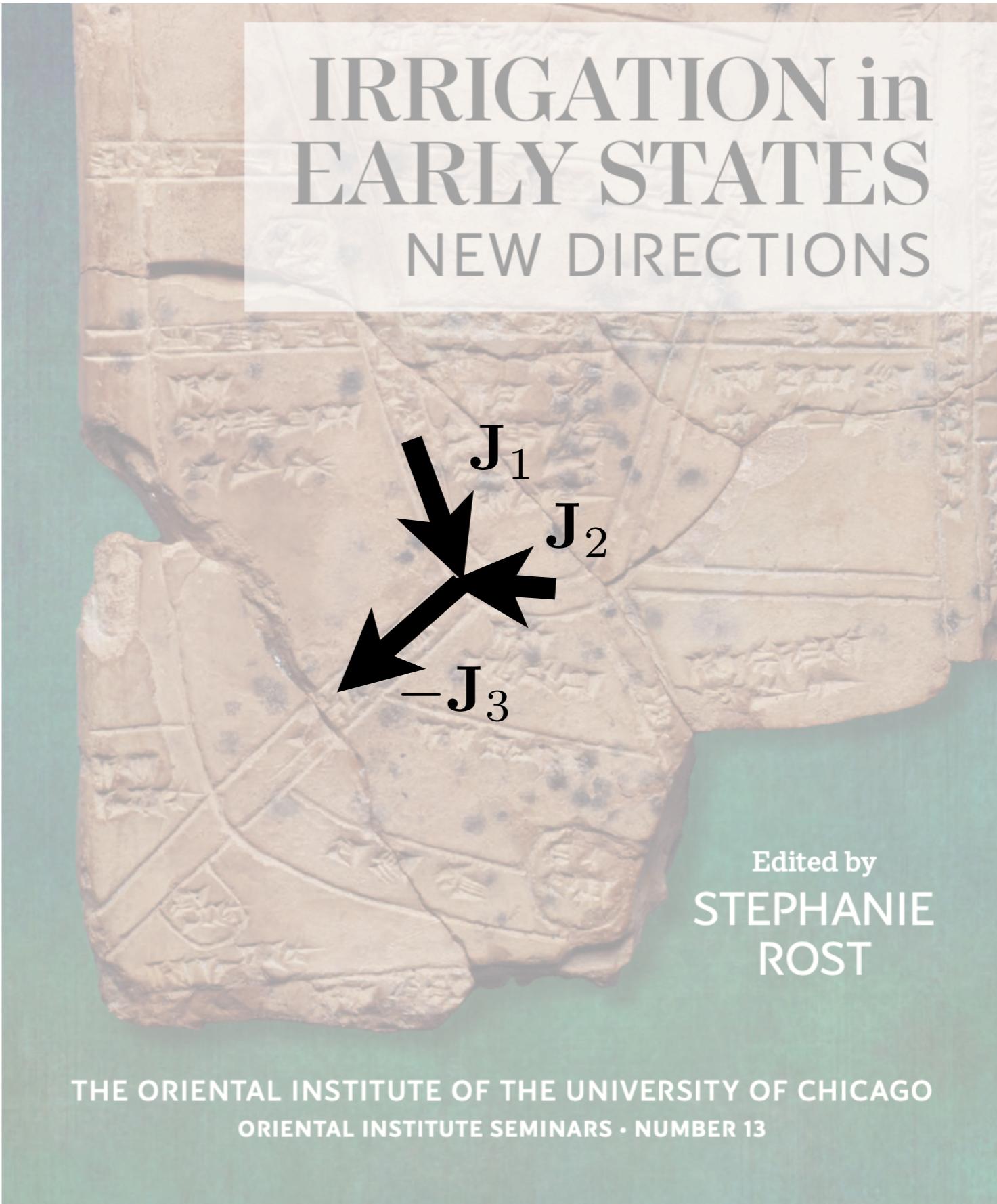
Vein skeleton of a *Hydrangea*, wikipedia



London's hydraulic Network 1960, Power water networks

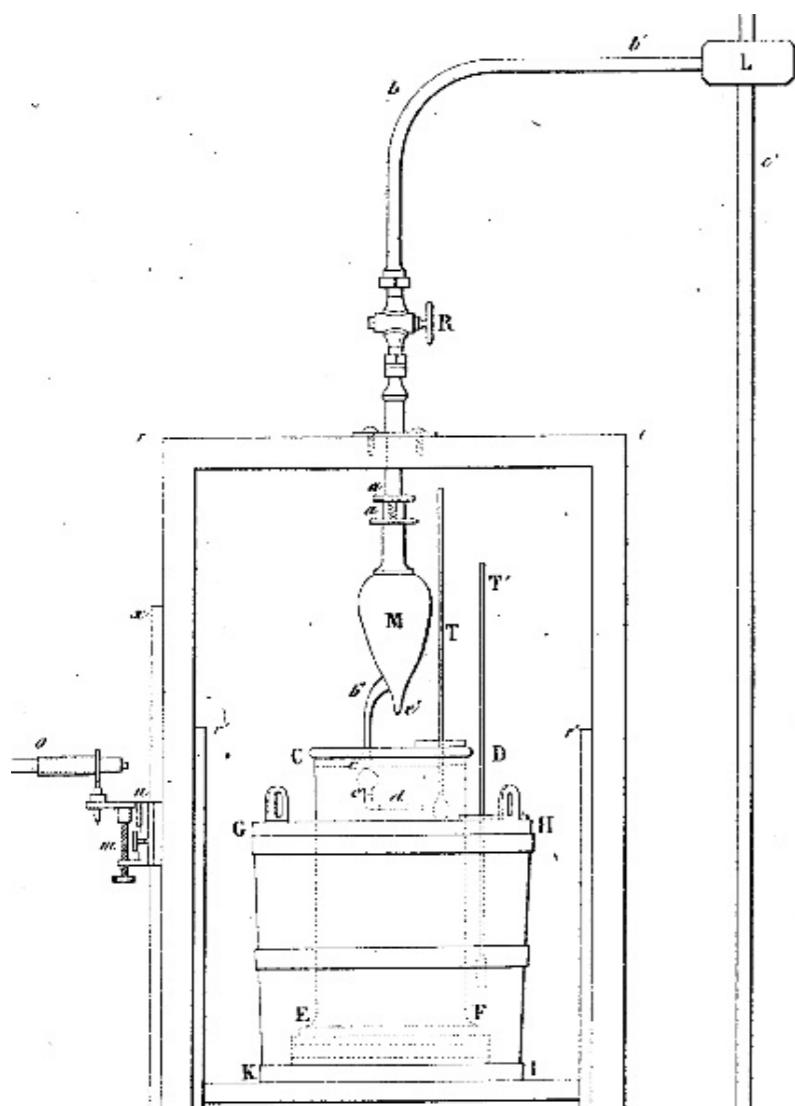
Mass conservation

$$\sum_{\text{node } i} \mathbf{J}_i = 0$$

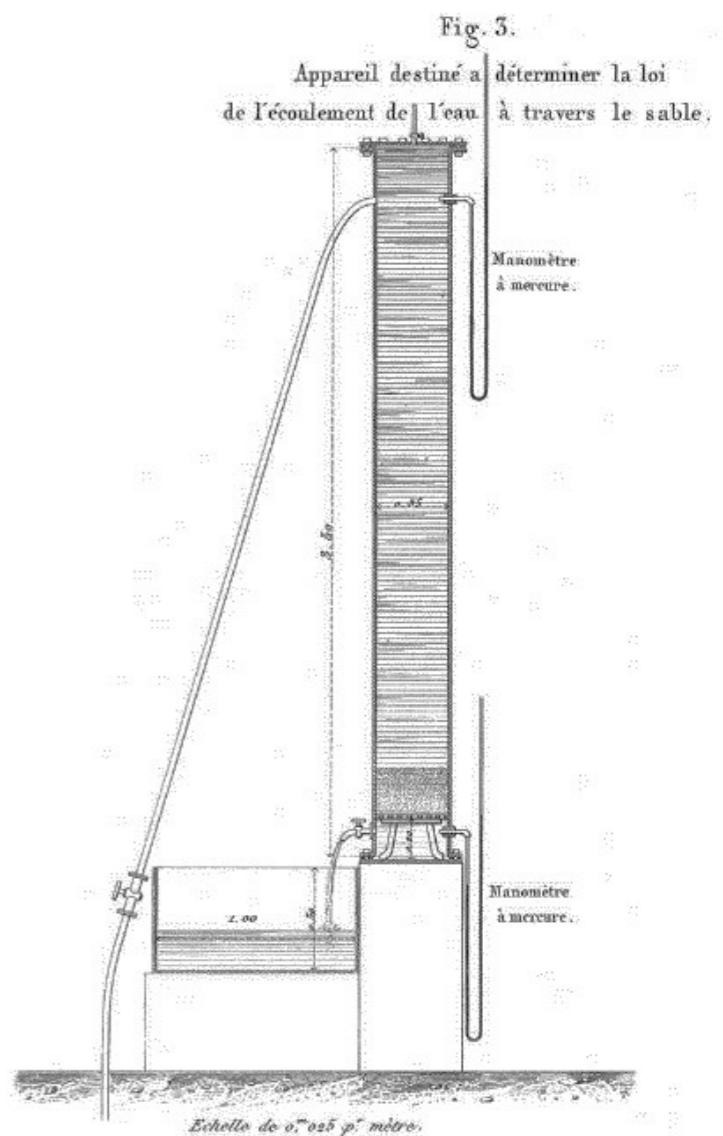


Constitutive relation

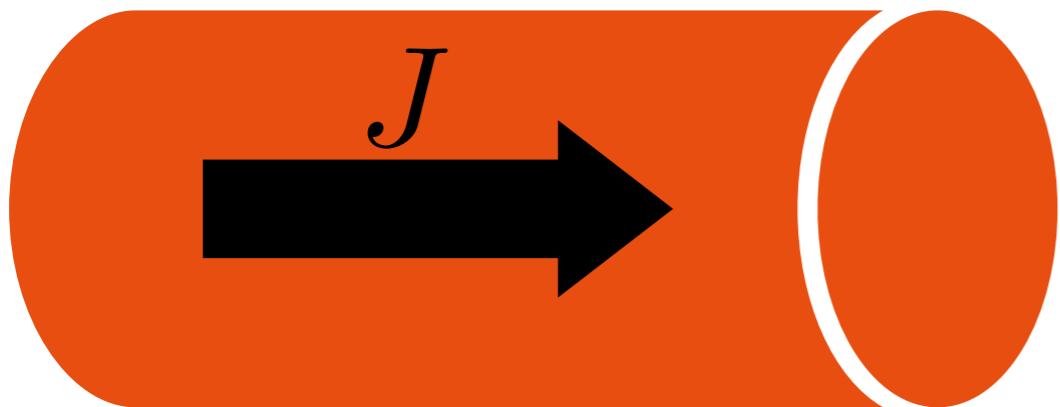
Poiseuille (1840)



Darcy (1856)



Darcy's law



$$J = -K \Delta P$$

Flux

Pressure drop

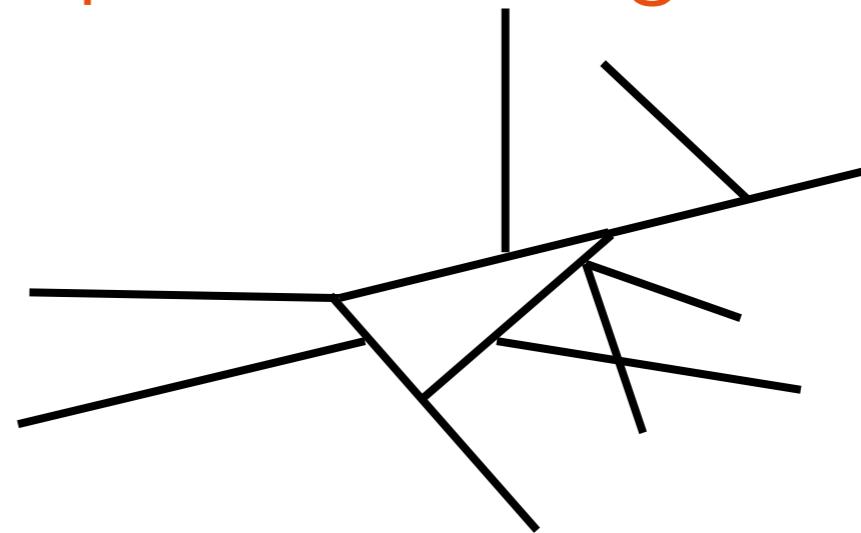
Hydraulics Newtonian fluids

Linear relations

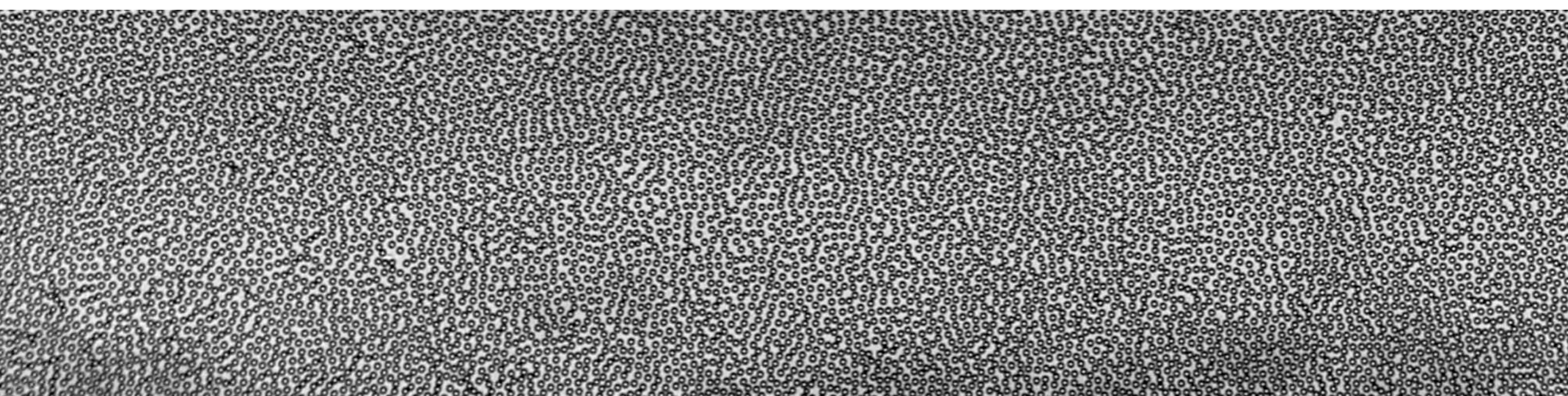
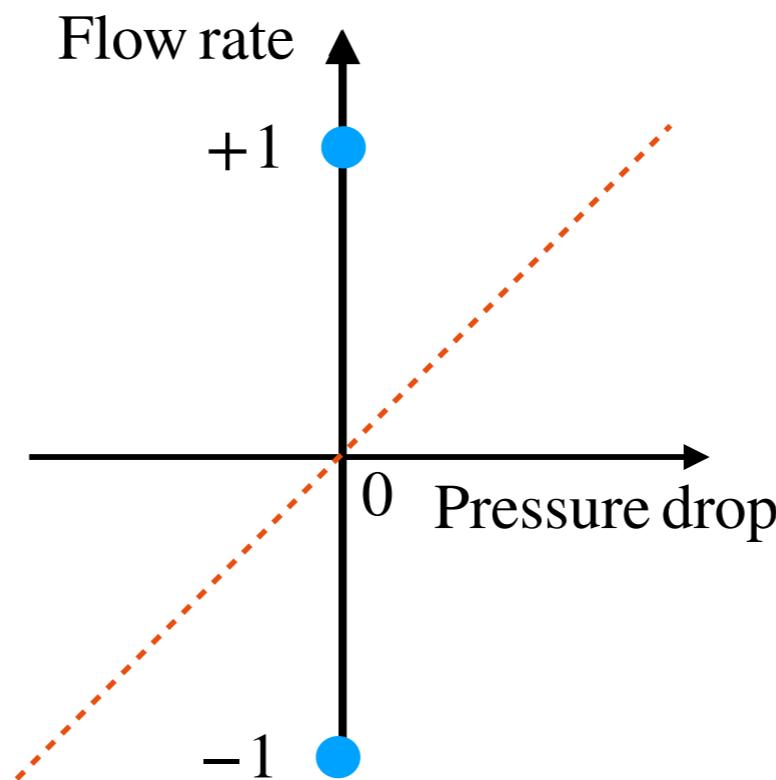
$$J = -K \Delta P$$

$$\sum_{\text{node } i} \mathbf{J}_i = 0$$

Pipe network geometry

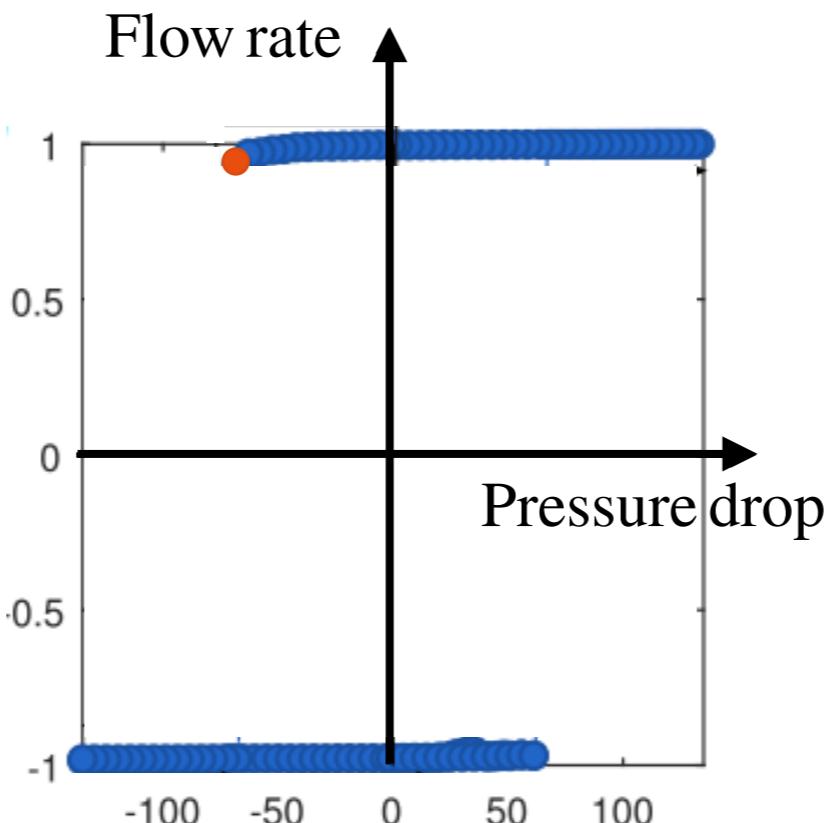


Confined Active Flows: Nonlinear

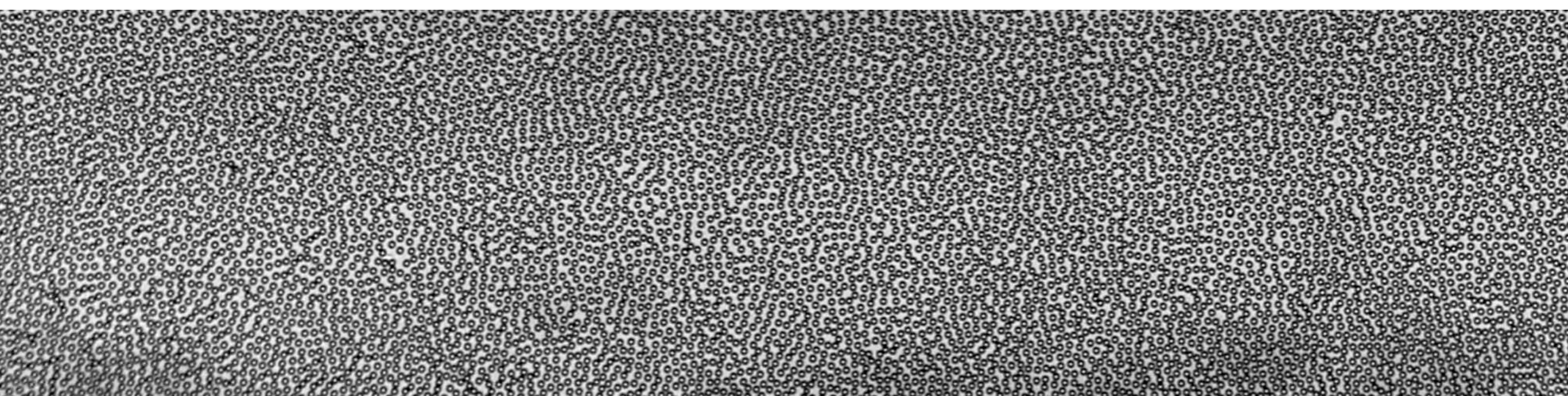


Colloidal rollers

Active fluids: Bistable flows



$$J = \pm J_0$$



Colloïdal rollers

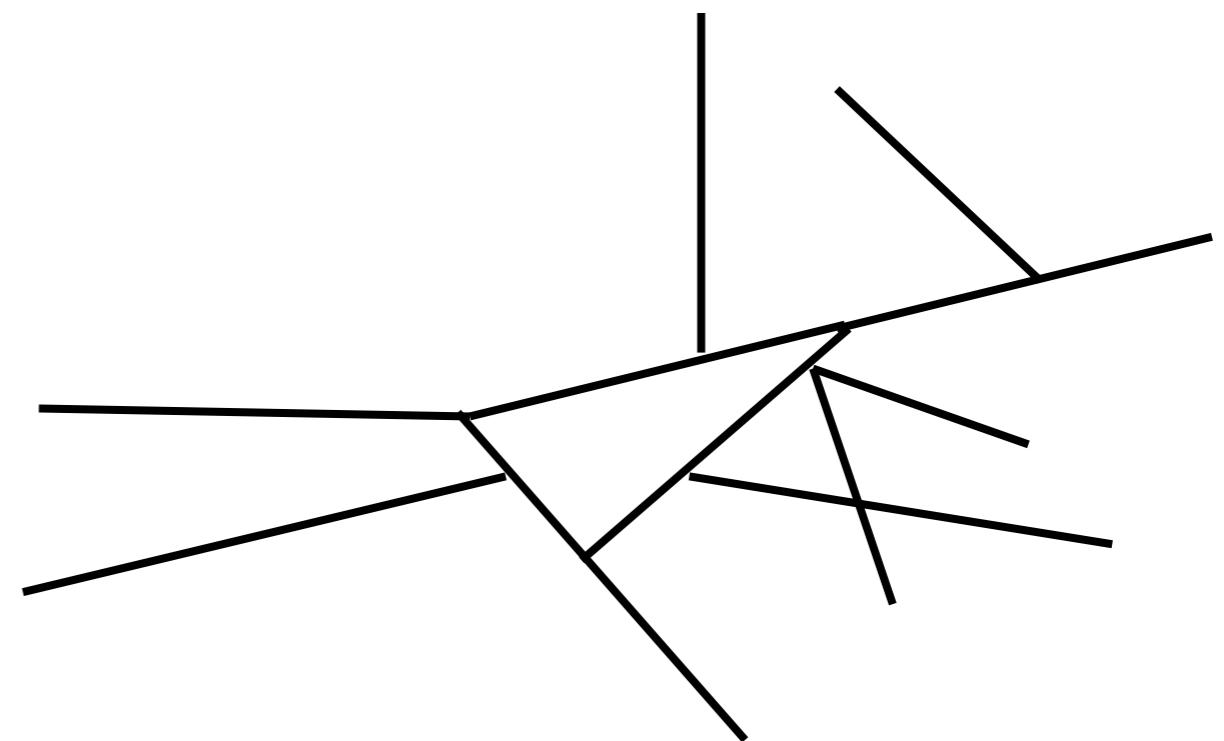
Active Hydraulics

Two constraints

$$J_i = \pm J_0$$

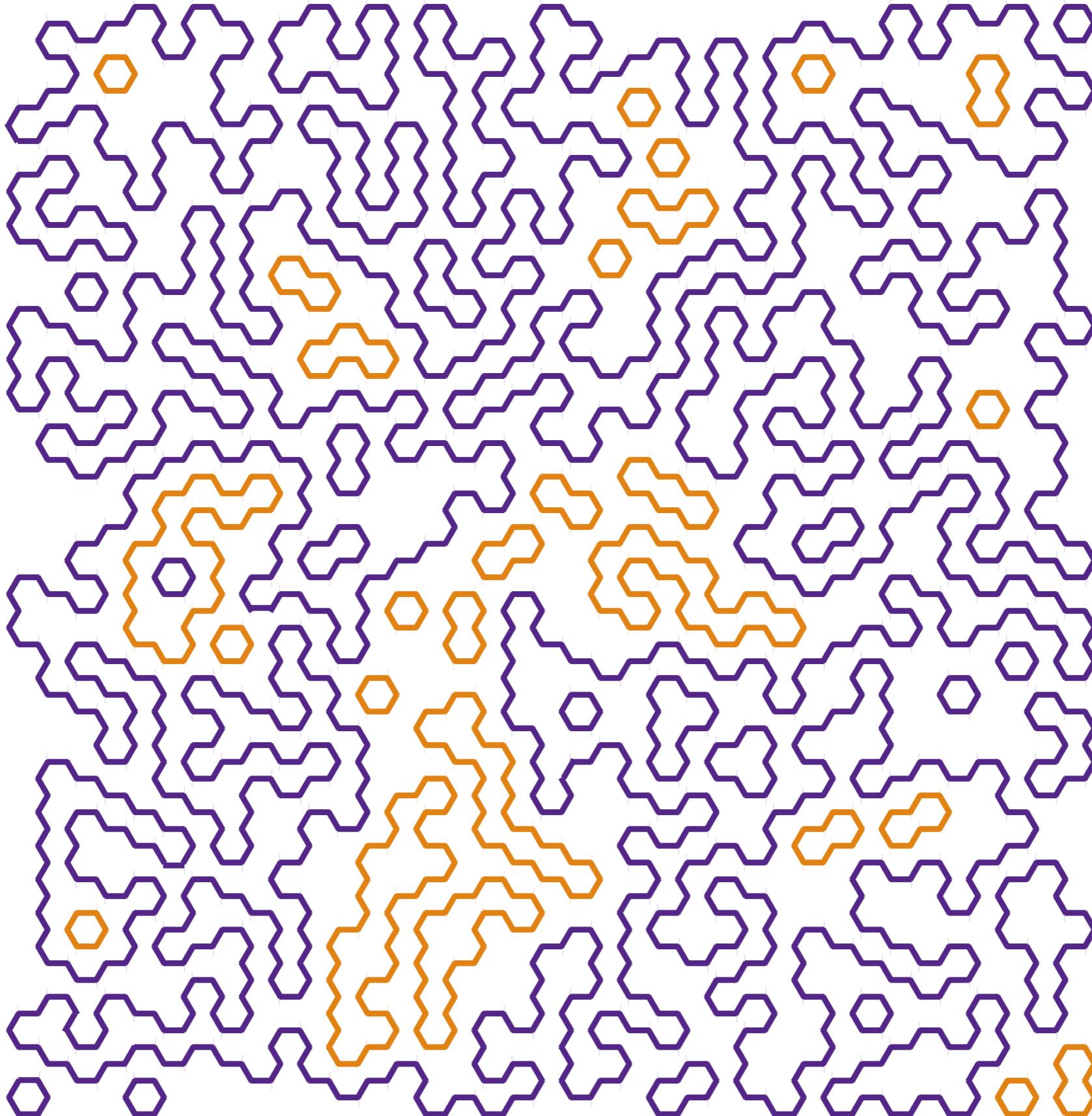
$$\sum_{\text{node } i} \mathbf{J}_i = 0$$

Pipe-network geometry



Vertex problem

Emergent flow patterns?



Simplest Geometry

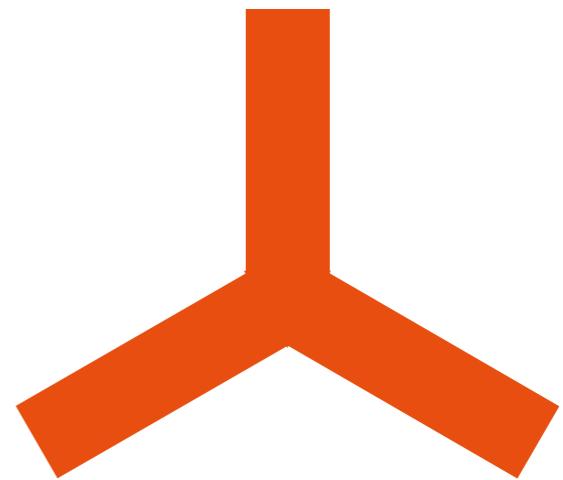
Bivalent Units



Pipe

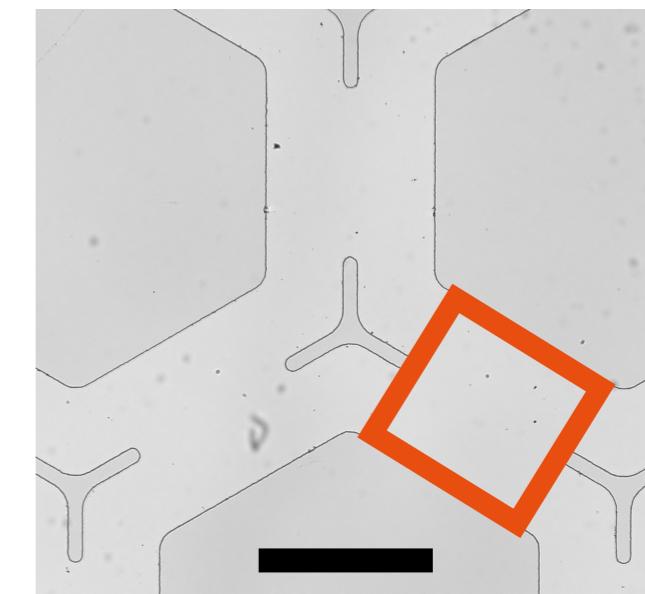
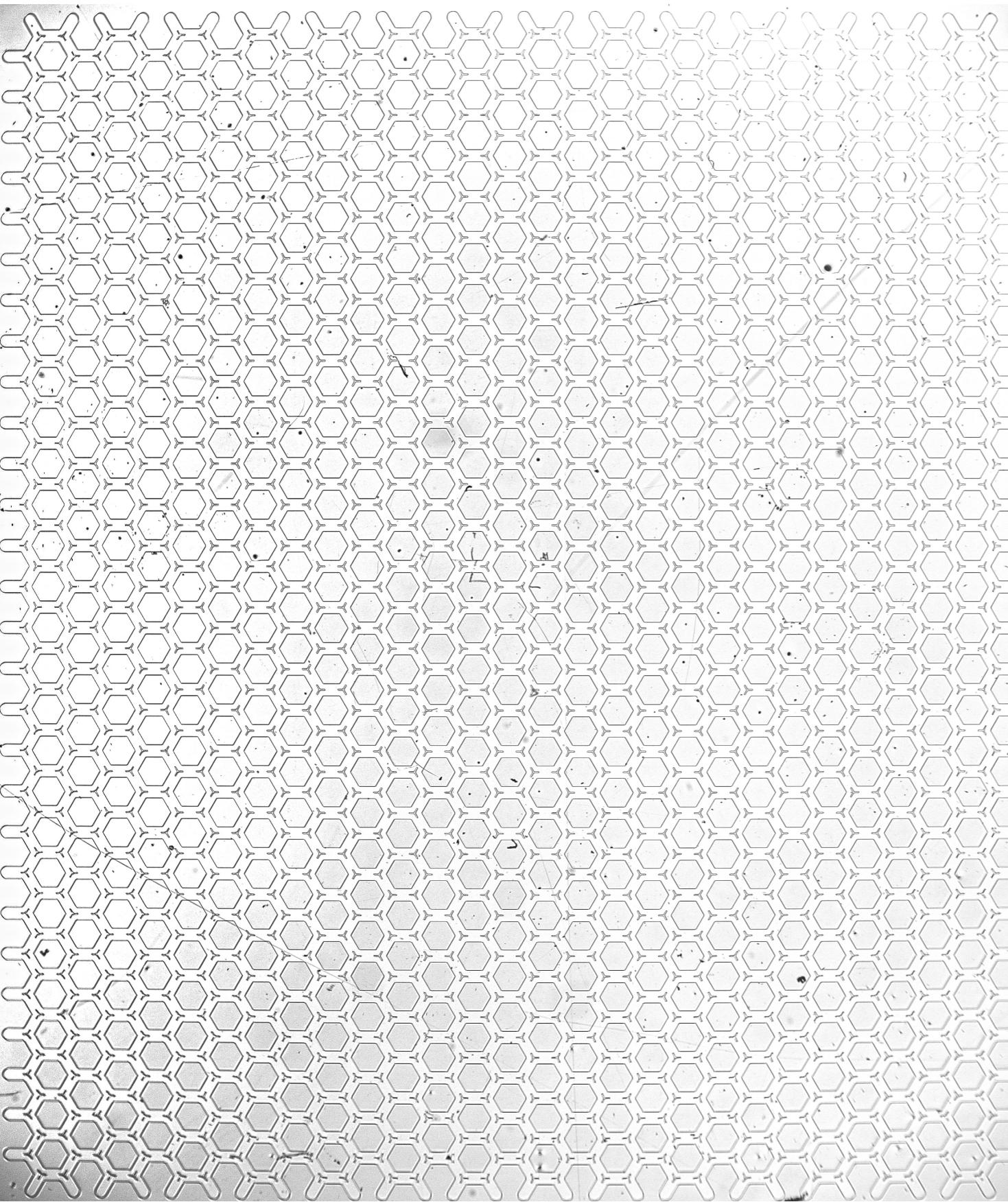


Trivalent Units



Network

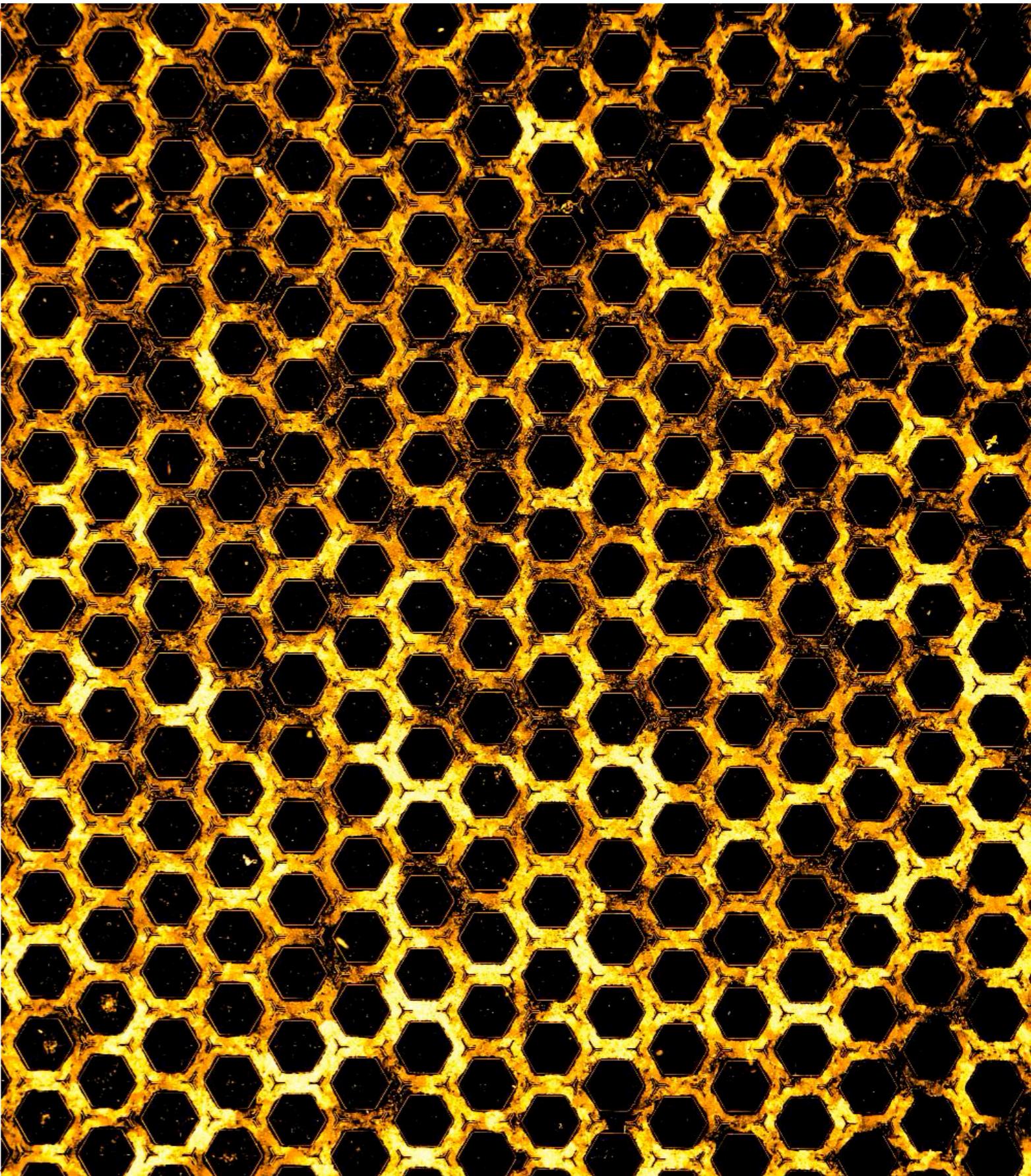
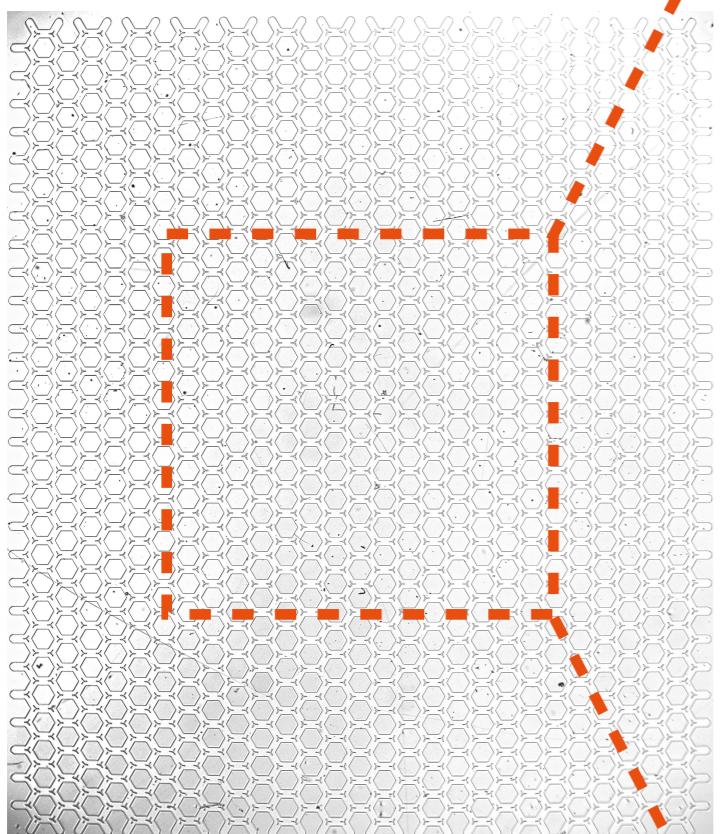
Honeycomb Lattice



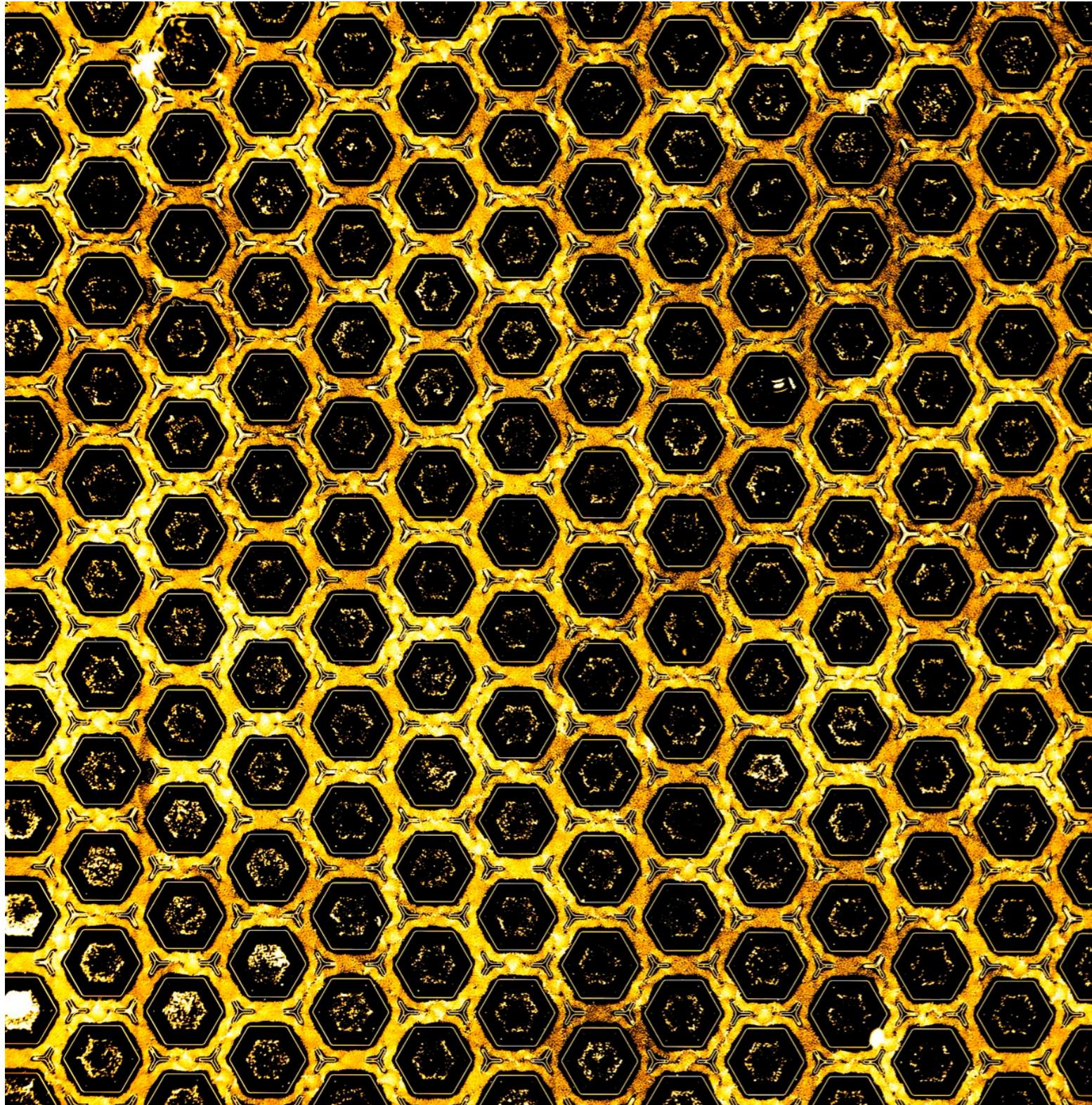
0.2 mm

Aspect ratio: 0.7–1.7

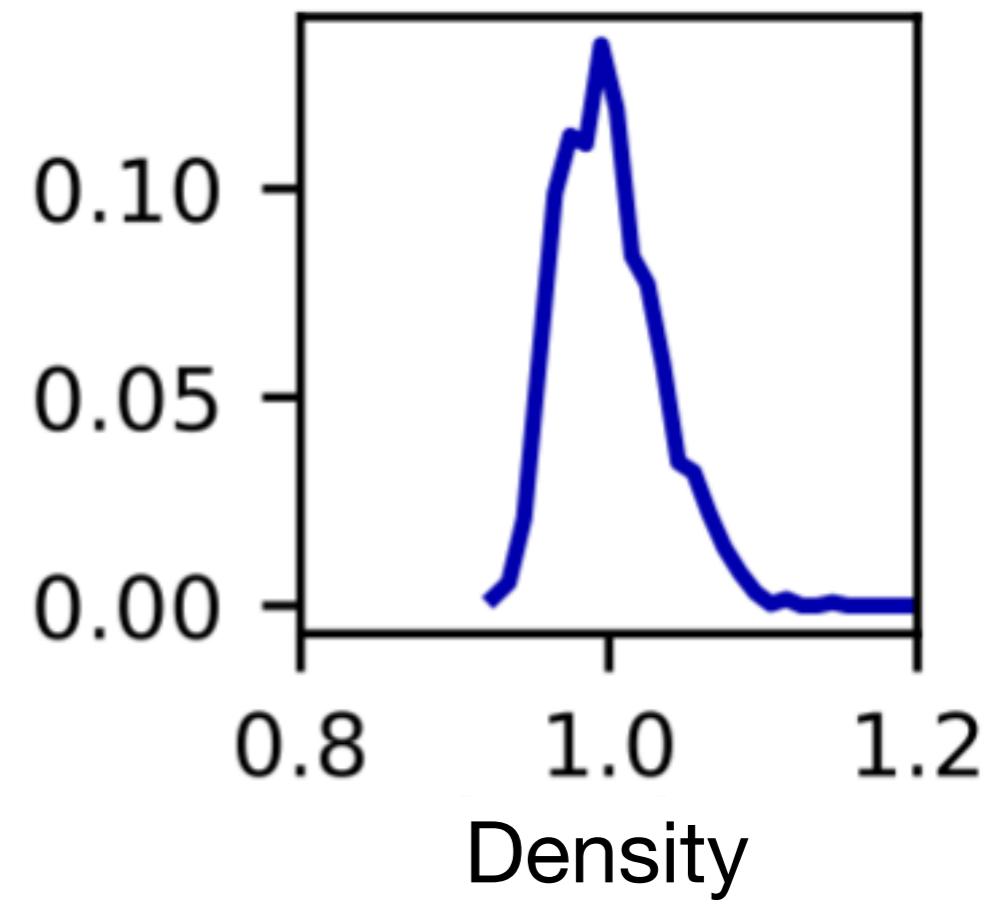
Colloidal Roller Fluid



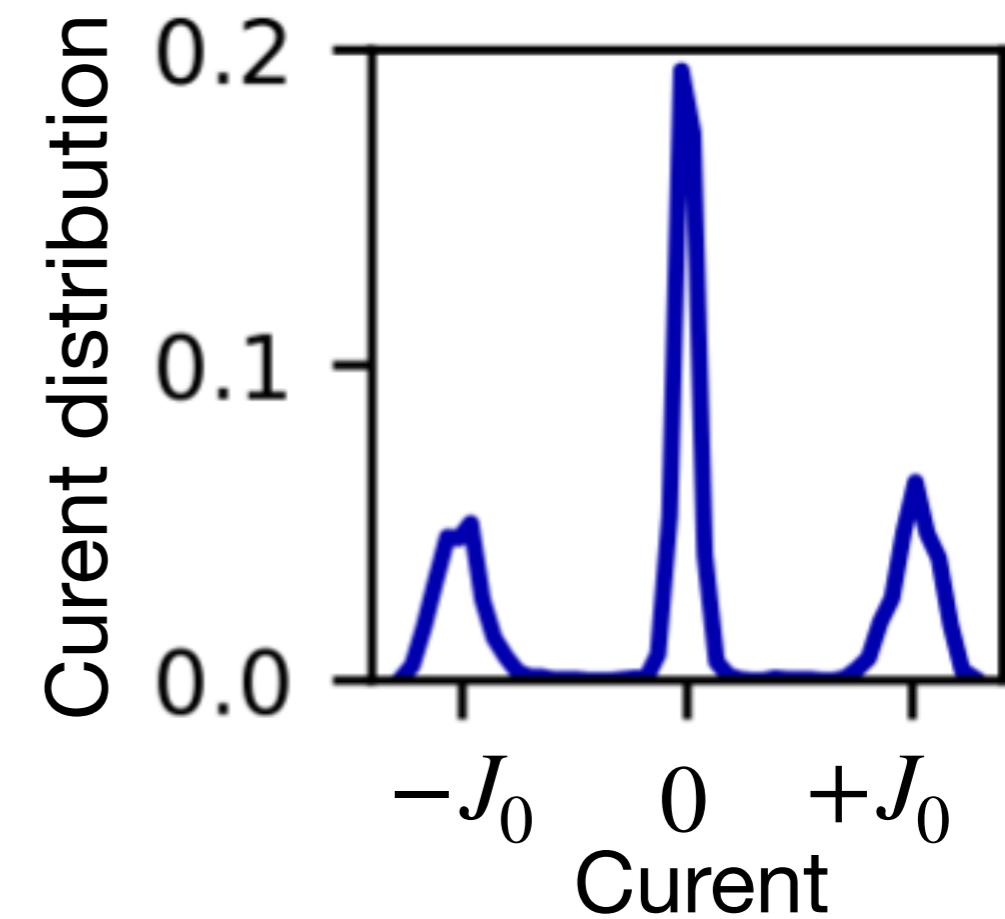
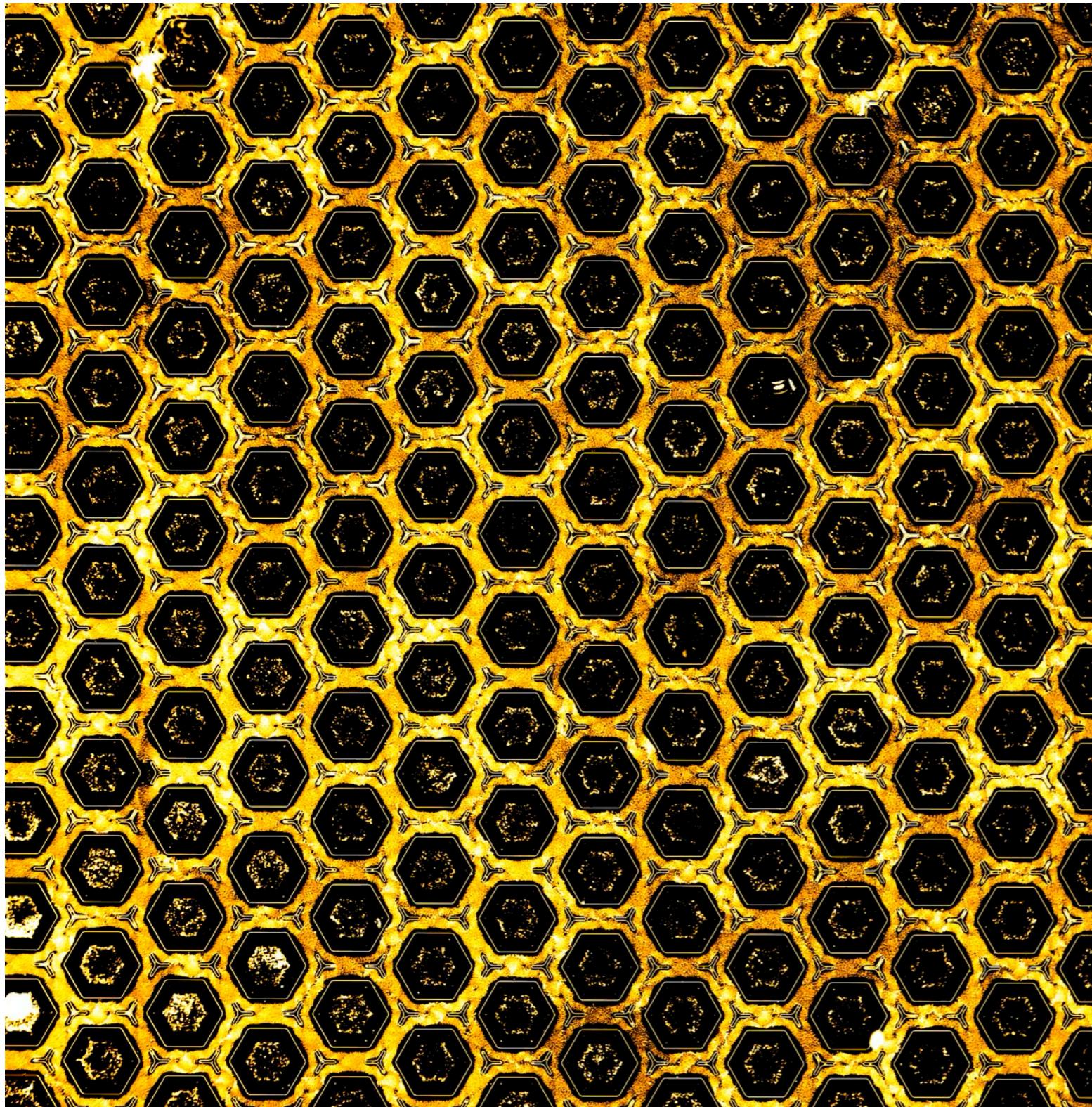
Steady state: Uniform packing fraction



Density distribution



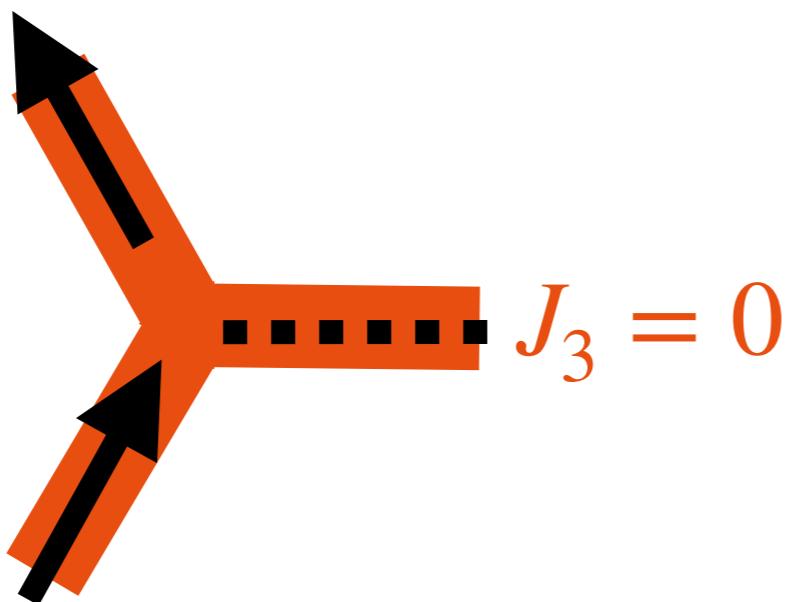
Steady state: Current statistics



$$J_i \neq \pm J_0$$

Geometrical Frustration

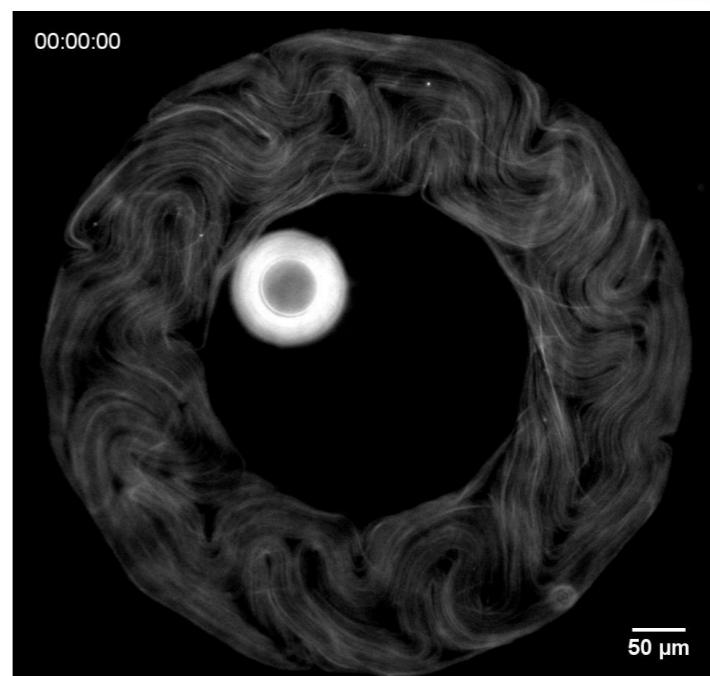
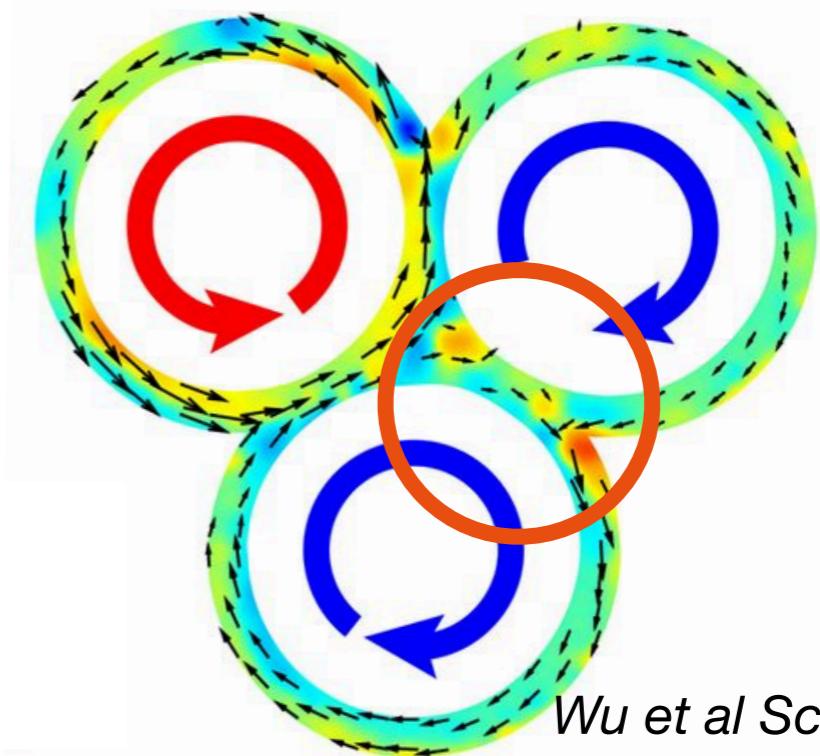
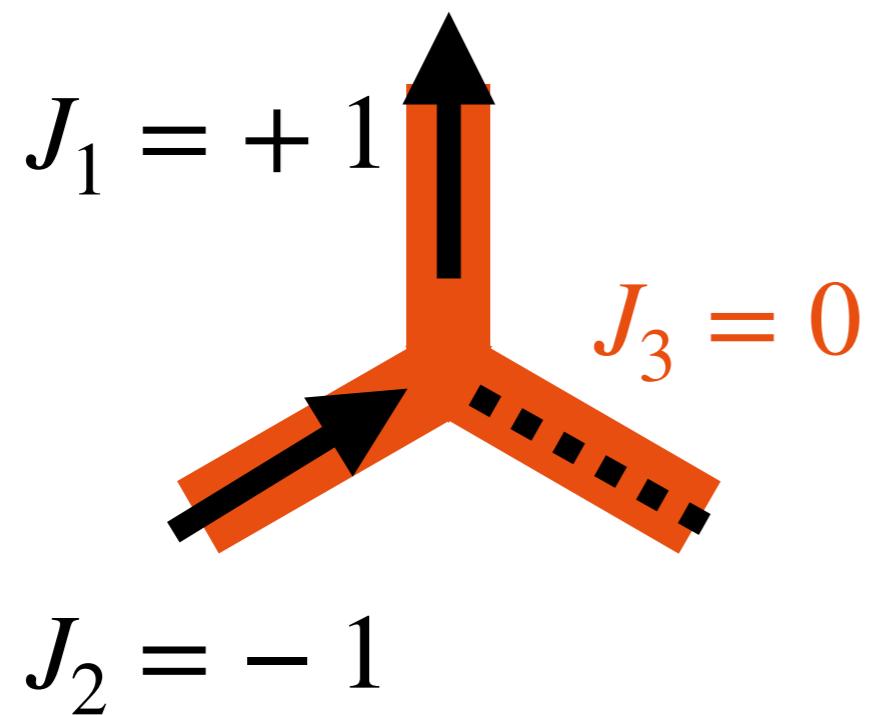
$$J_2 = -1$$



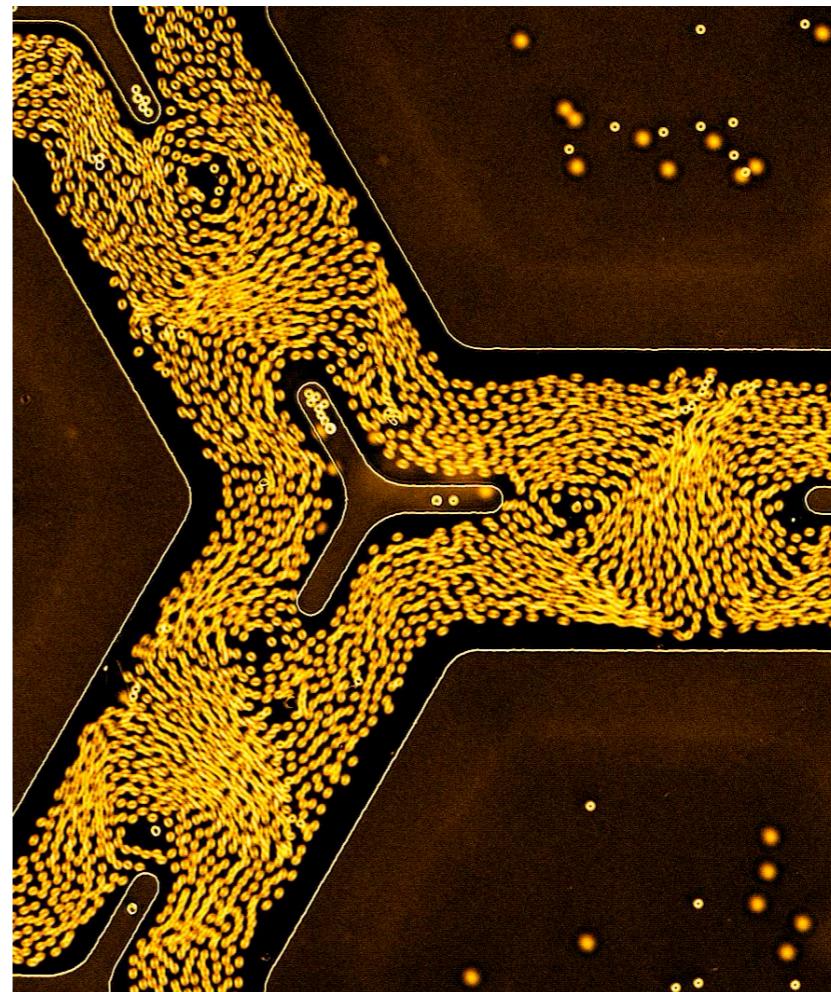
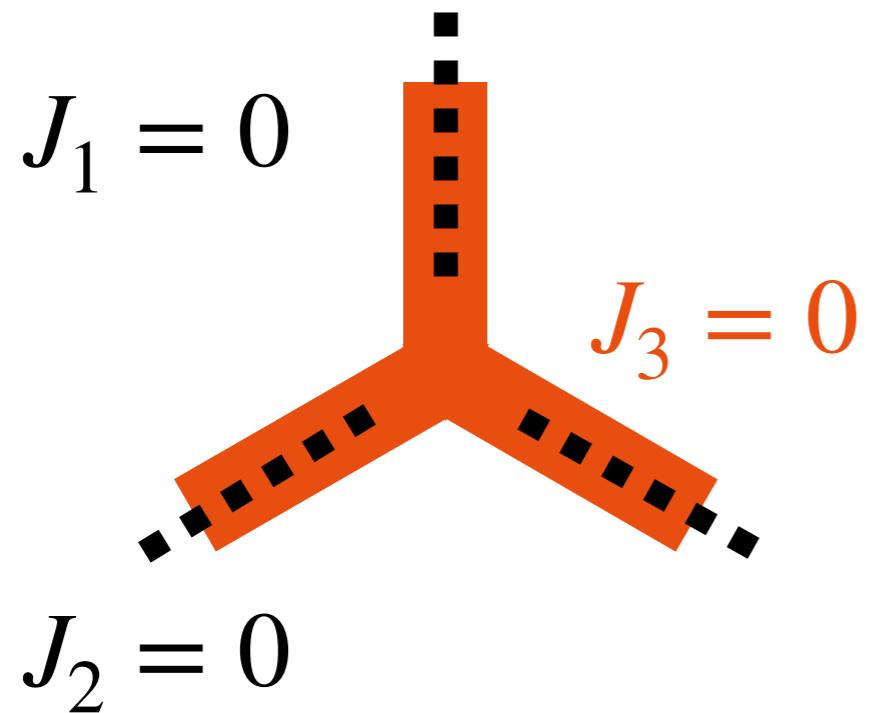
$$J_1 = +1$$

Activity forbids uniform laminar flows

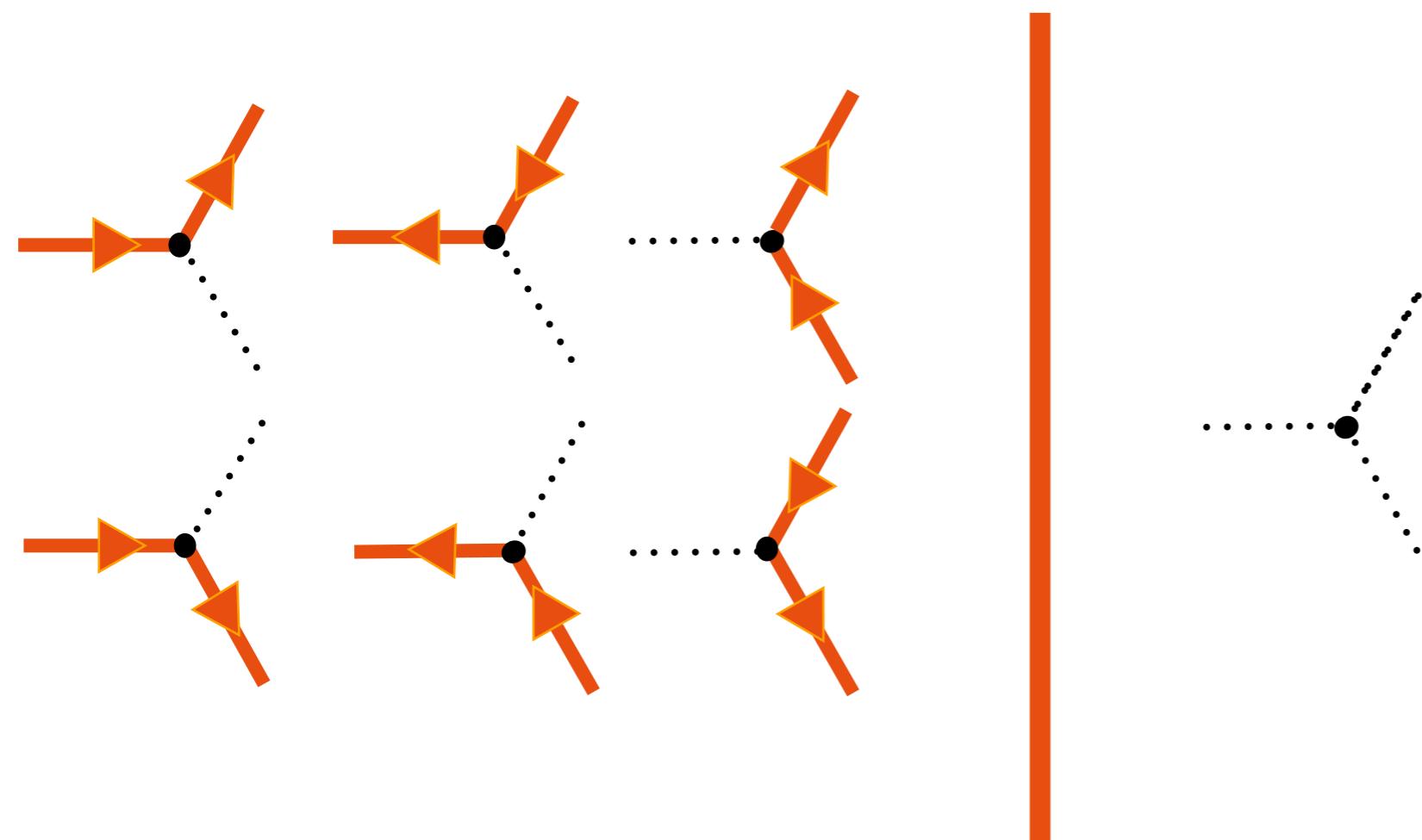
Geometrical Frustration



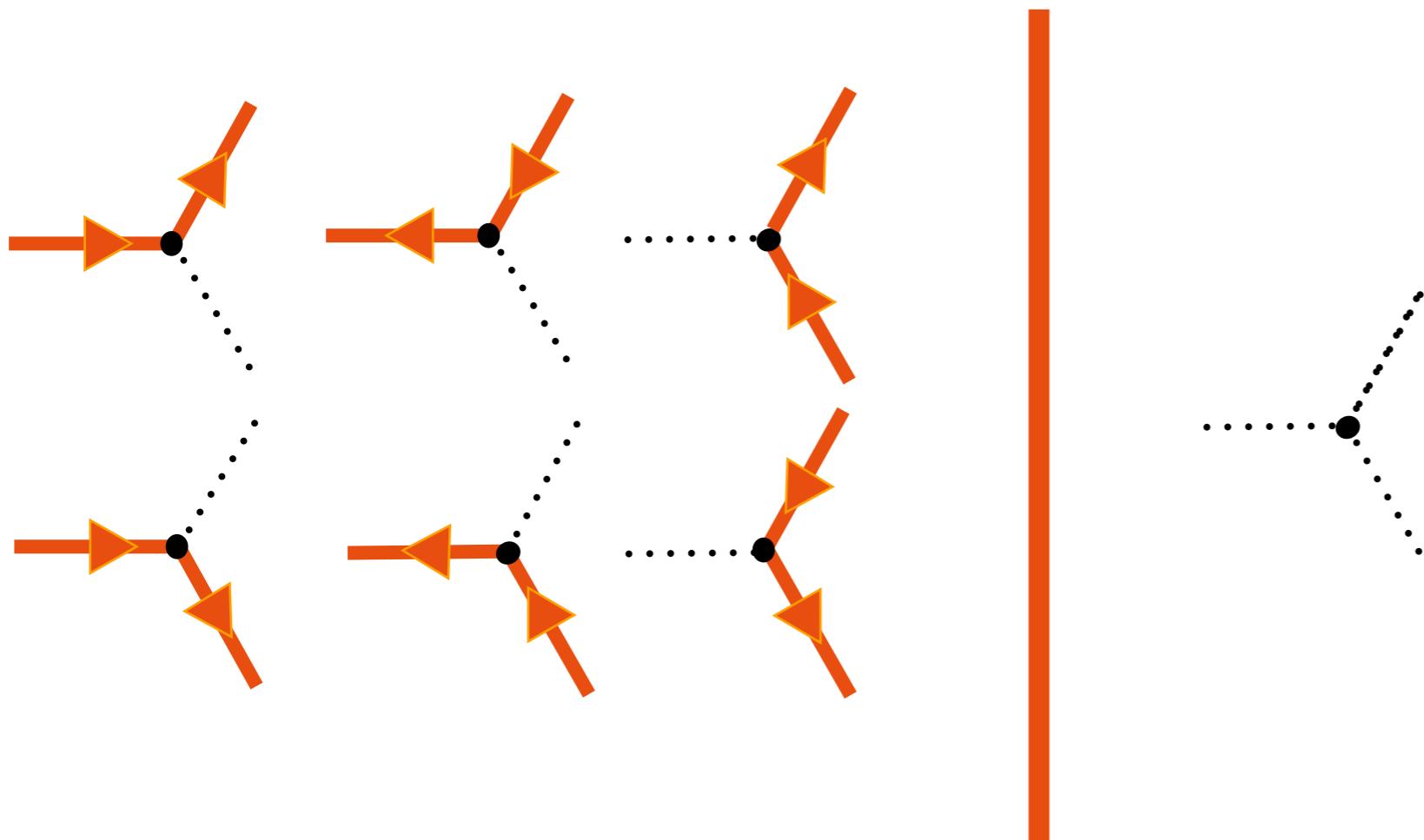
Geometrical Frustration



Seven vertex configurations

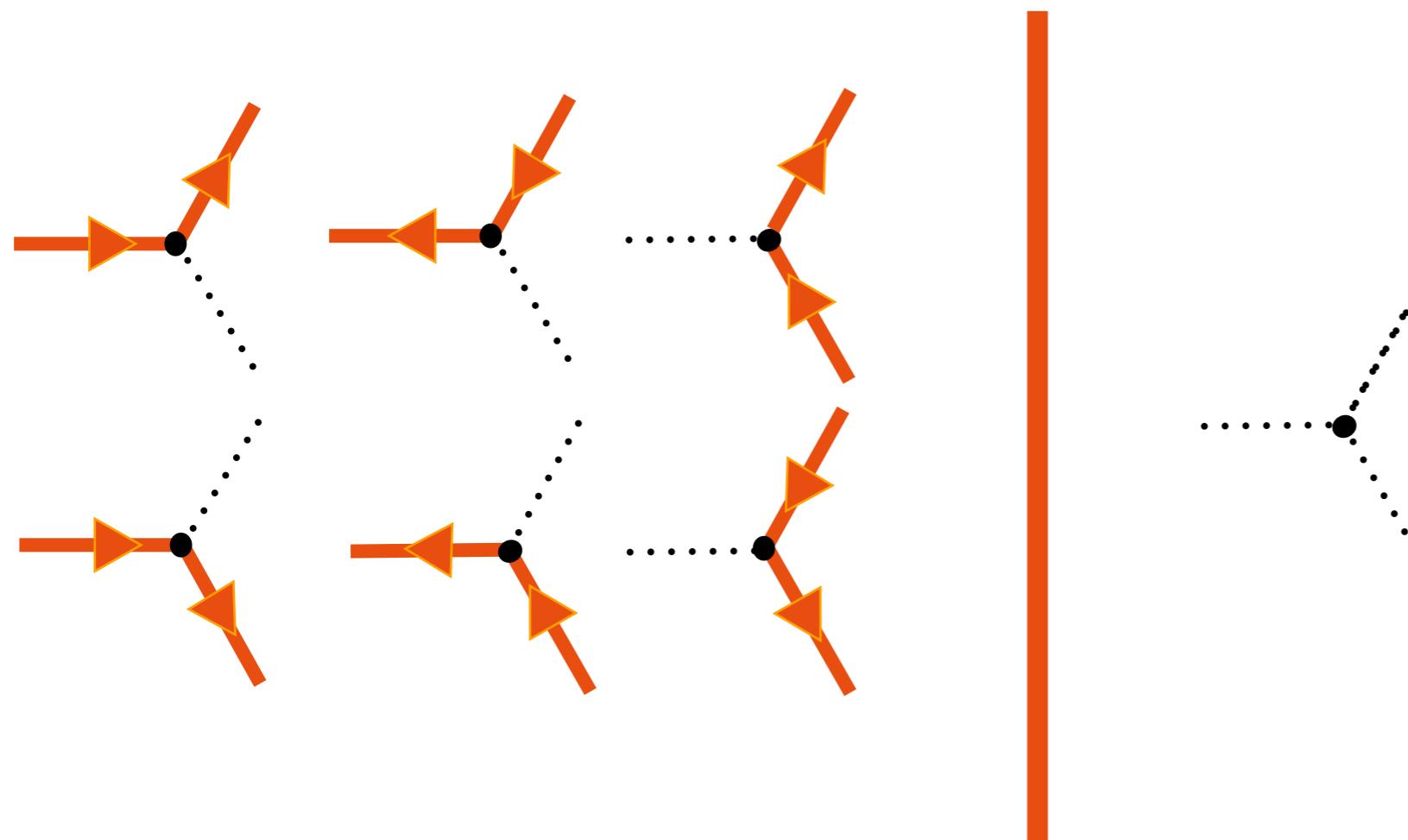


Active Fluidic Network Theory



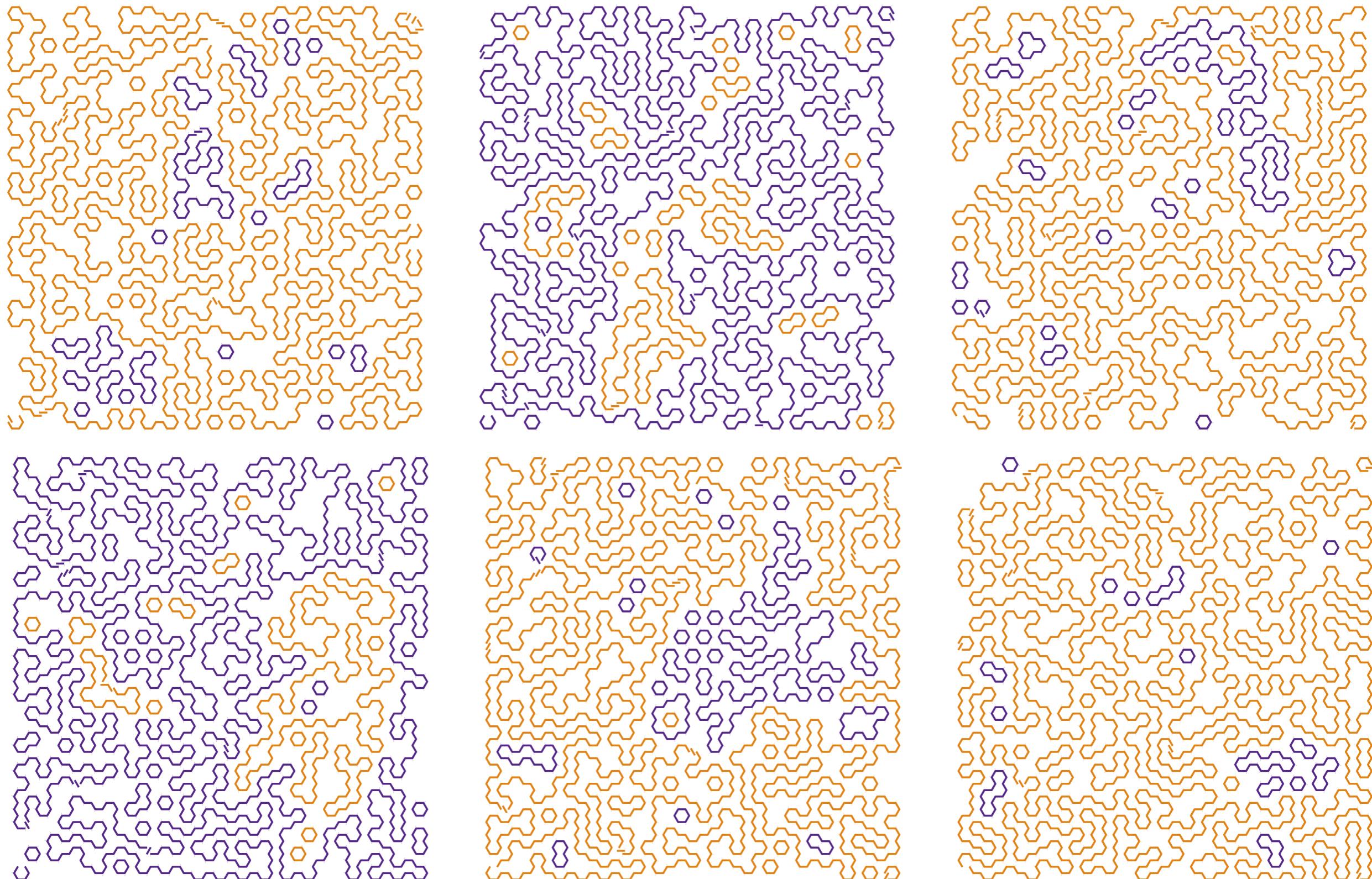
F. Woodhouse and J. Dunkel

Seven vertex configurations



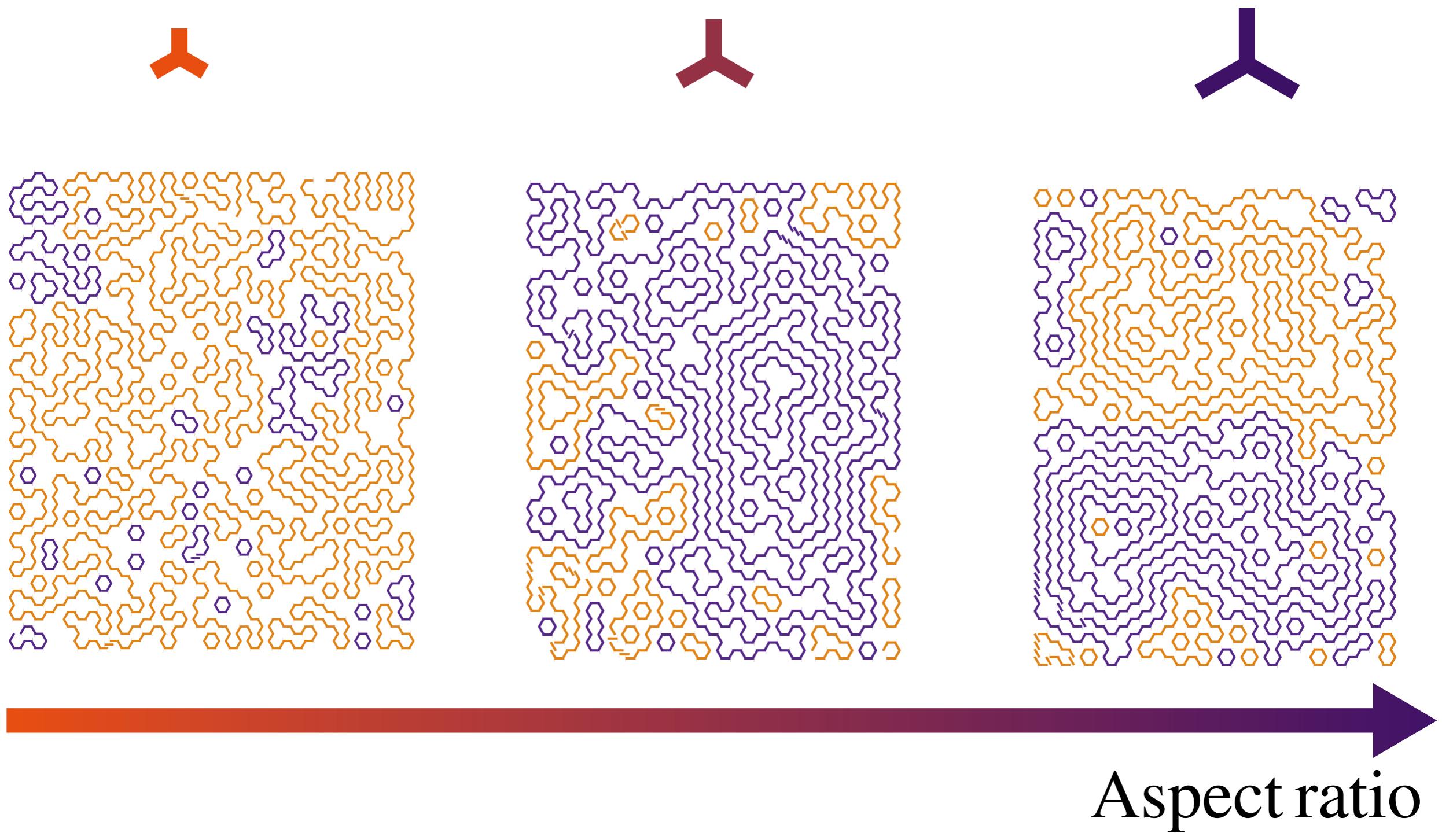
Generators of self-avoiding random walks

Streamlines: Self-avoiding loops



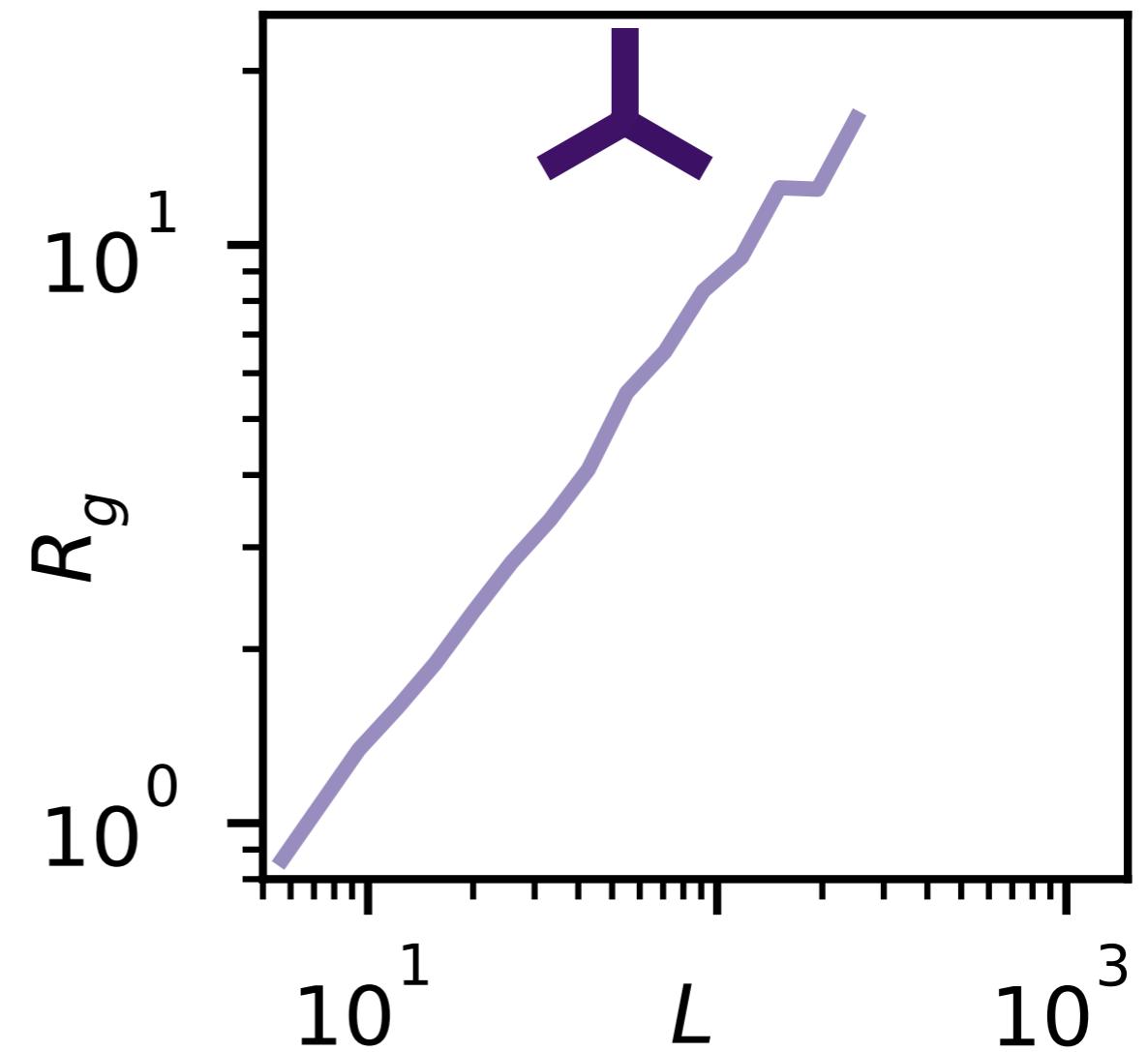
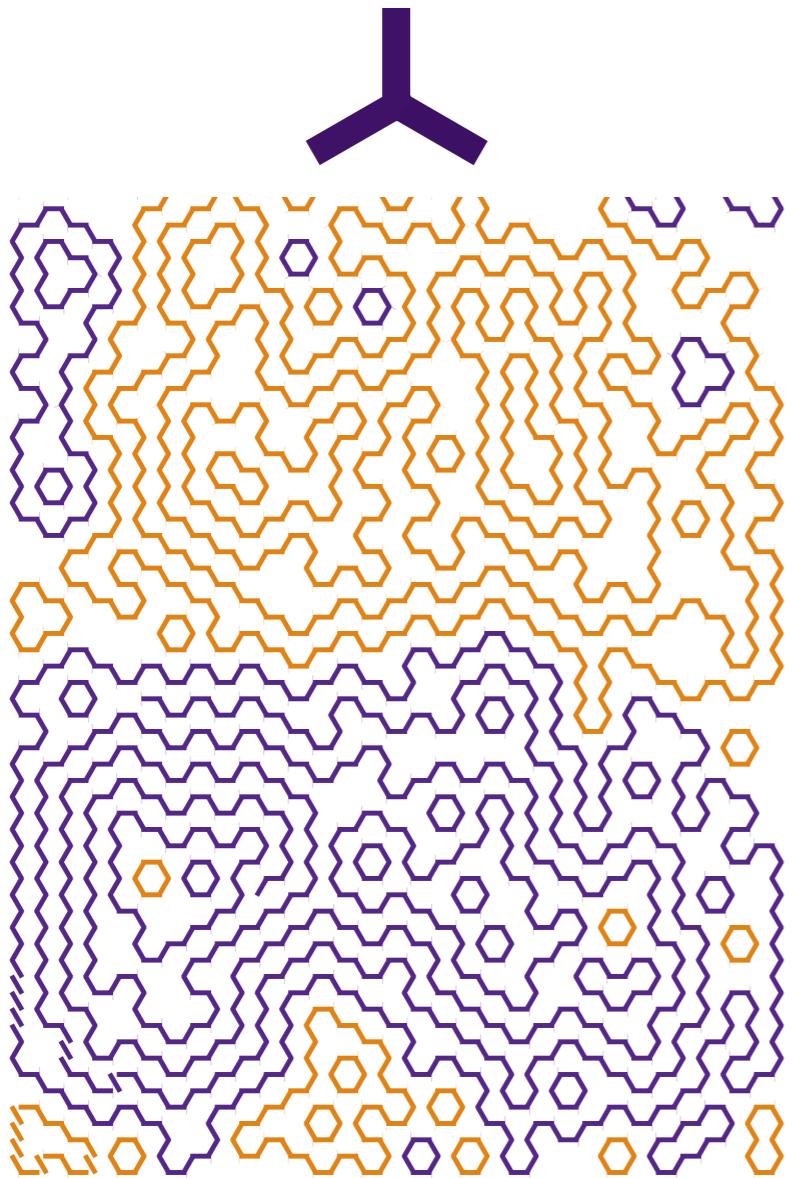
Steady state degeneracy

Impact of the channel geometry



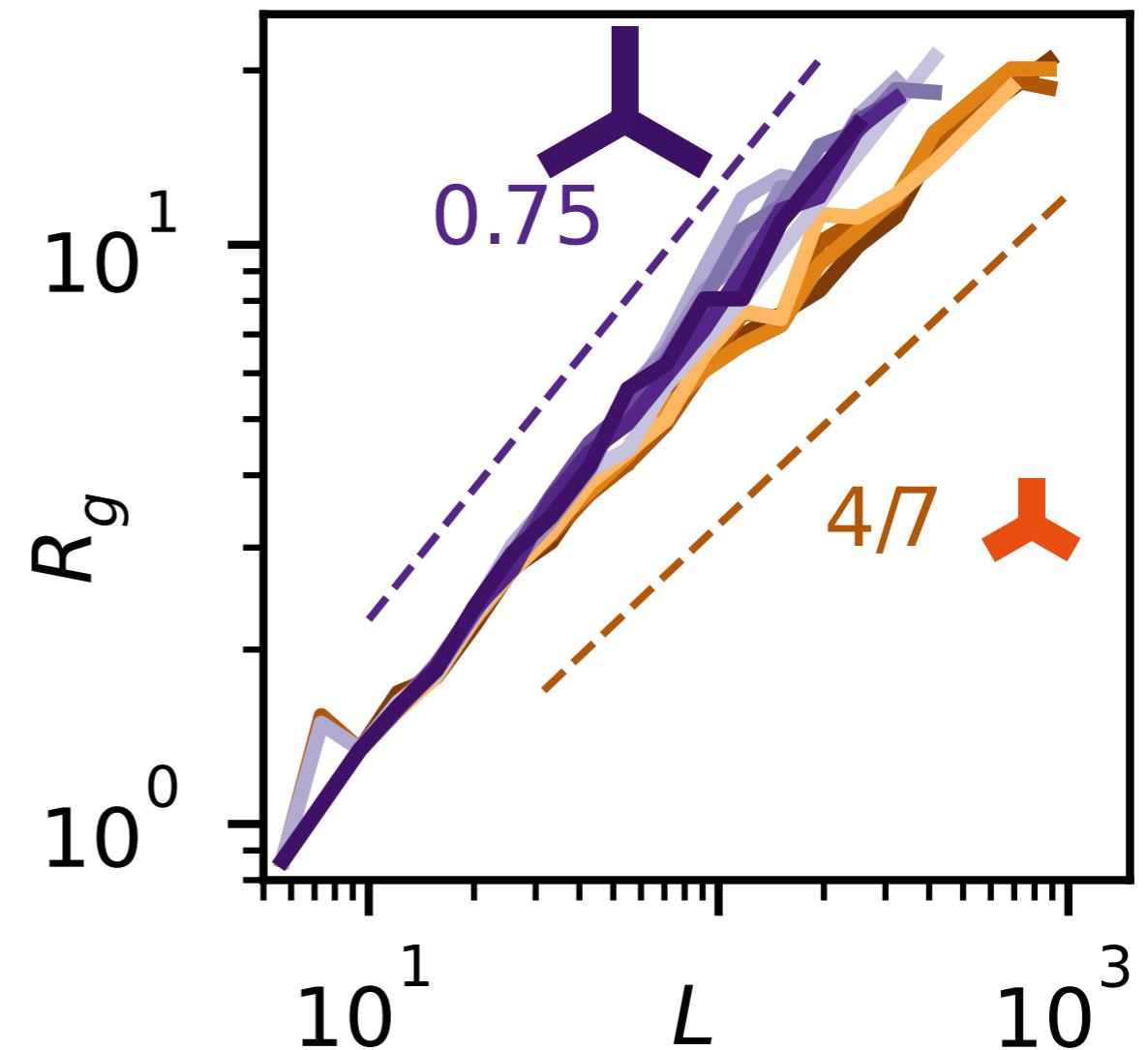
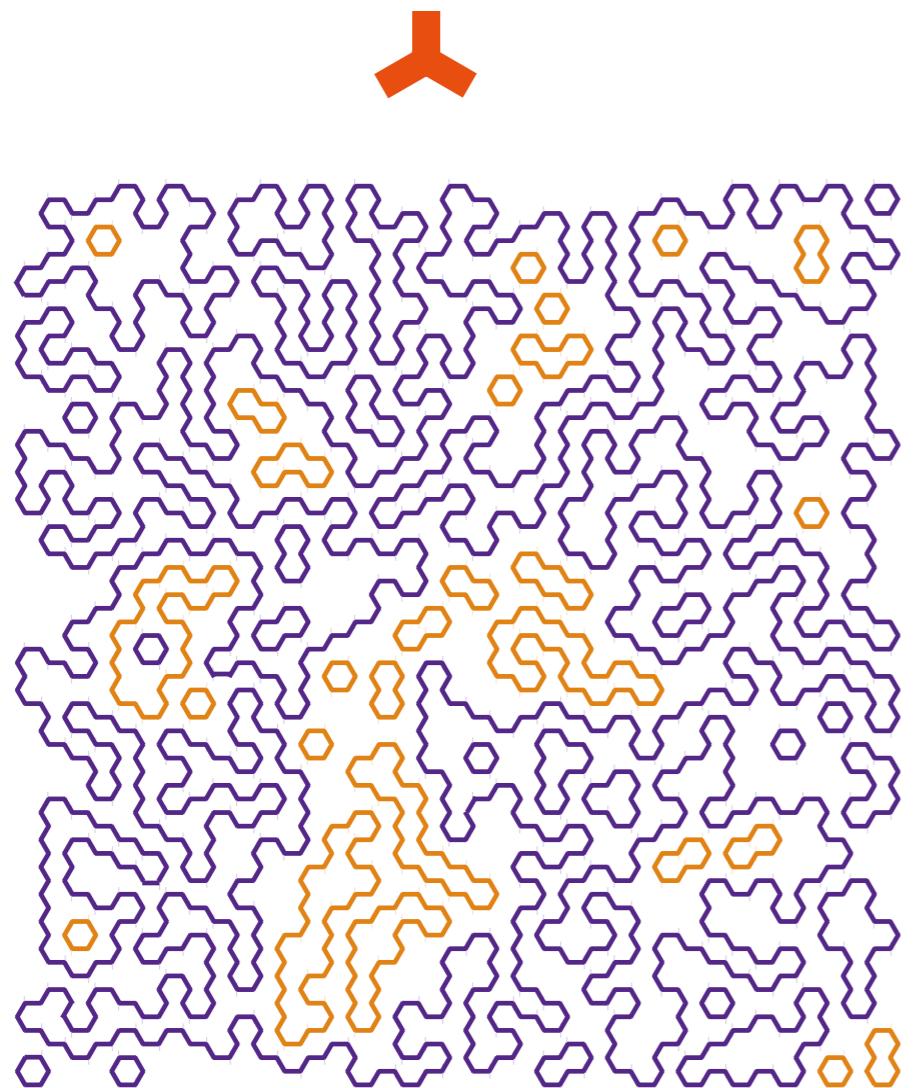
Structural change

Gyration radius



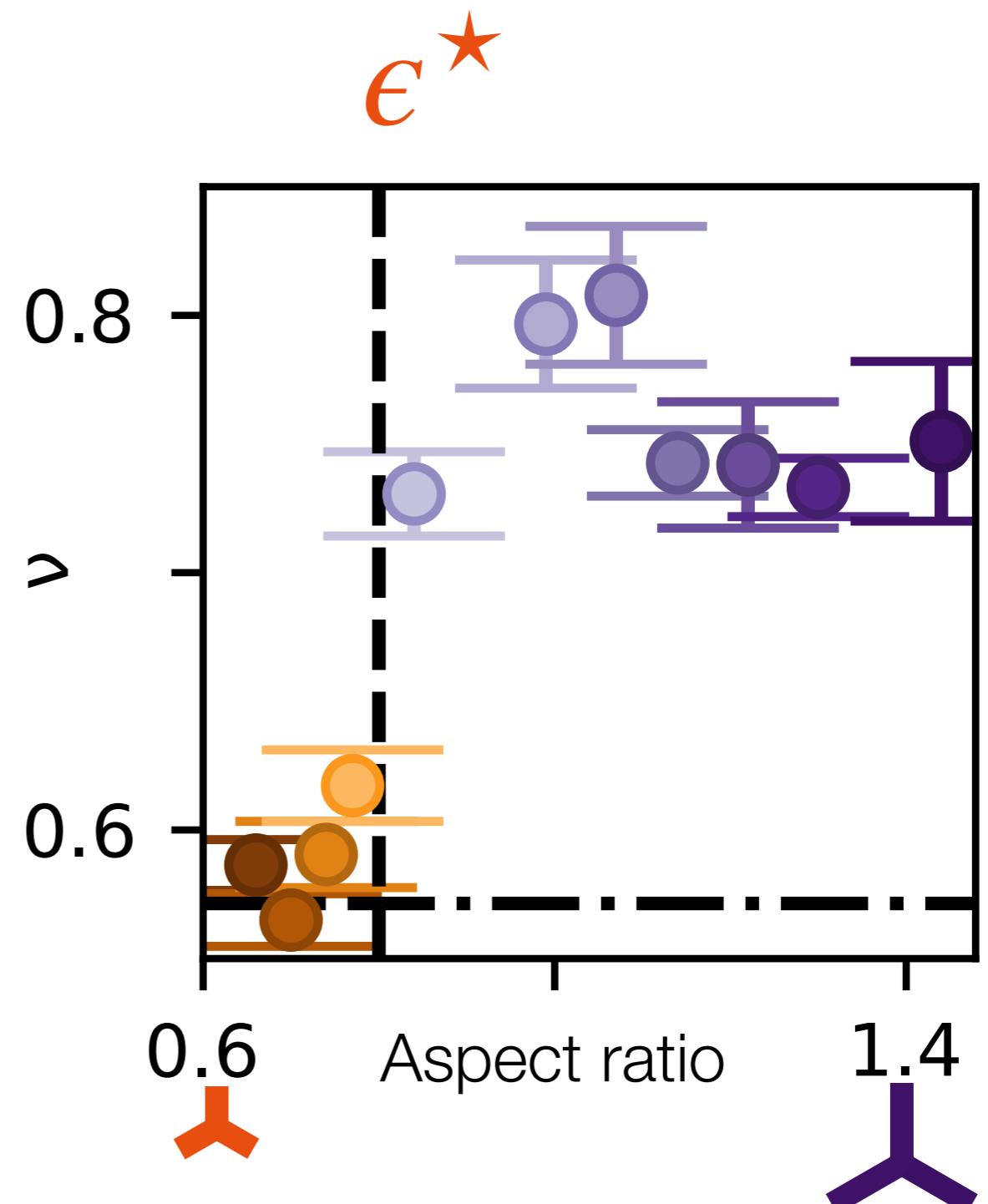
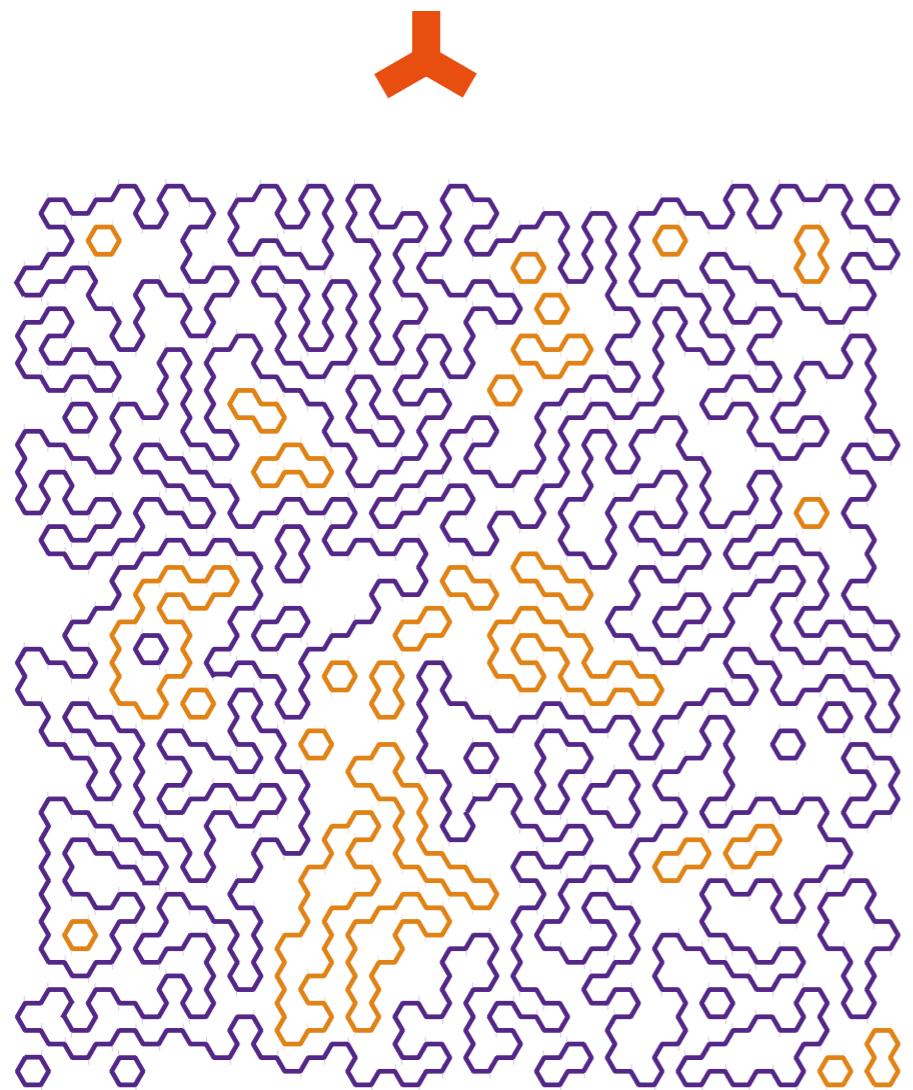
$$R_g \sim L^\nu$$

Gyration radius



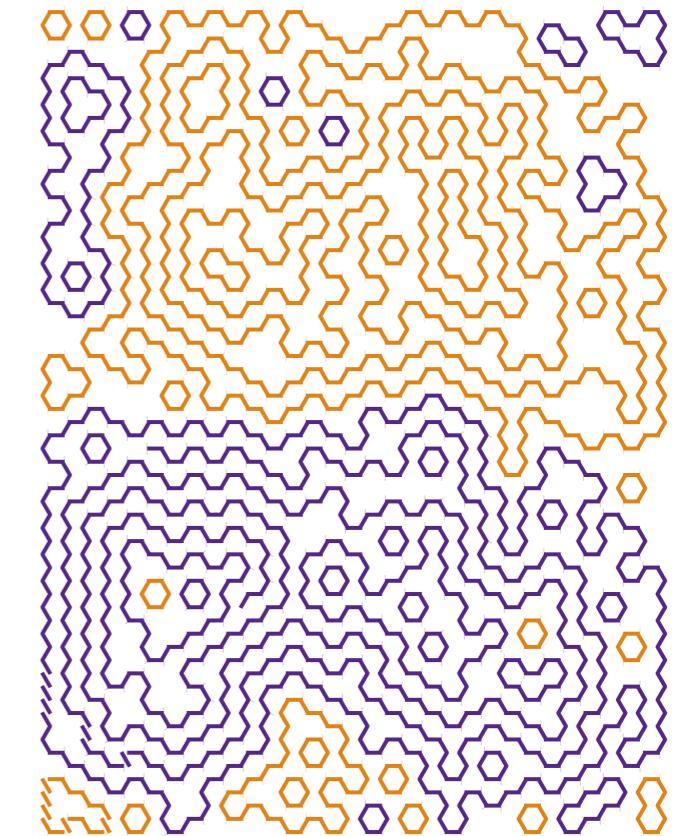
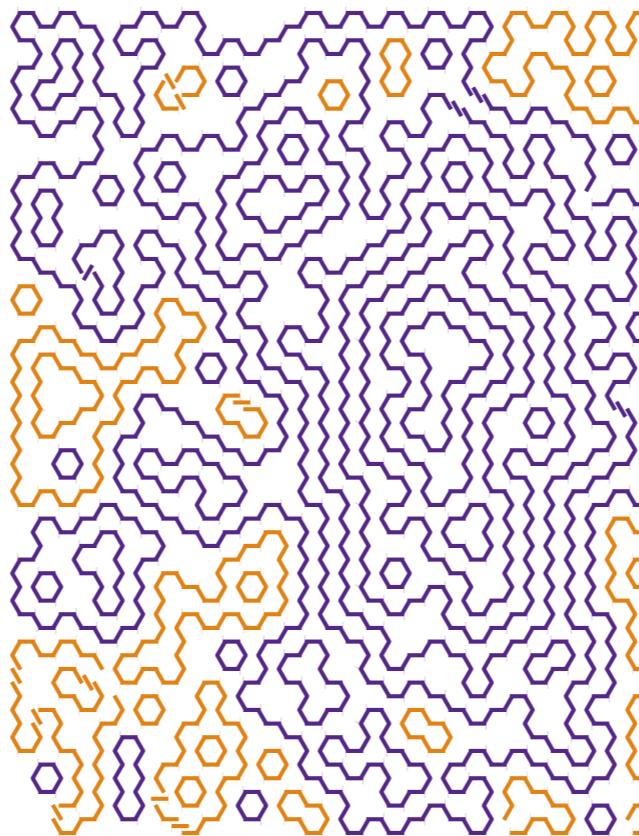
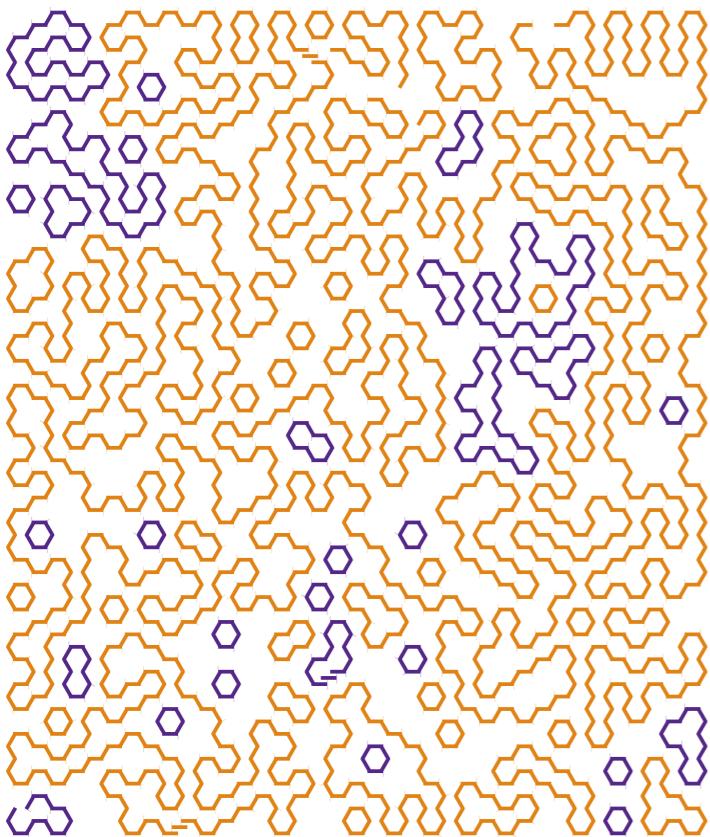
$$R_g \sim L^\nu$$

Gyration radius



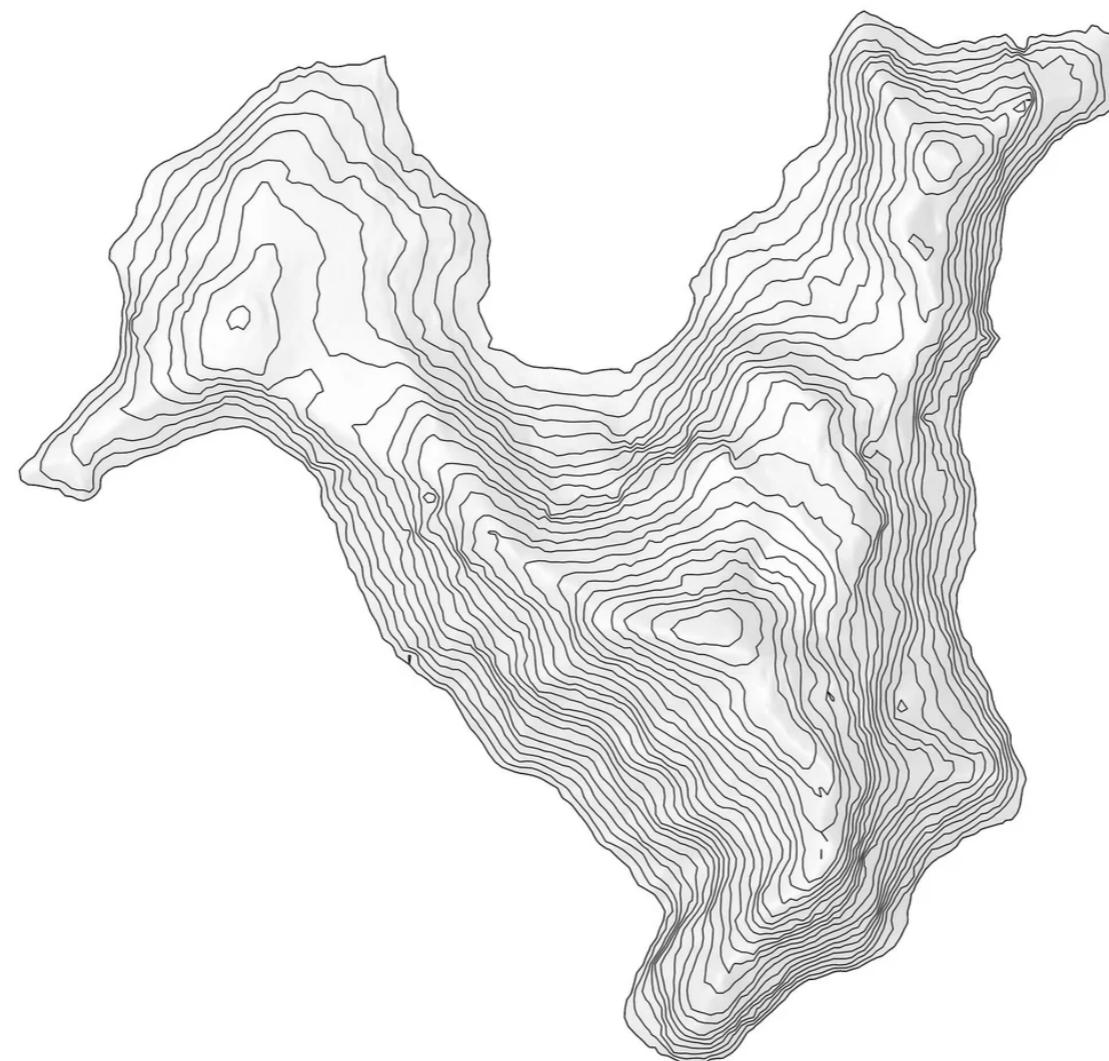
$$R_g \sim L^\nu$$

Segregated vs nested loops



Aspect ratio

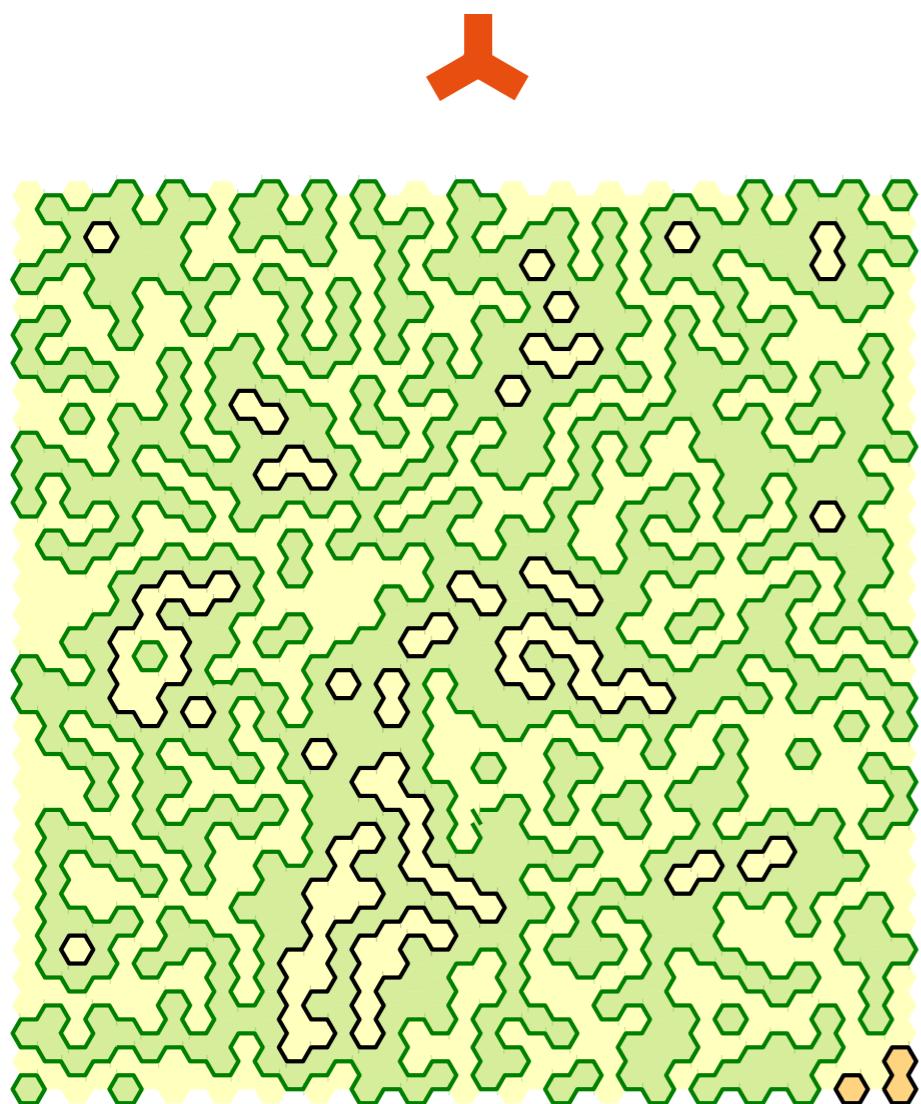
Steamlines as a landscape's contour map



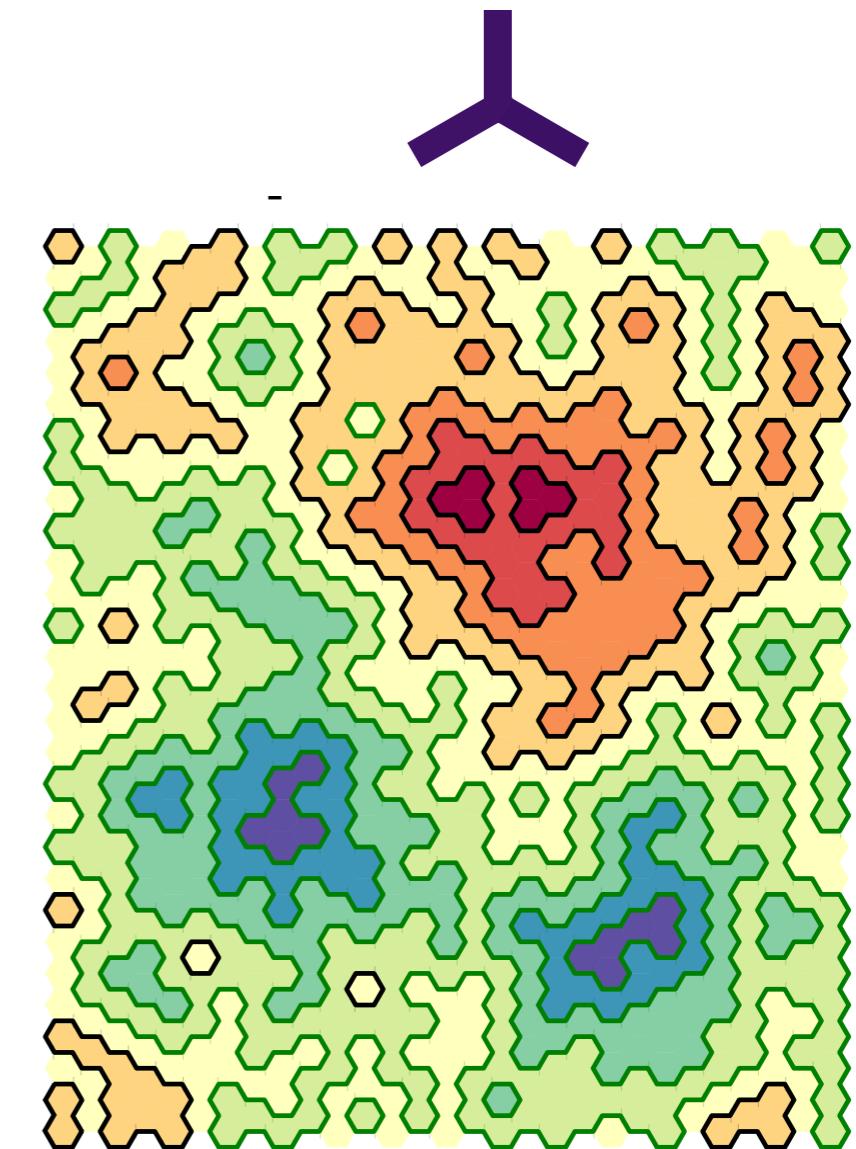
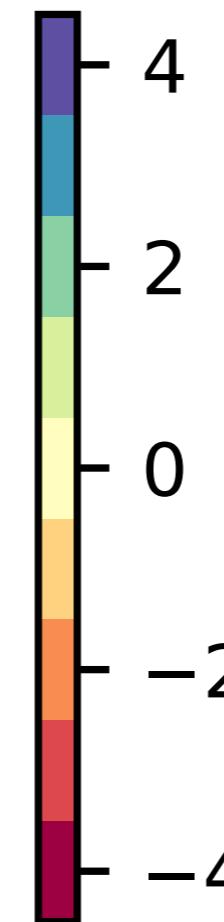
Mont Blanc

4,808m | 15,777ft

Streamlines as a contour map



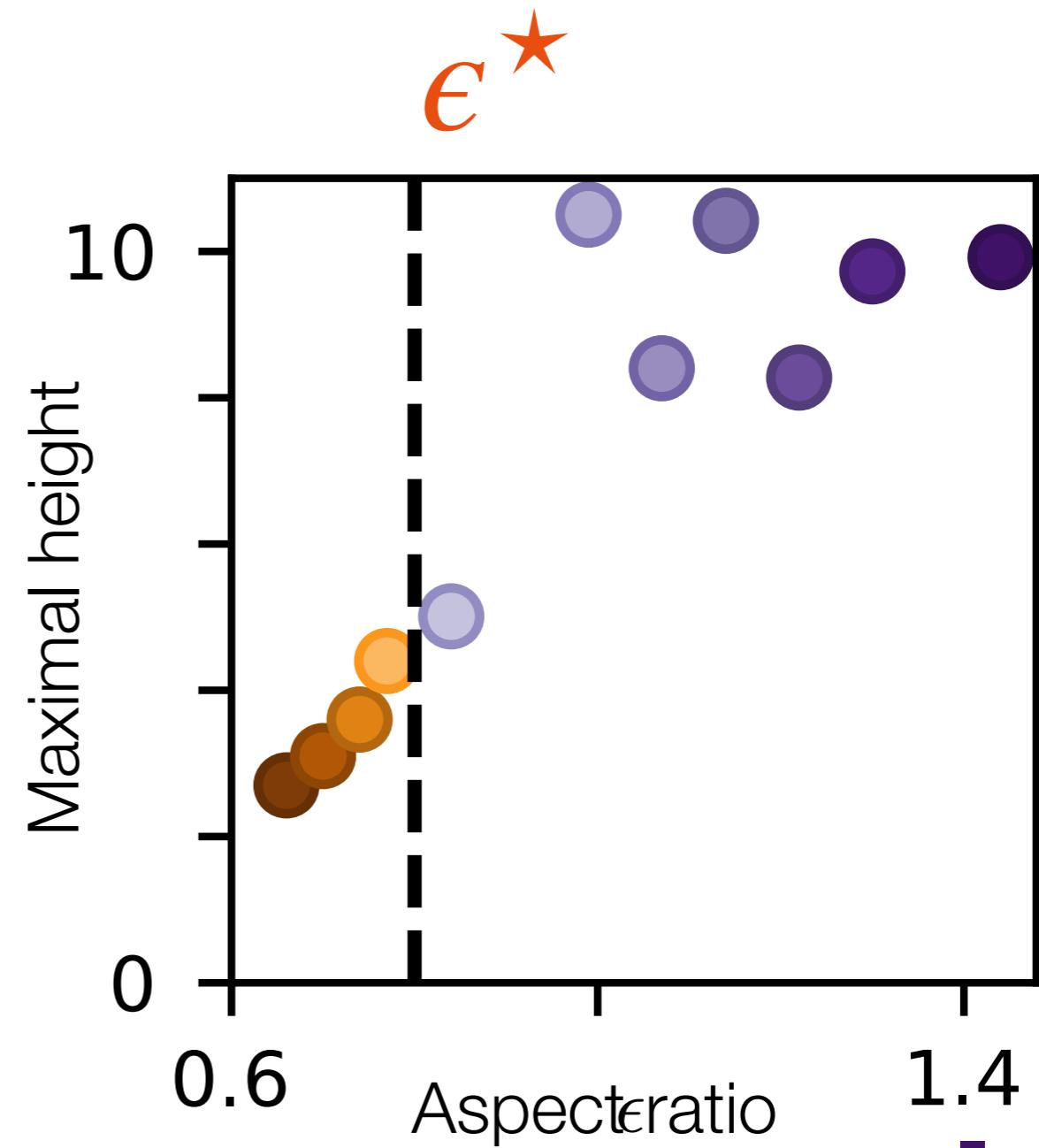
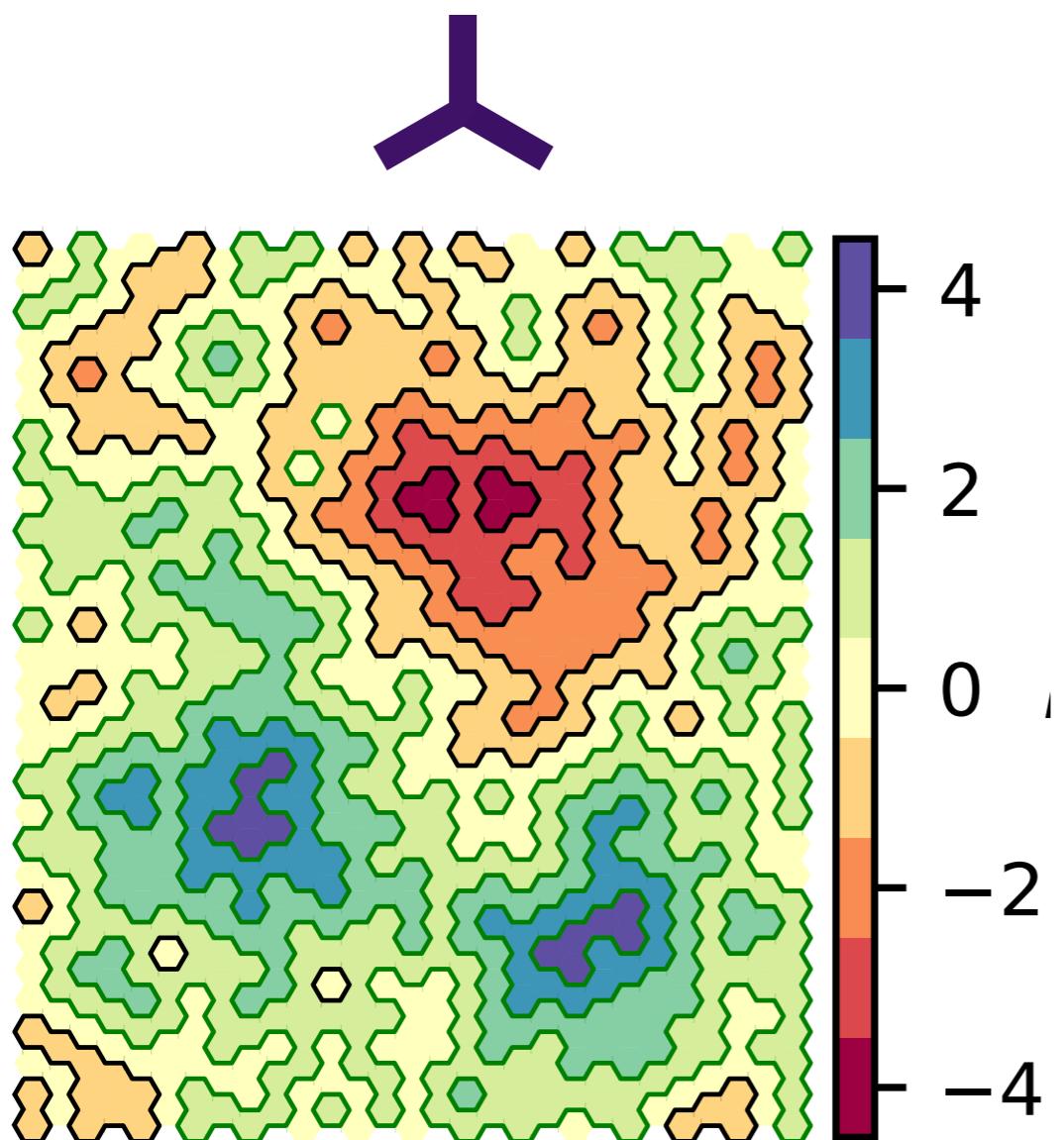
Height



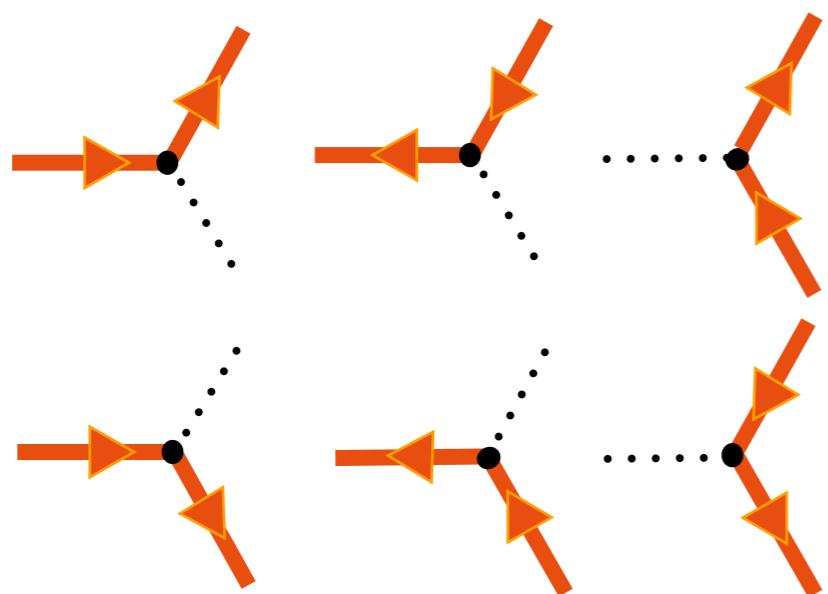
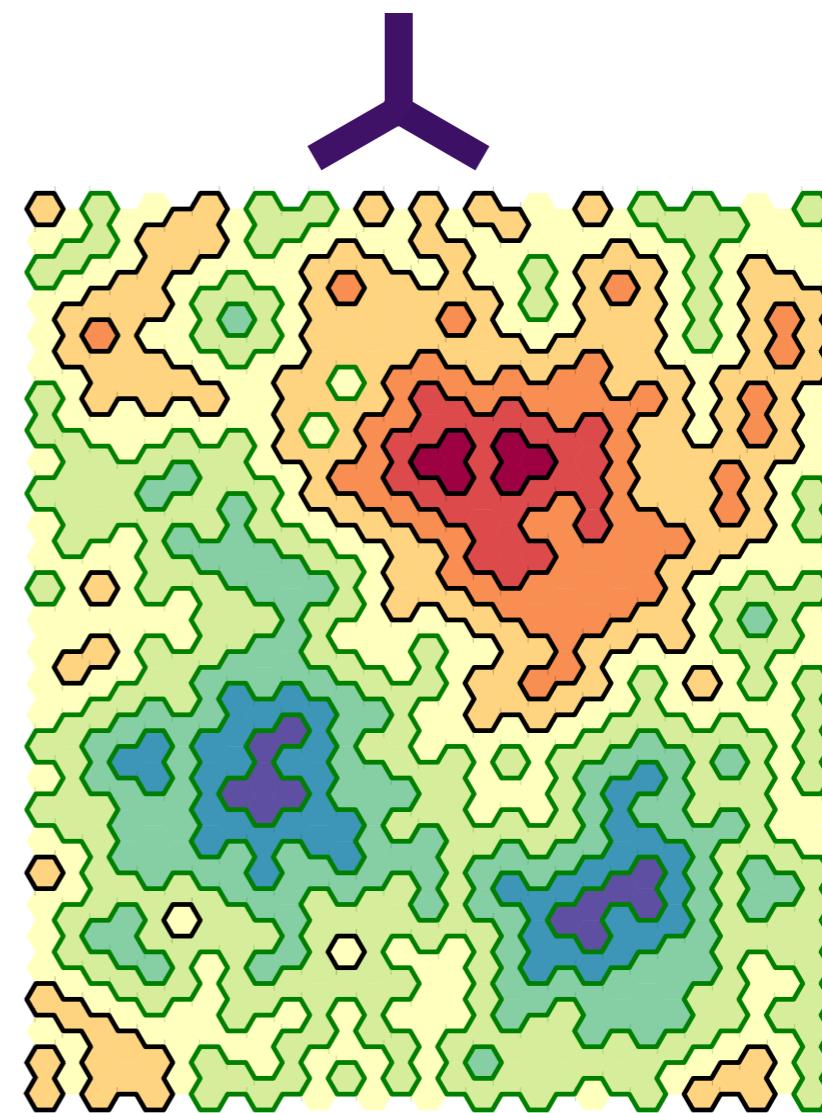
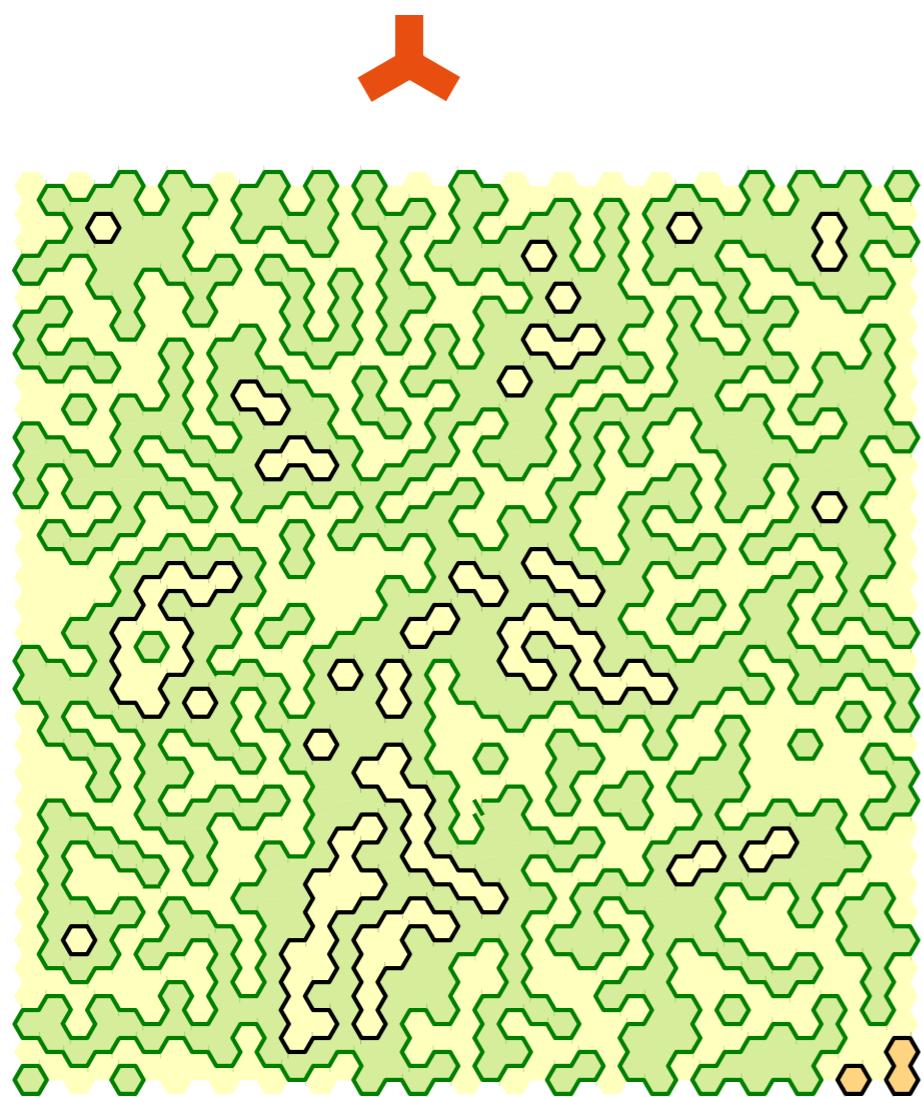
Aspect ratio

Nienhuis 1980's

Nesting Increases



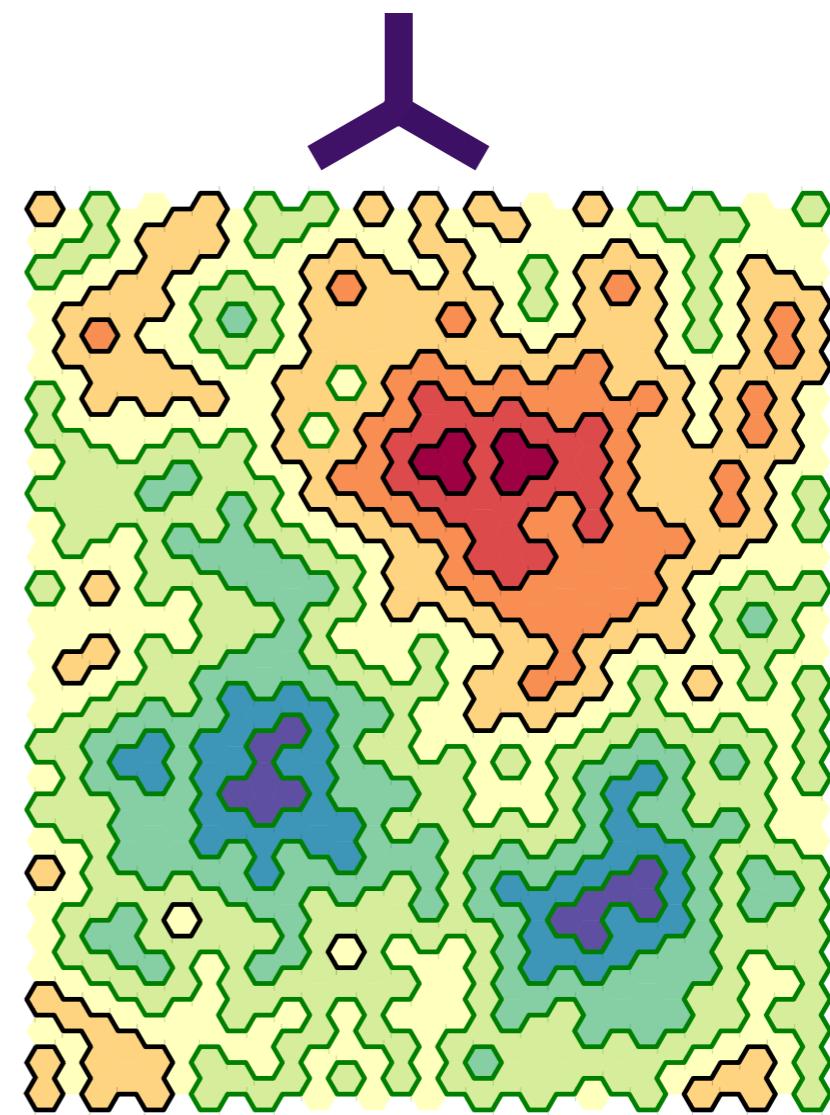
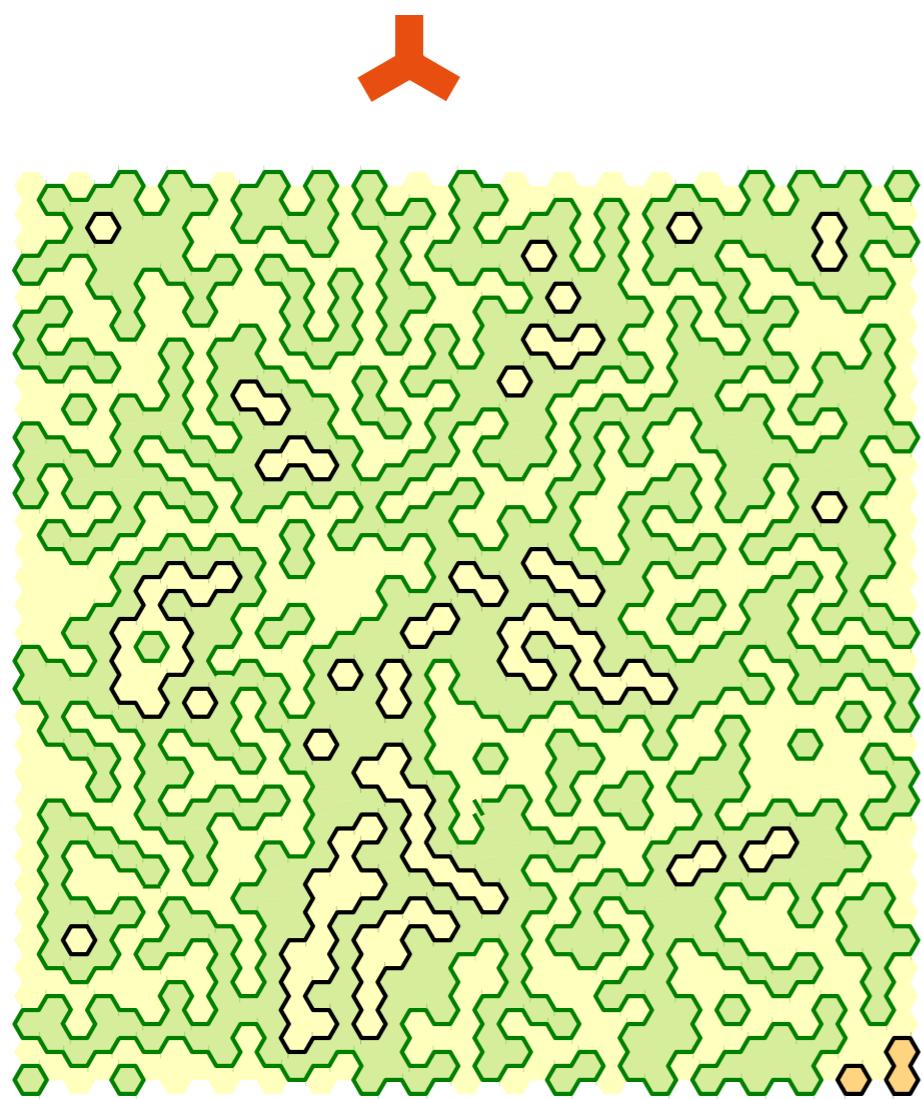
Structural Diversity ??



Agnostic to the channels' geometry...

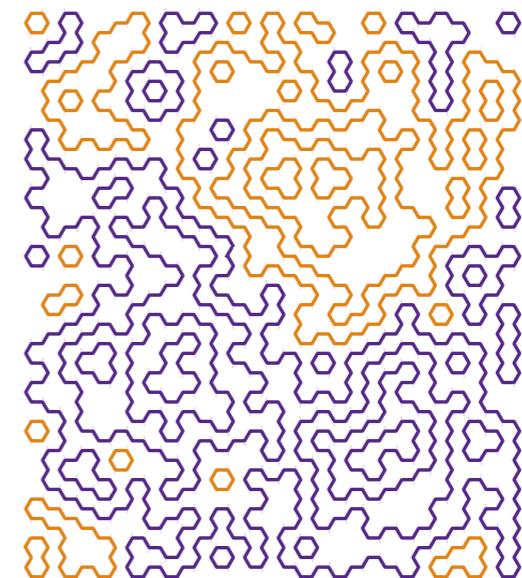
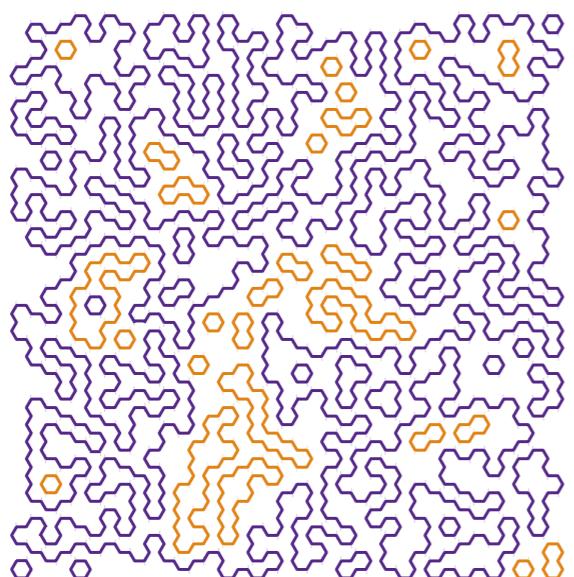
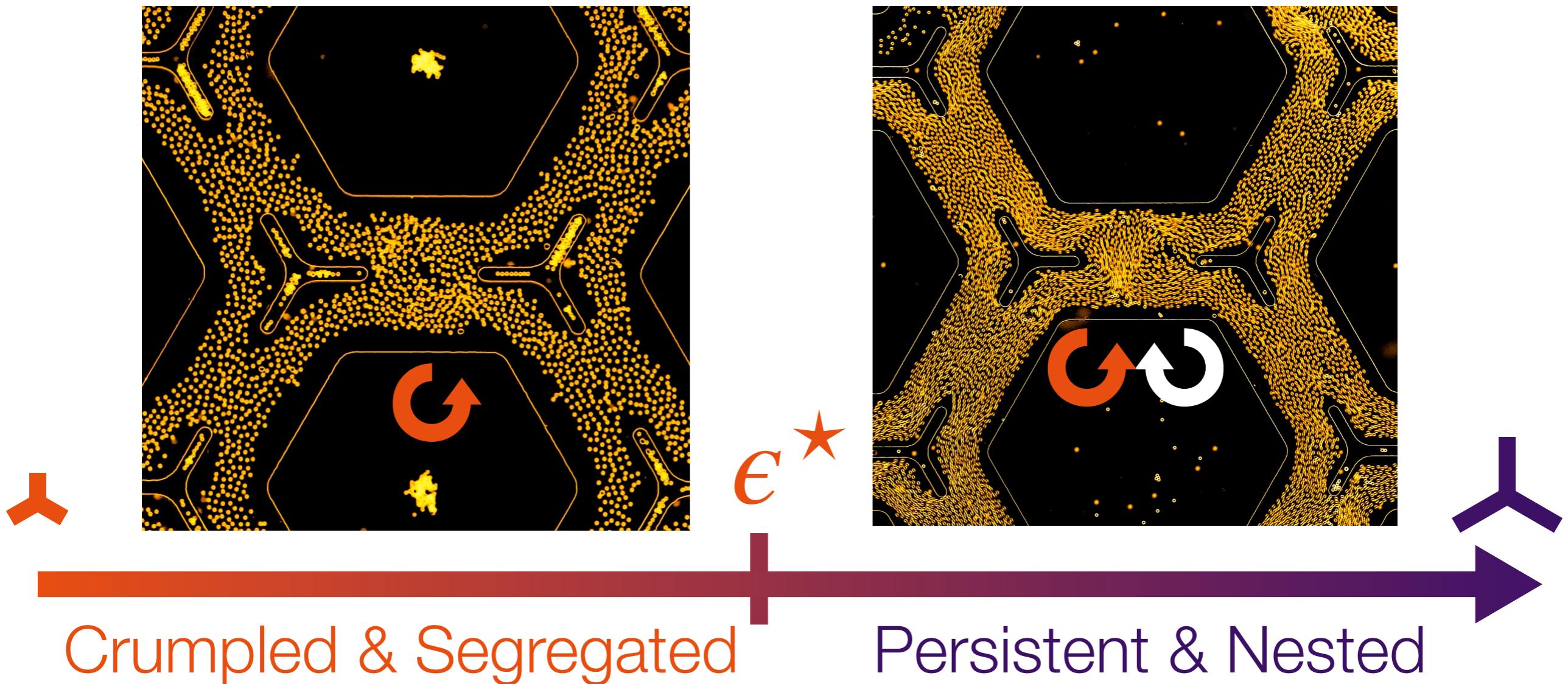
What's missing??

Structural Diversity ??



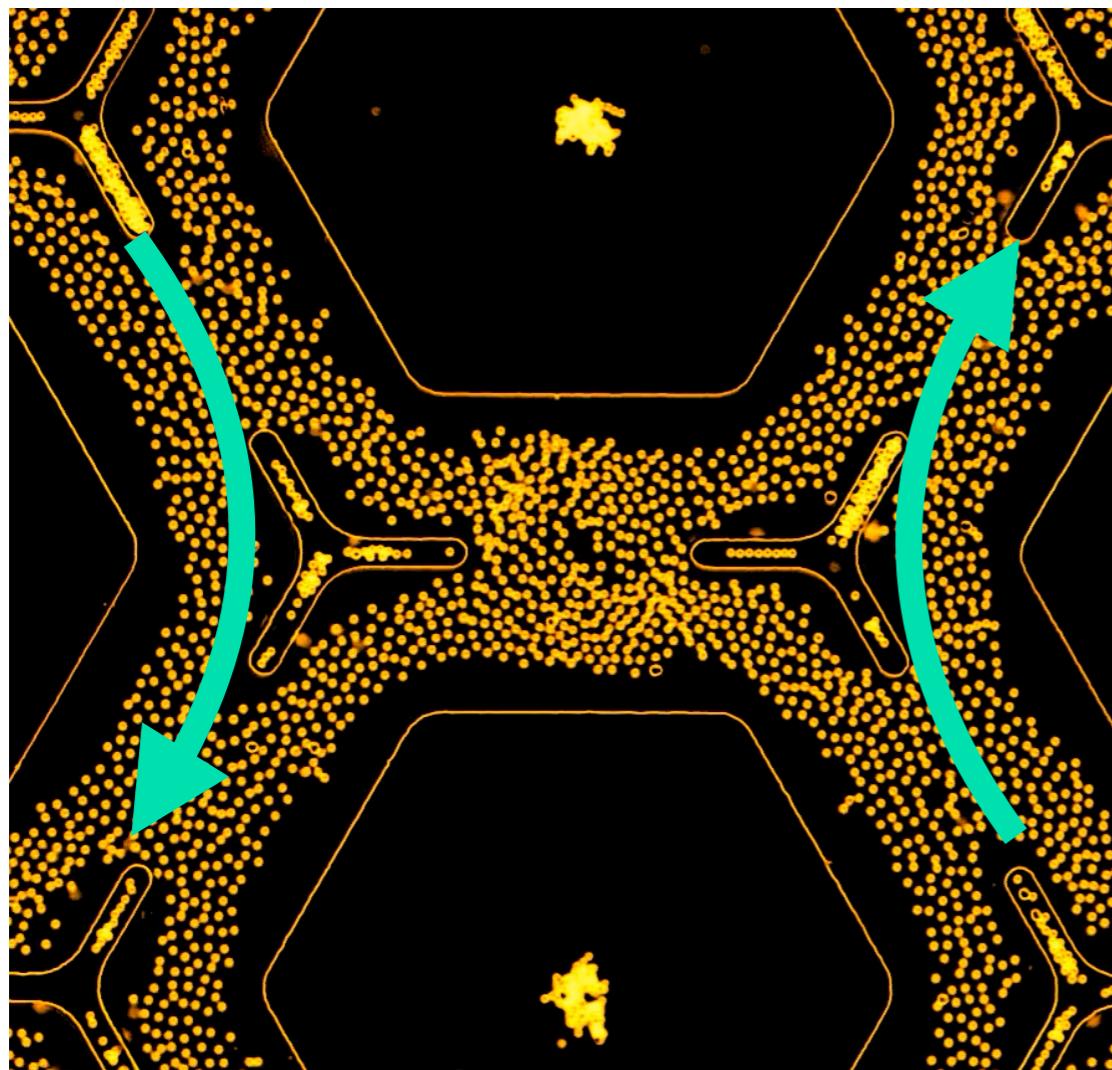
Interactions between stream lines

Structure of the zero-current channels

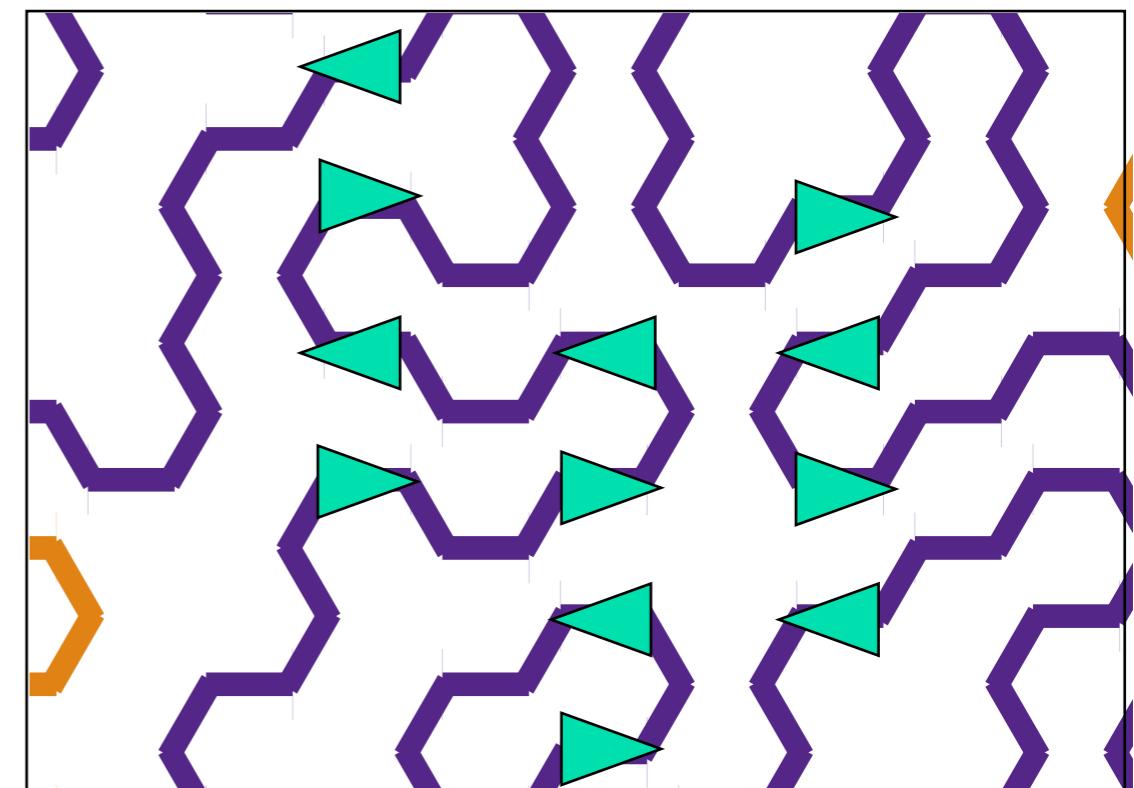


Coupling Symmetries

Antiferromagnetic

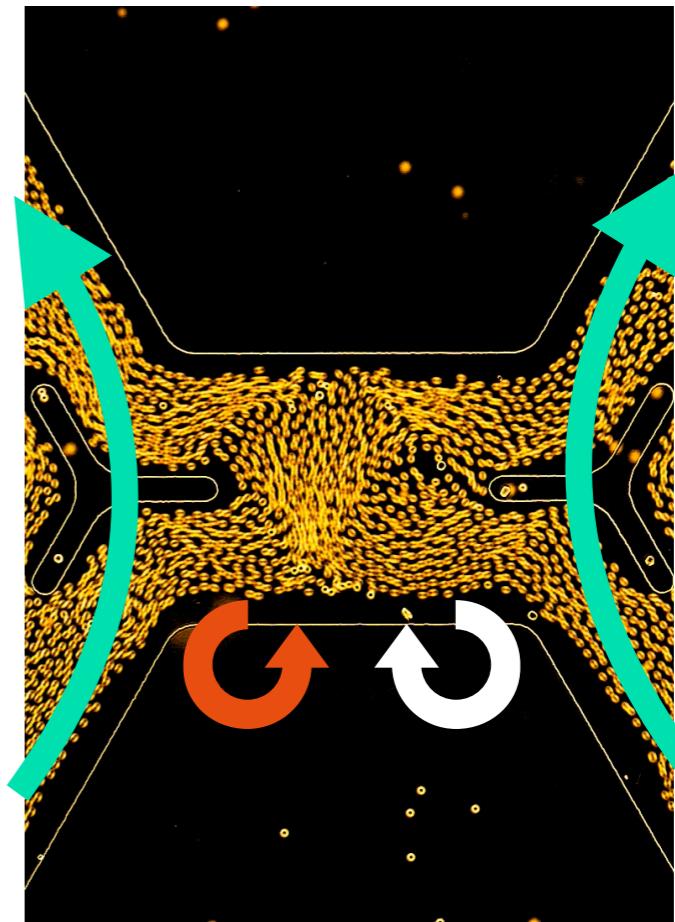


Favors
hairpins & crumples



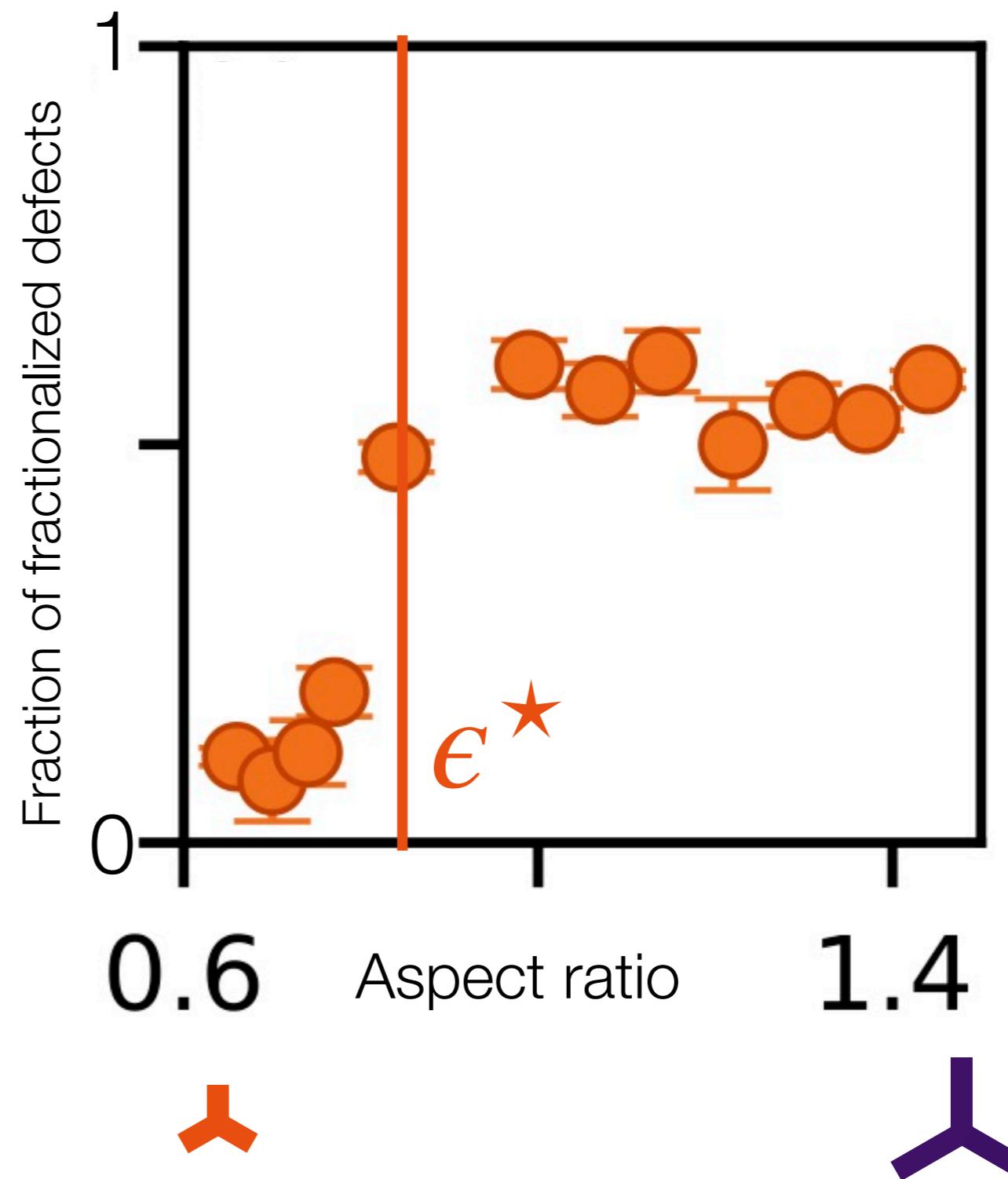
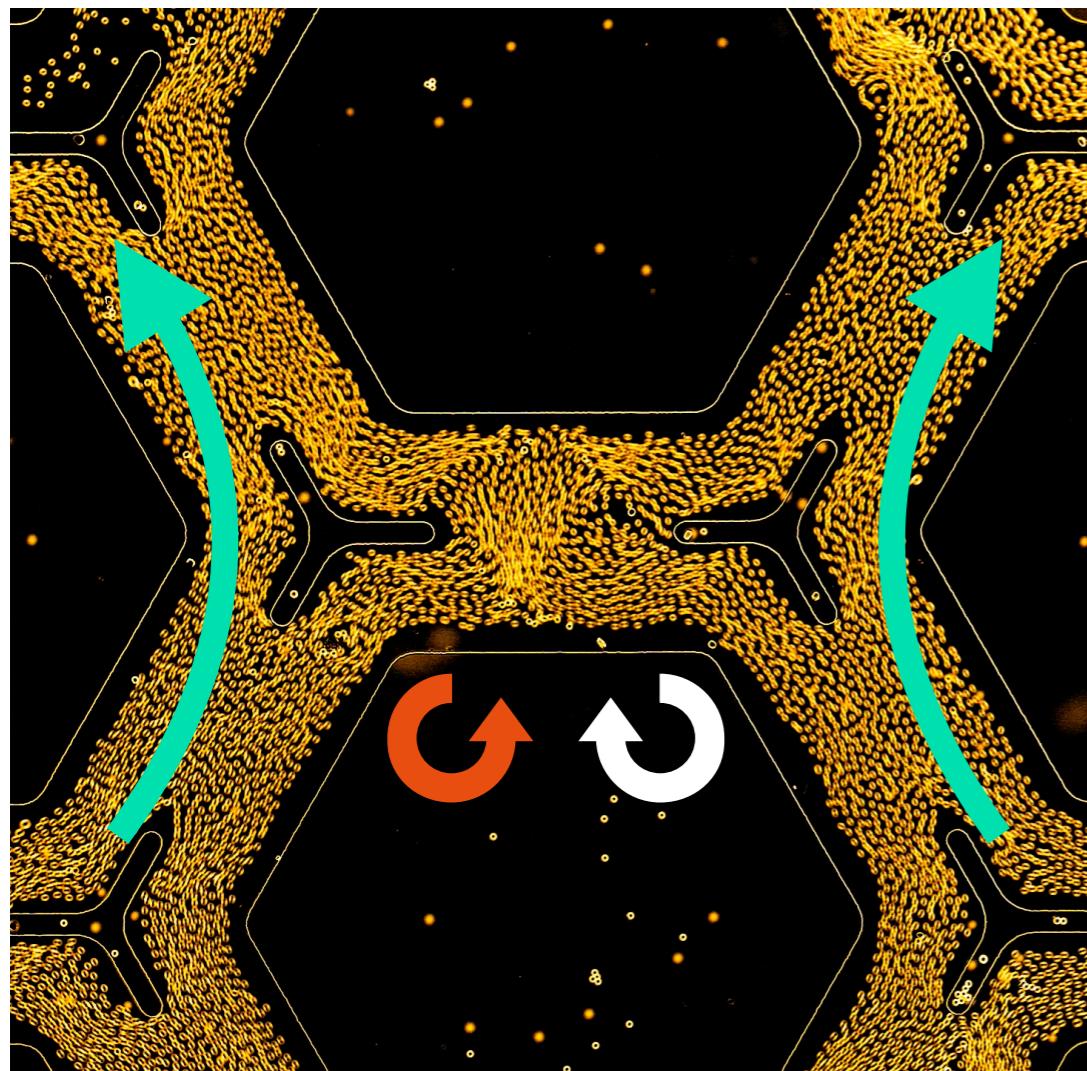
Coupling Symmetries?

Ferromagnetic



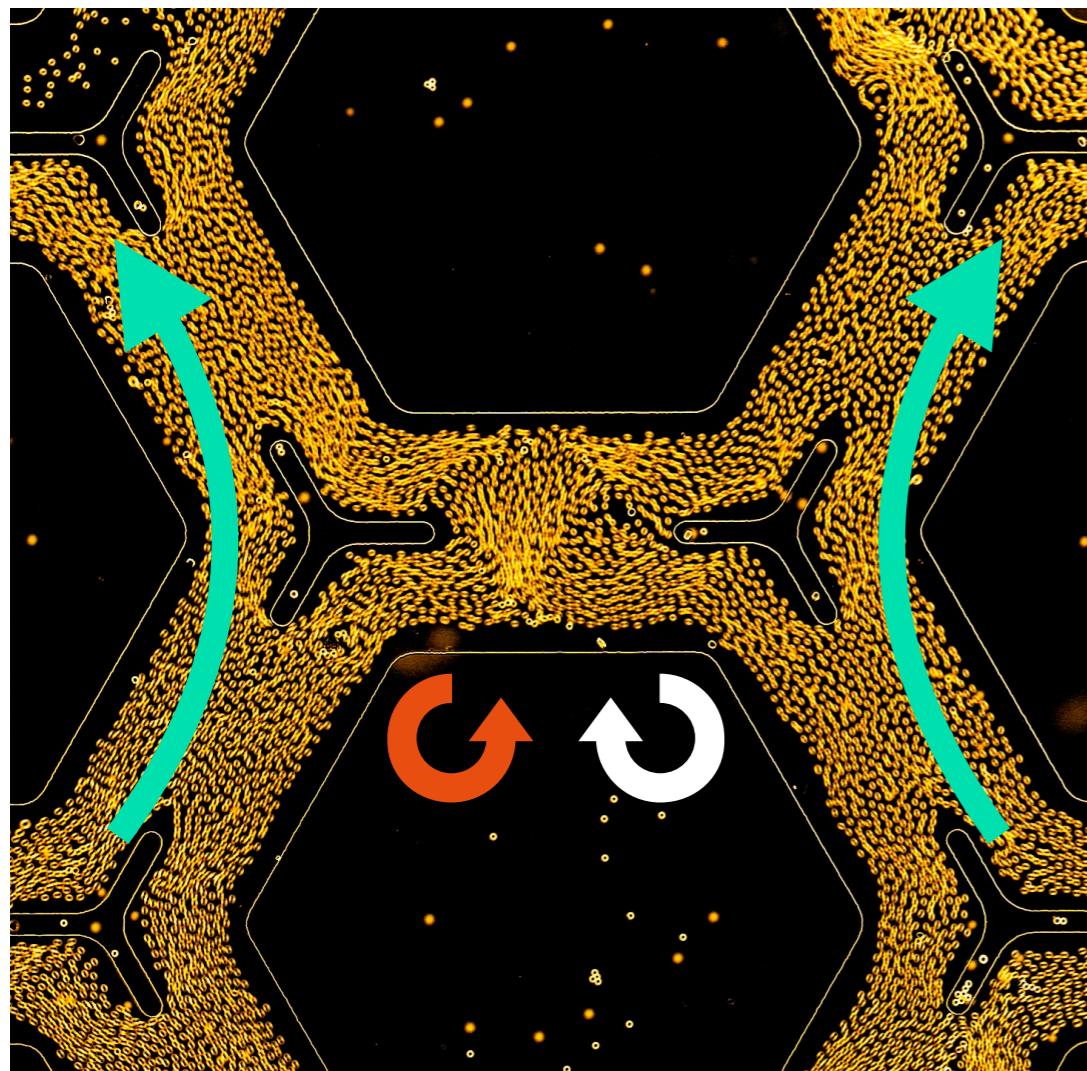
Ferromagnetic interactions prevails

Ferromagnetic

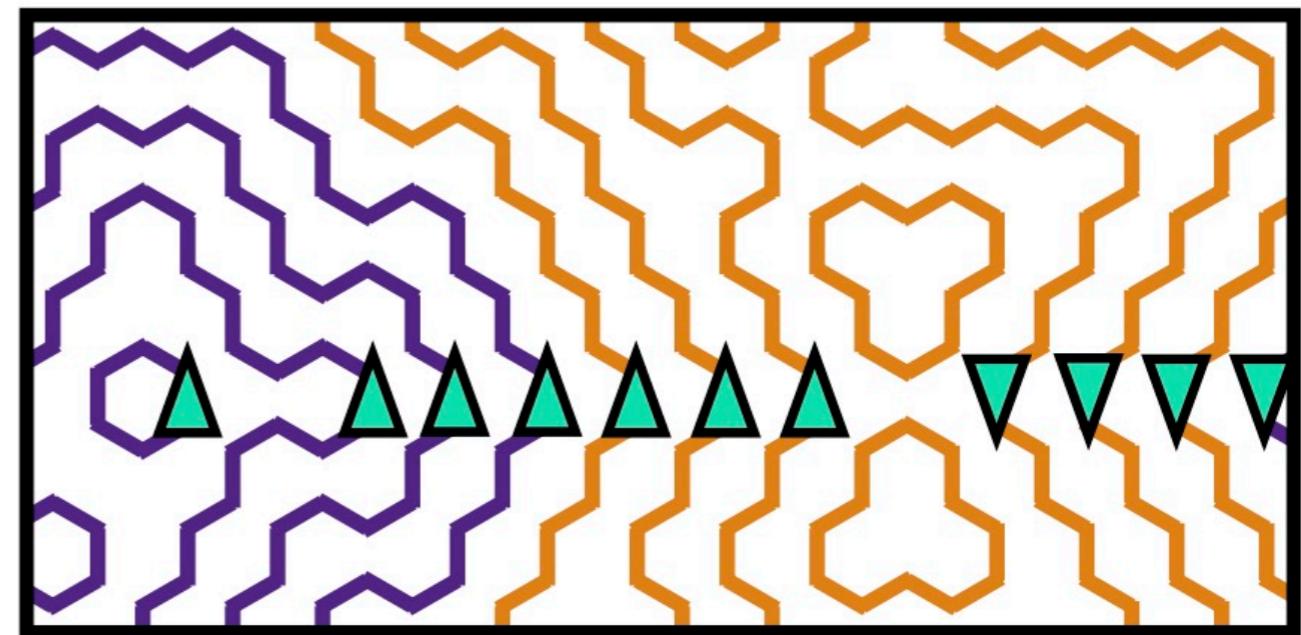


Ferromagnetic interactions prevails

Ferromagnetic



Favors
Persistent & nested loops

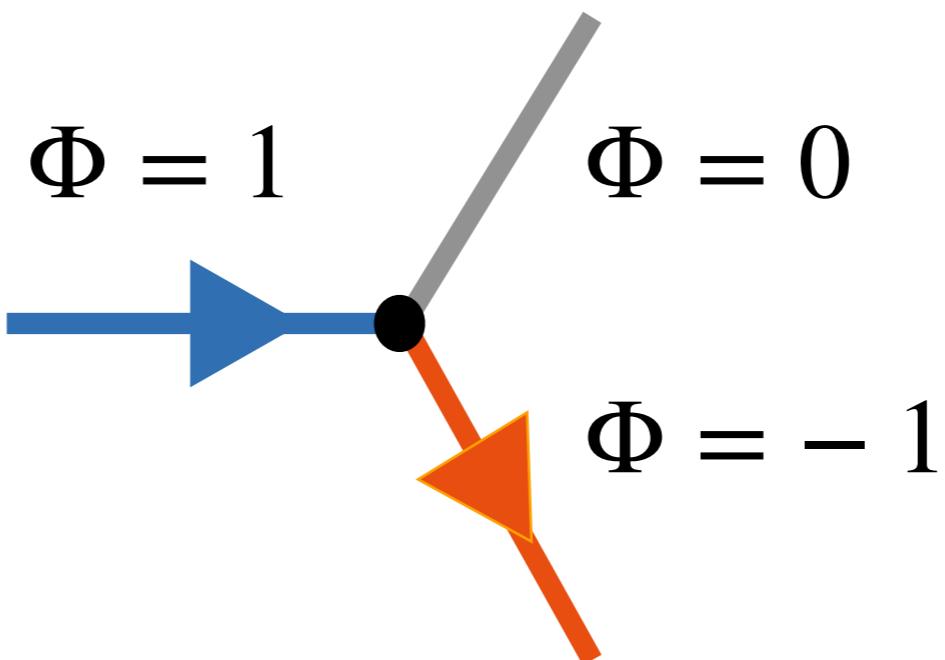


Active Hydraulics

1 – Mass conservation

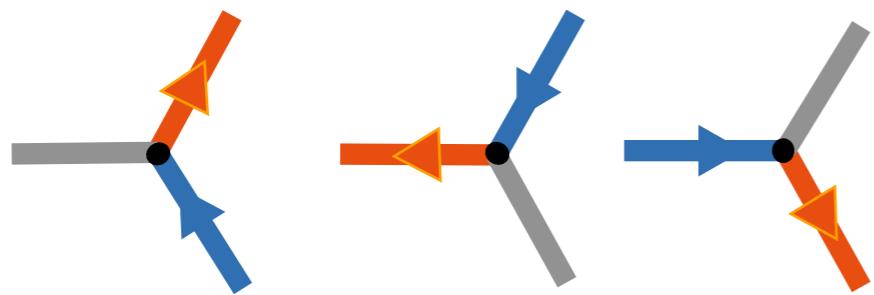
2 – Spontaneous flows

$$\sum_j \Phi_{ij} = 0 \quad \Phi_{ij} = \pm \Phi_0, 0$$

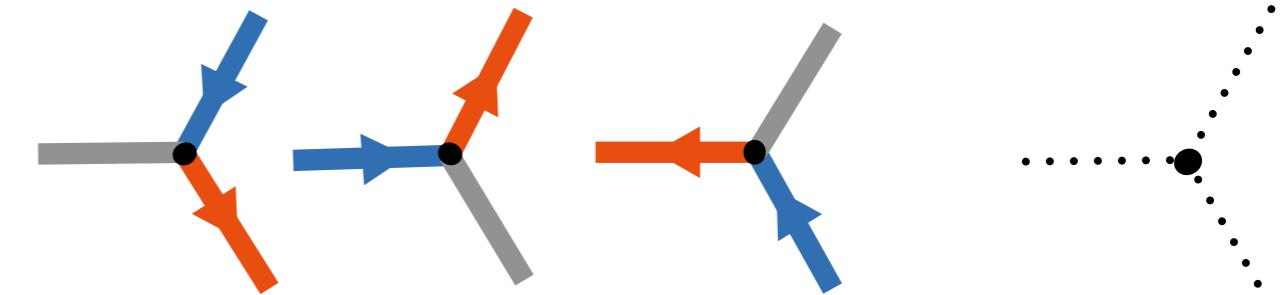


Active Hydraulics

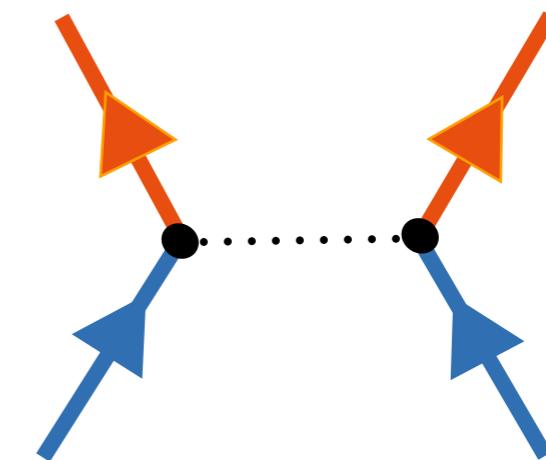
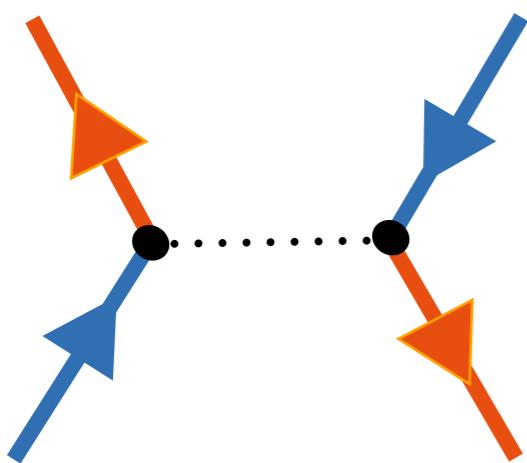
1 – Mass conservation



2 – Spontaneous flows

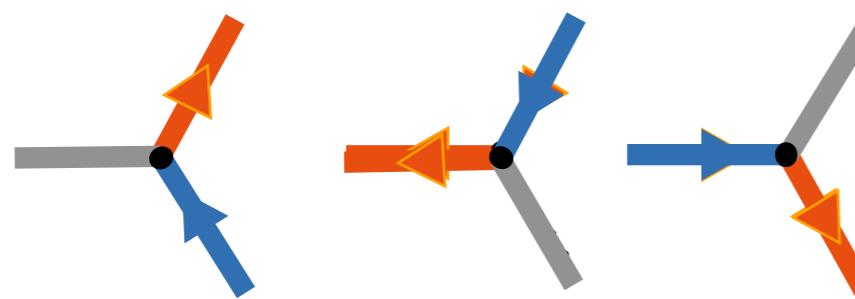


3 – Topological-defect-mediated interactions



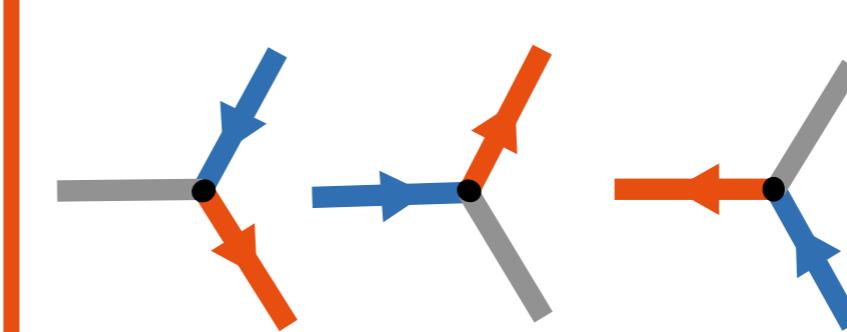
Three Coloring model

1 – Mass conservation



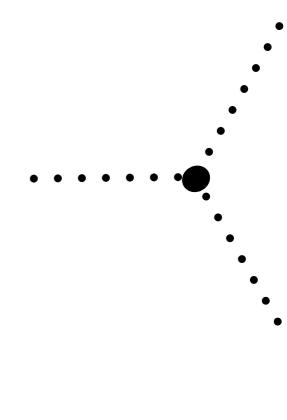
$$\sigma = +1$$

2 – Spontaneous flows



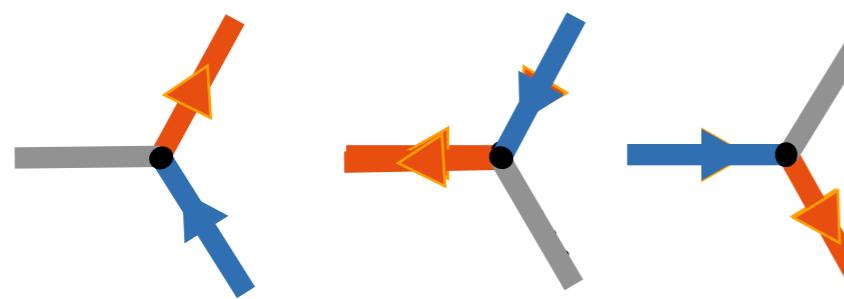
$$\sigma = -1$$

$$\sigma = 0$$

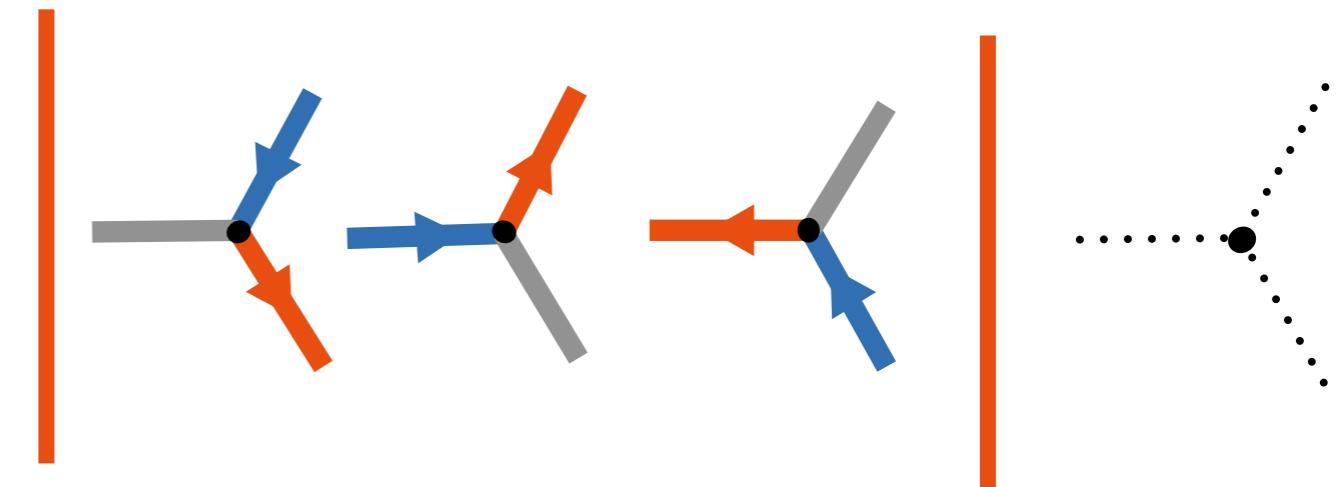


Active Hydraulics

1 – Mass conservation

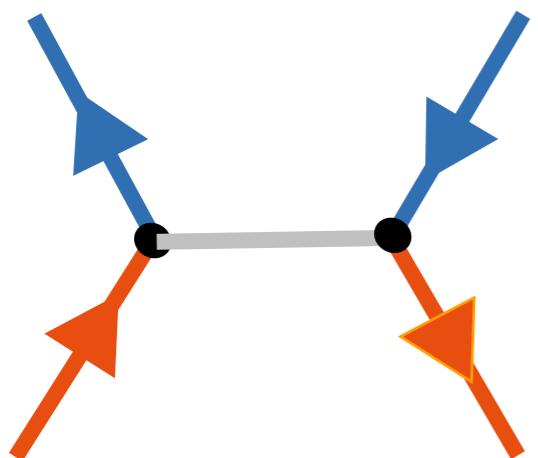


2 – Spontaneous flows

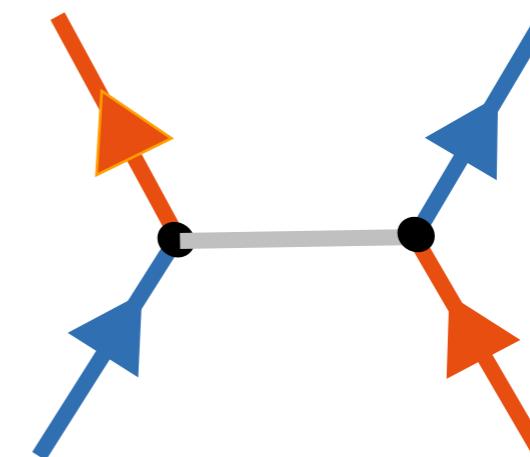


3 – Topological-defect-mediated interactions

$$\sigma_1 \sigma_2 = -1$$



$$\sigma_1 \sigma_2 = +1$$



Predicting flow patterns

Edge current:

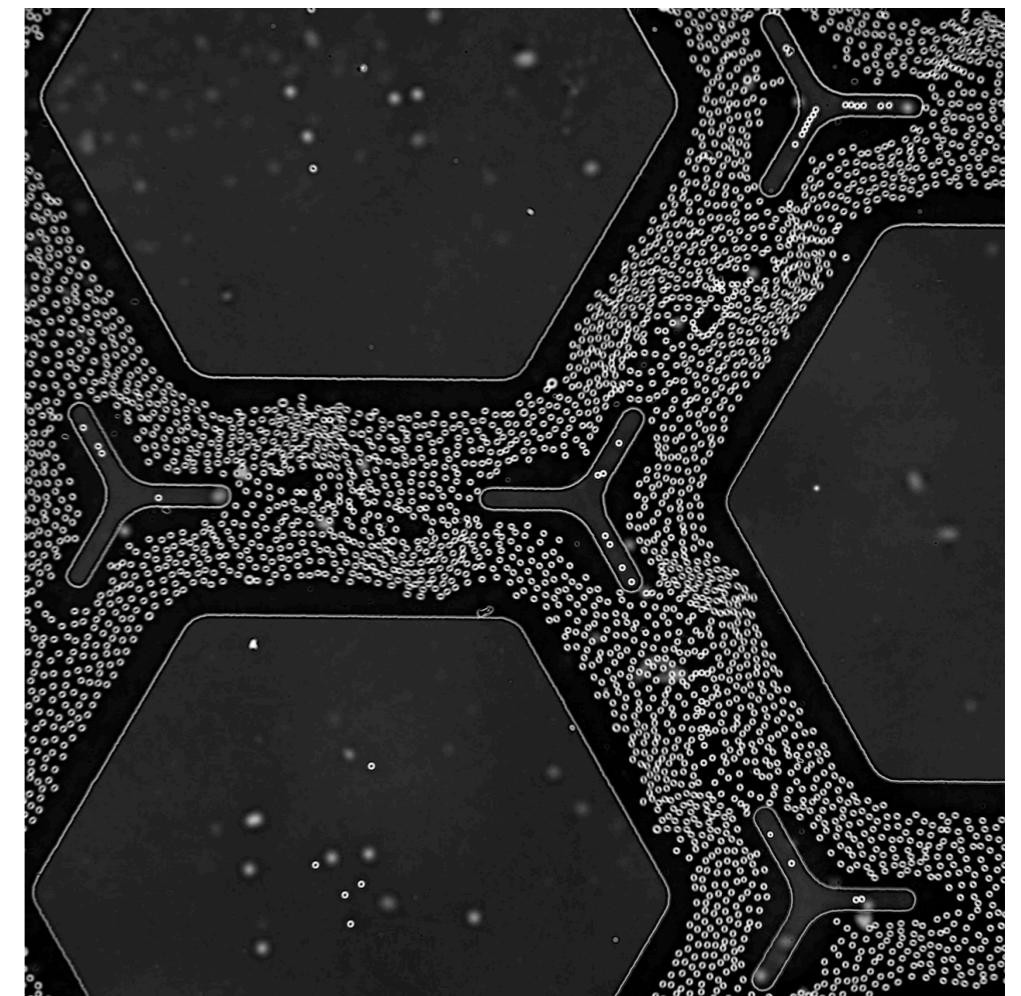
$$\Phi_{ij} = \pm 1,0$$

Node handedness

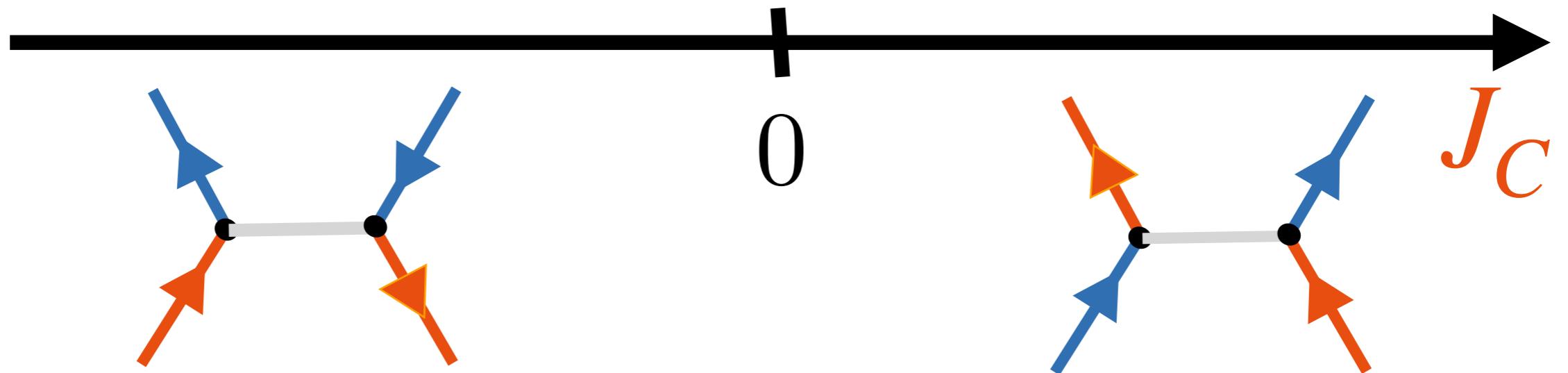
$$\sigma_i = \pm 1,0$$

Promote Spontaneous flows

$$\mathcal{H} = -J_A \sum_{\langle i,j \rangle} \Phi_{ij}^2$$



Stramline Interactions



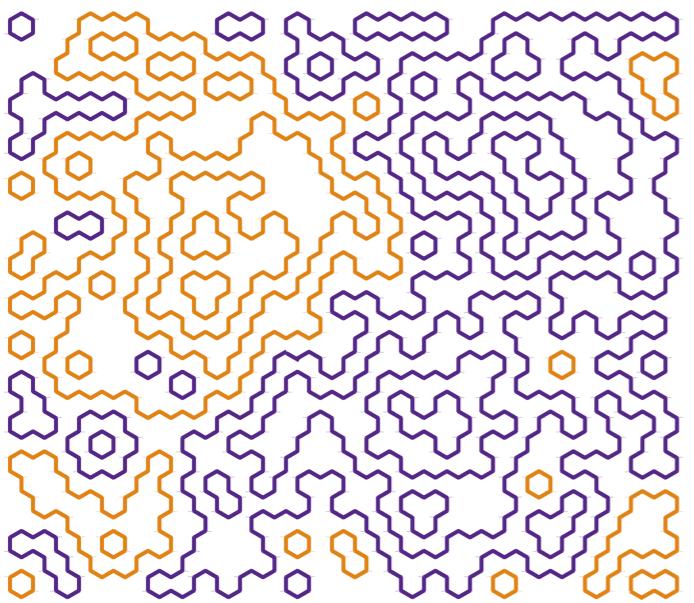
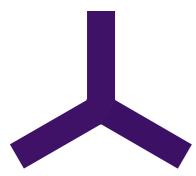
$$\mathcal{H} = -J_A \sum_{\langle i,j \rangle} \Phi_{ij}^2 - J_C \sum_{\langle i,j \rangle} \delta_{\Phi_{ij},0} \sigma_i \sigma_j.$$

Minimize given the mass-conservation constraint

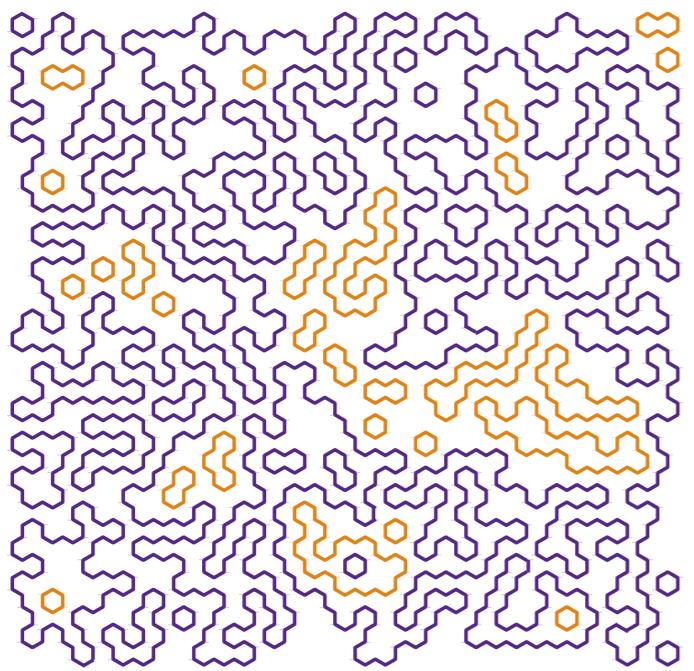
$$\sum_j \Phi_{ij} = 0$$

Experiments

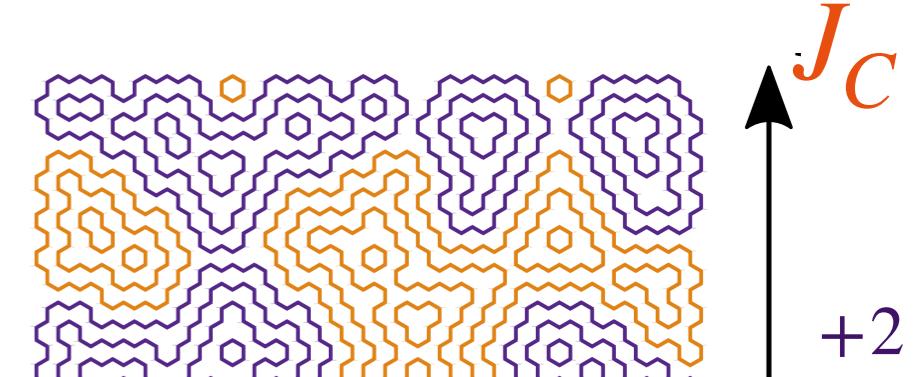
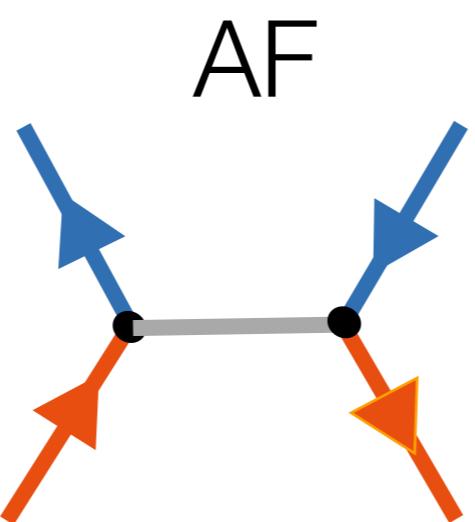
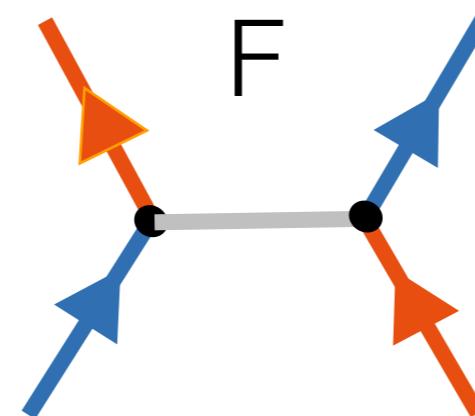
Theory



↑
 ϵ^*

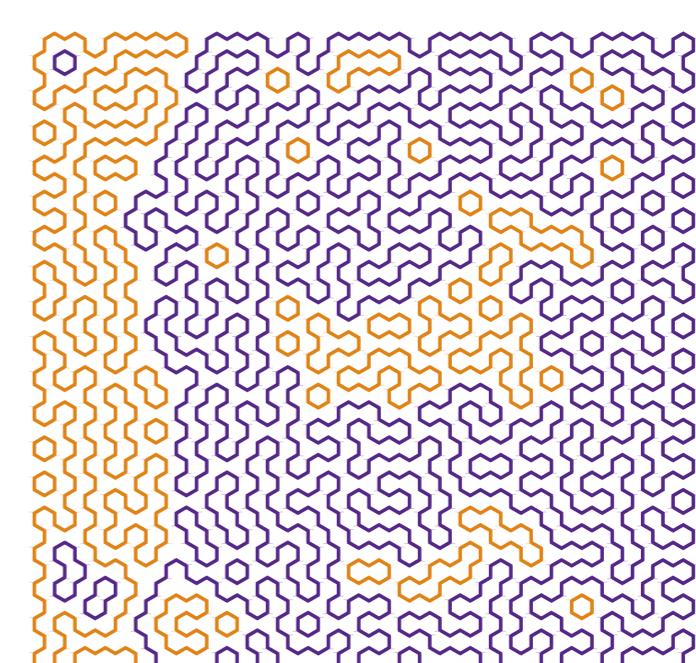


*3



J_C

+2

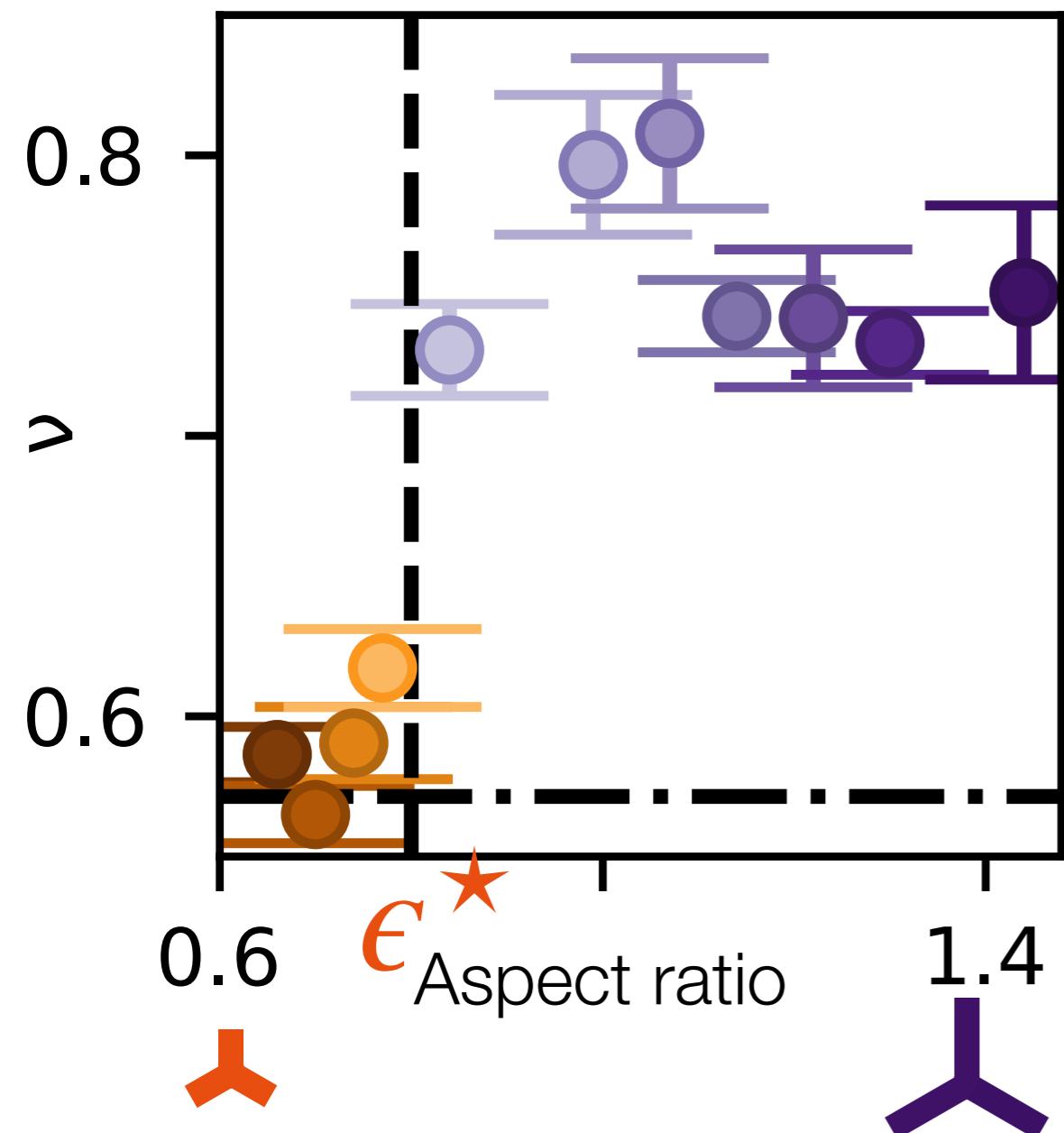


0

-2

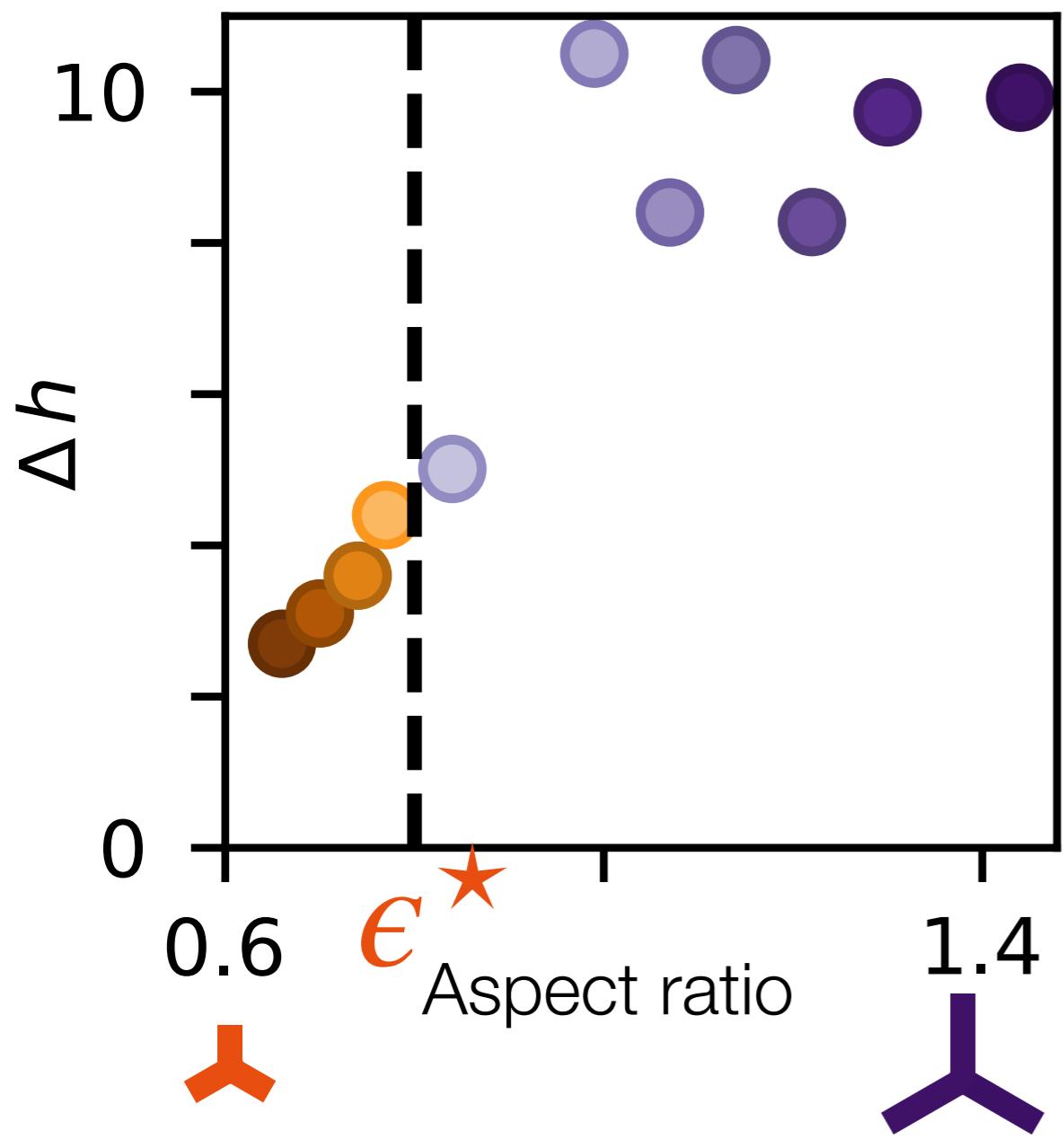
Crumpling of the stream lines

Experiments

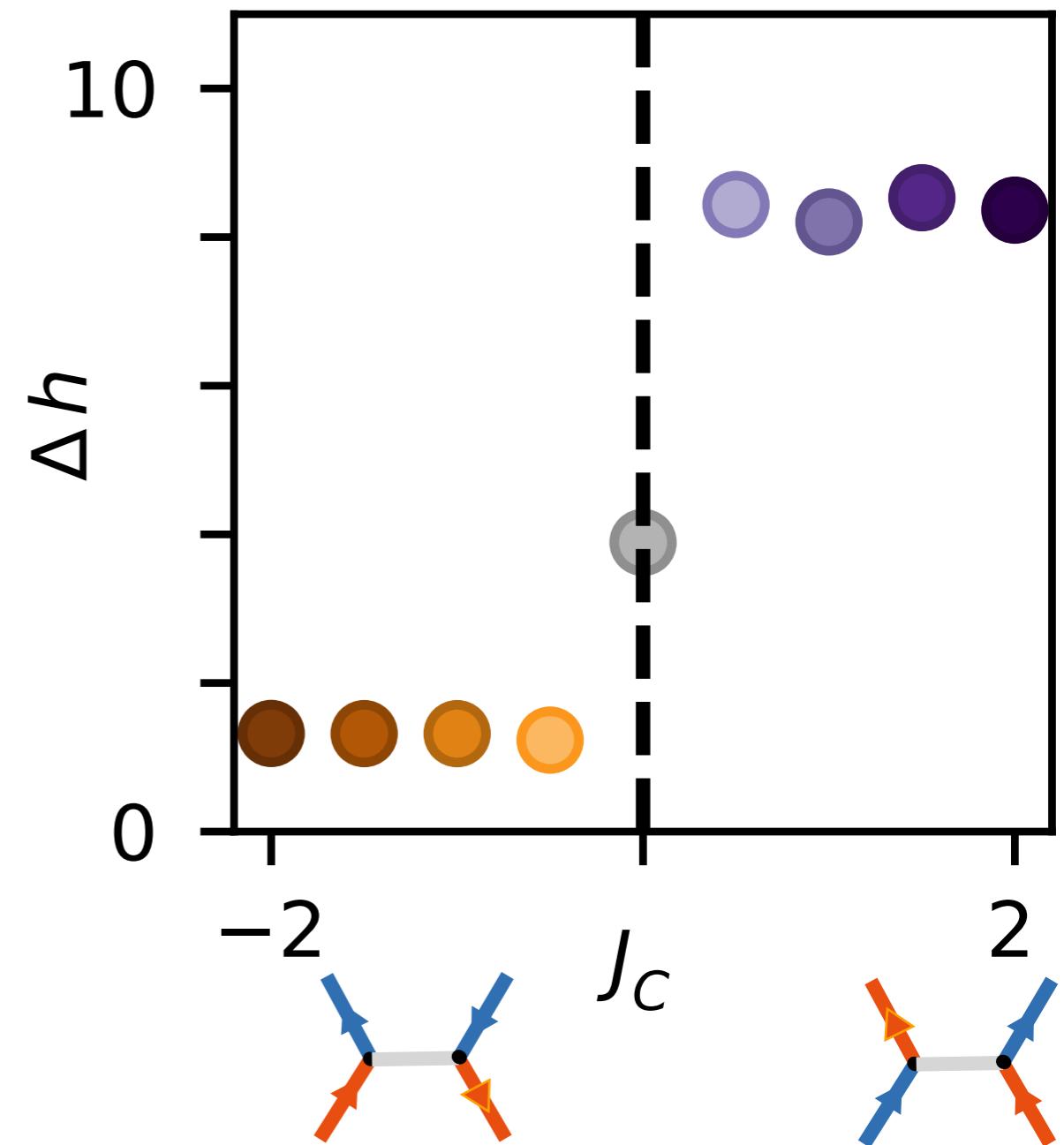


Nesting of the stream lines

Experiments



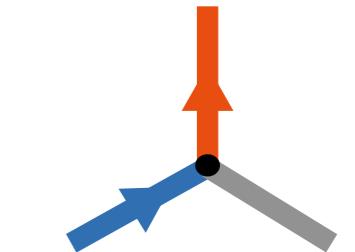
Simulations



Active Hydraulics

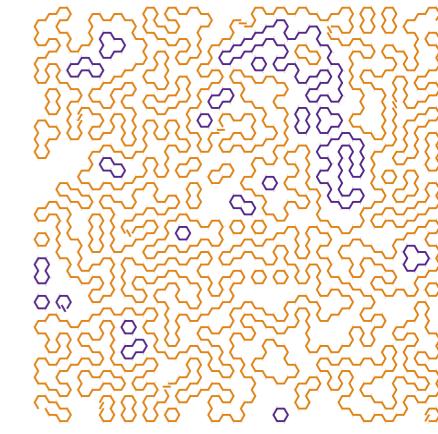
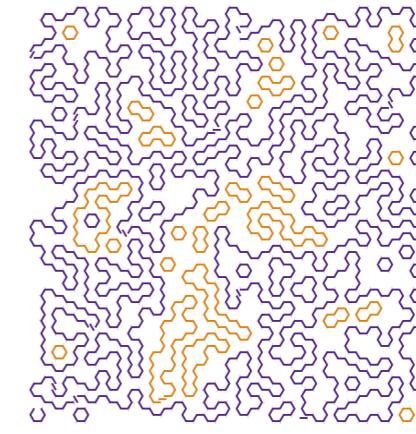
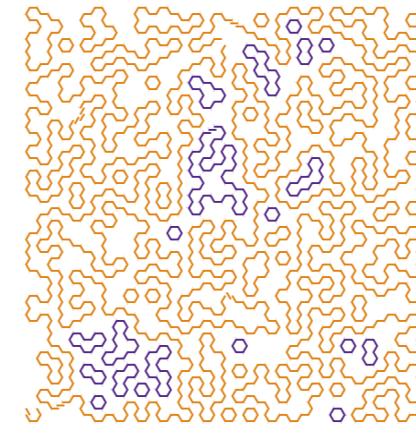
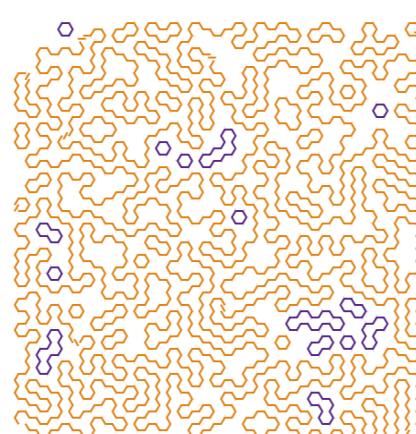
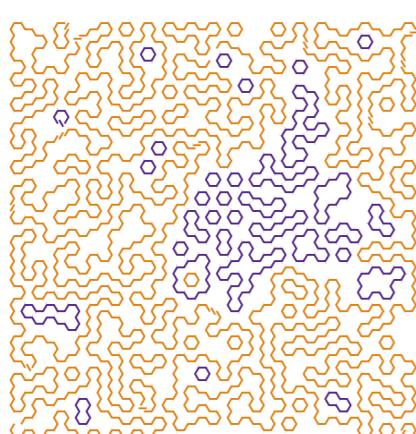
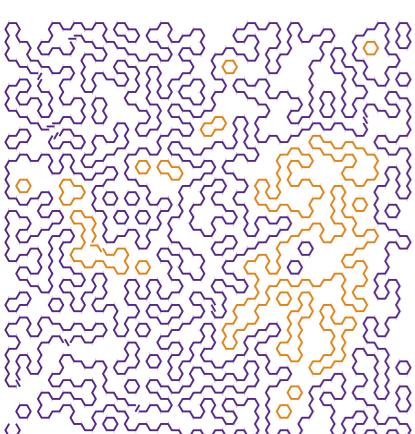
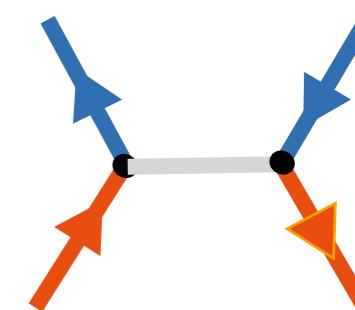
1 – Mass conservation

$$\sum_{\text{node } i} \mathbf{J}_i = 0$$
$$J_i = \pm J_0, 0$$

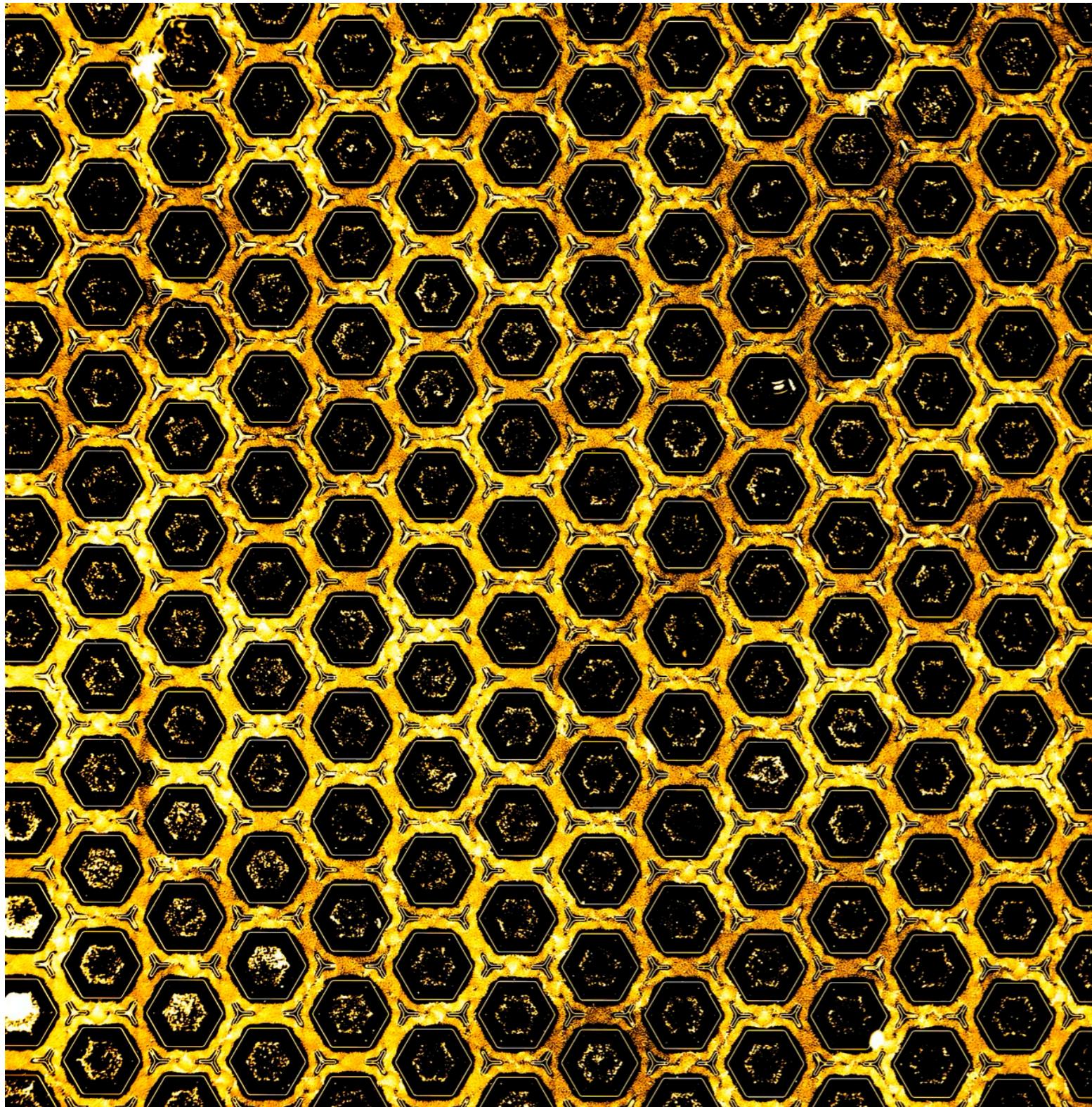


2 – Spontaneous flows

3 – Defect-mediated interactions



Active Hydraulics



Crowd Hydrodynamics

Without any assumption
about pedestrian behavior



Crowds as continua

Conservation laws
&
Constitutive relations



Experimental measurements



Controlled perturbations



2,000 runners

Response to a boundary perturbation



Hydrodynamic model

Chicago marathon

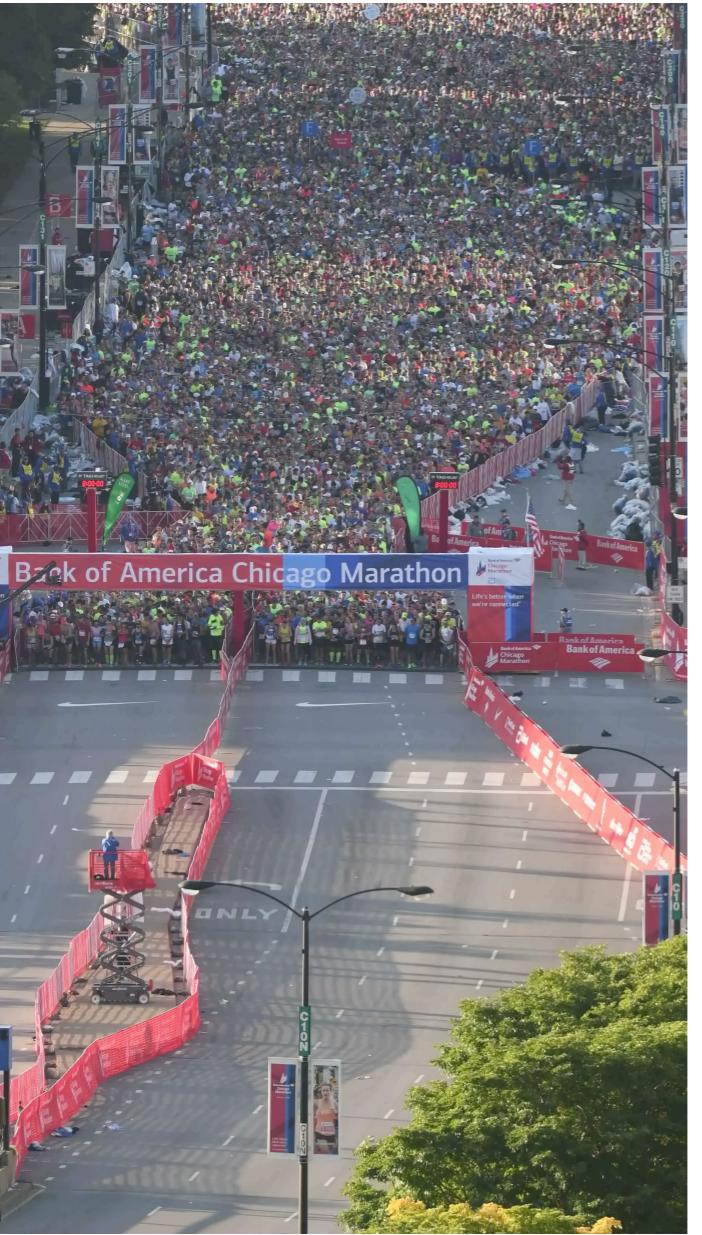


Chicago marathon

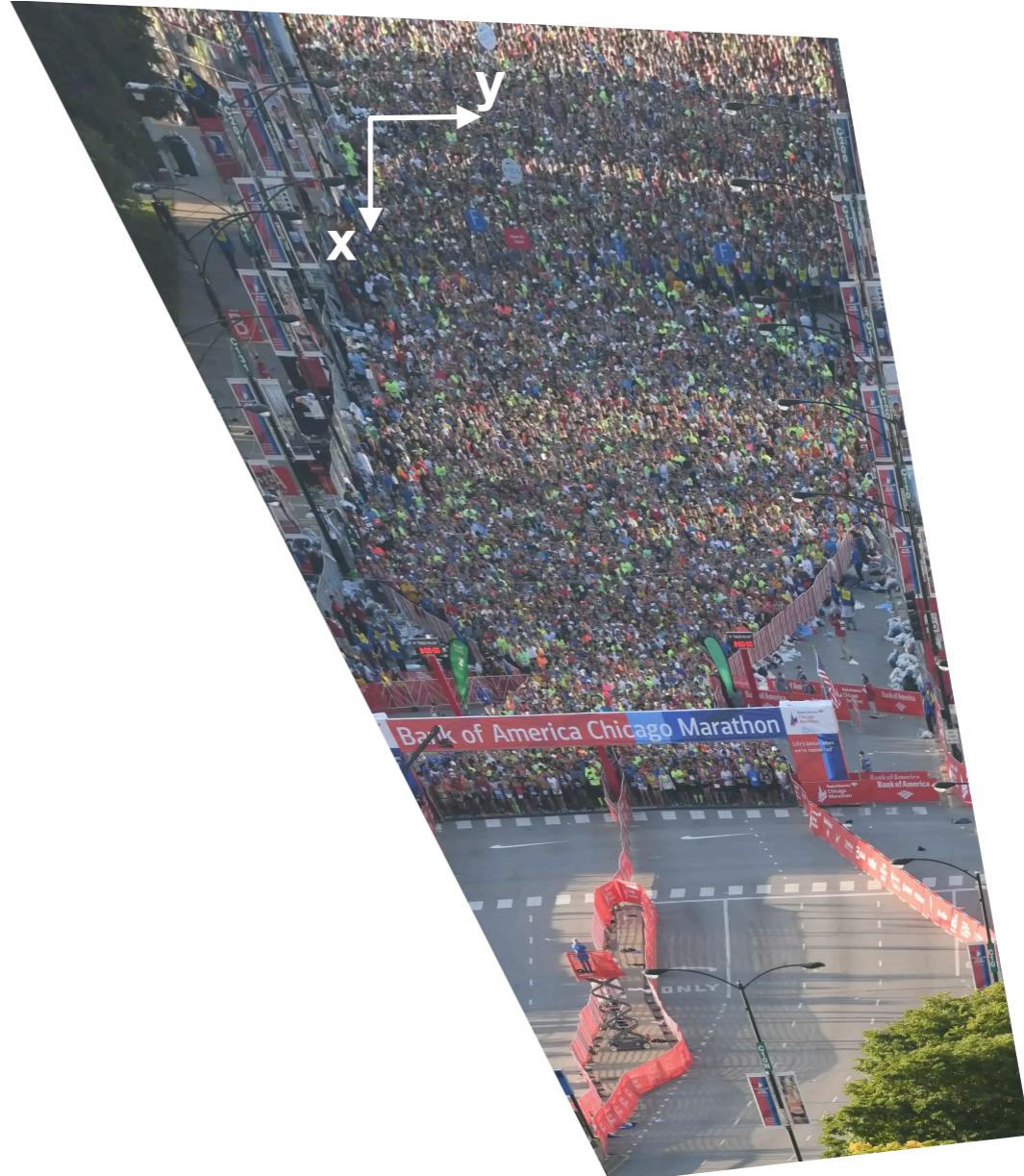


Image correction

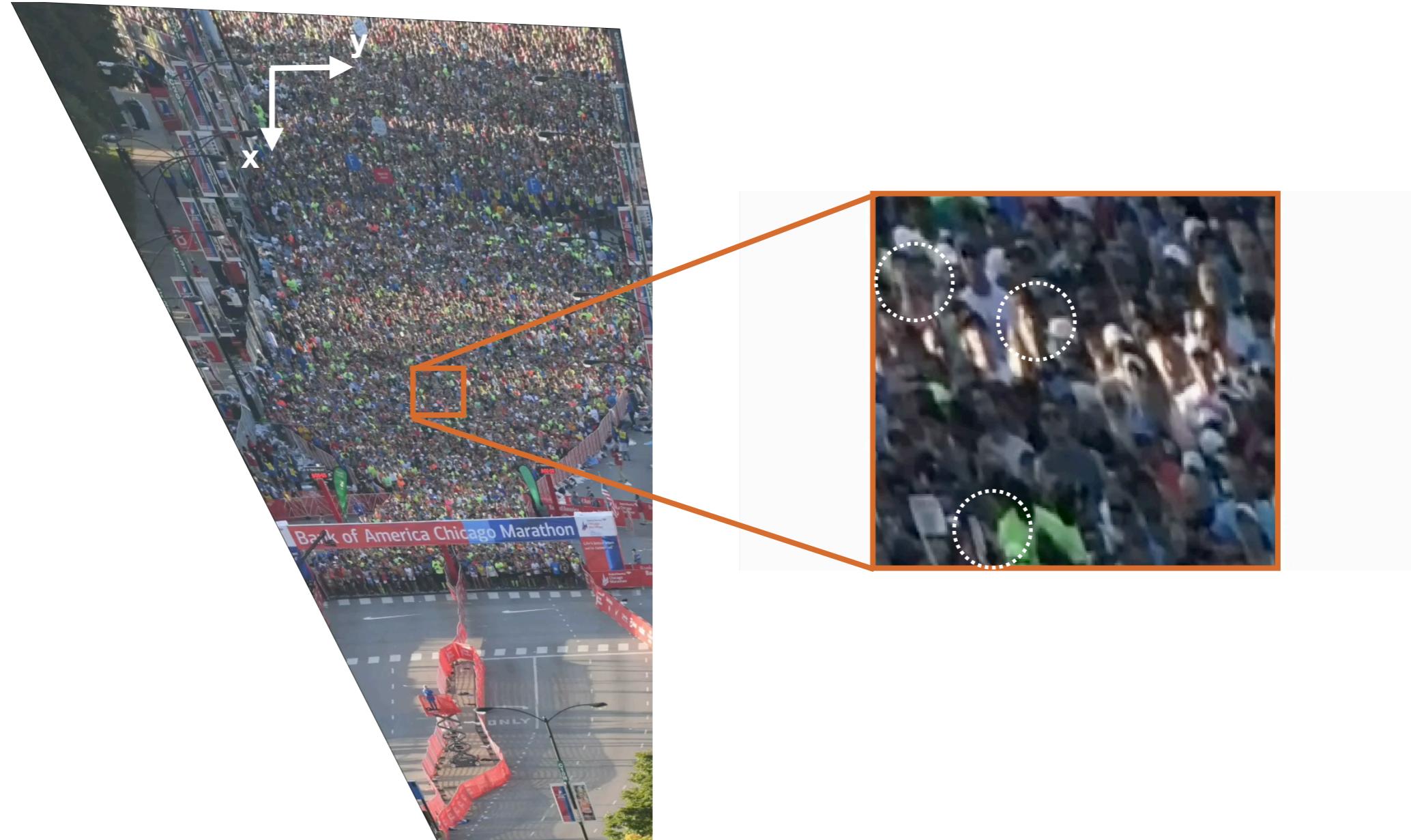
Raw image



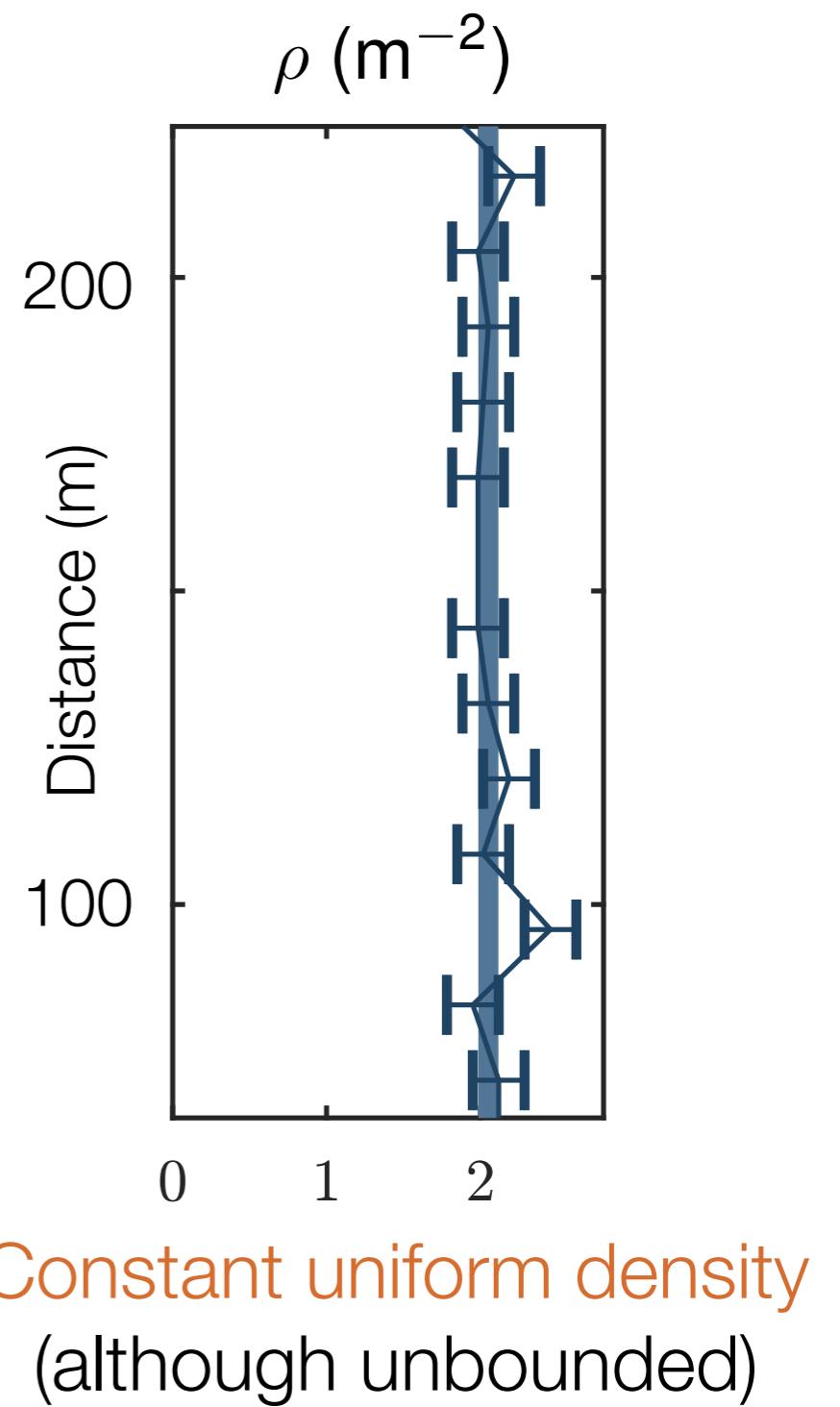
Corrected image



Density field



Crowd hydrostatics



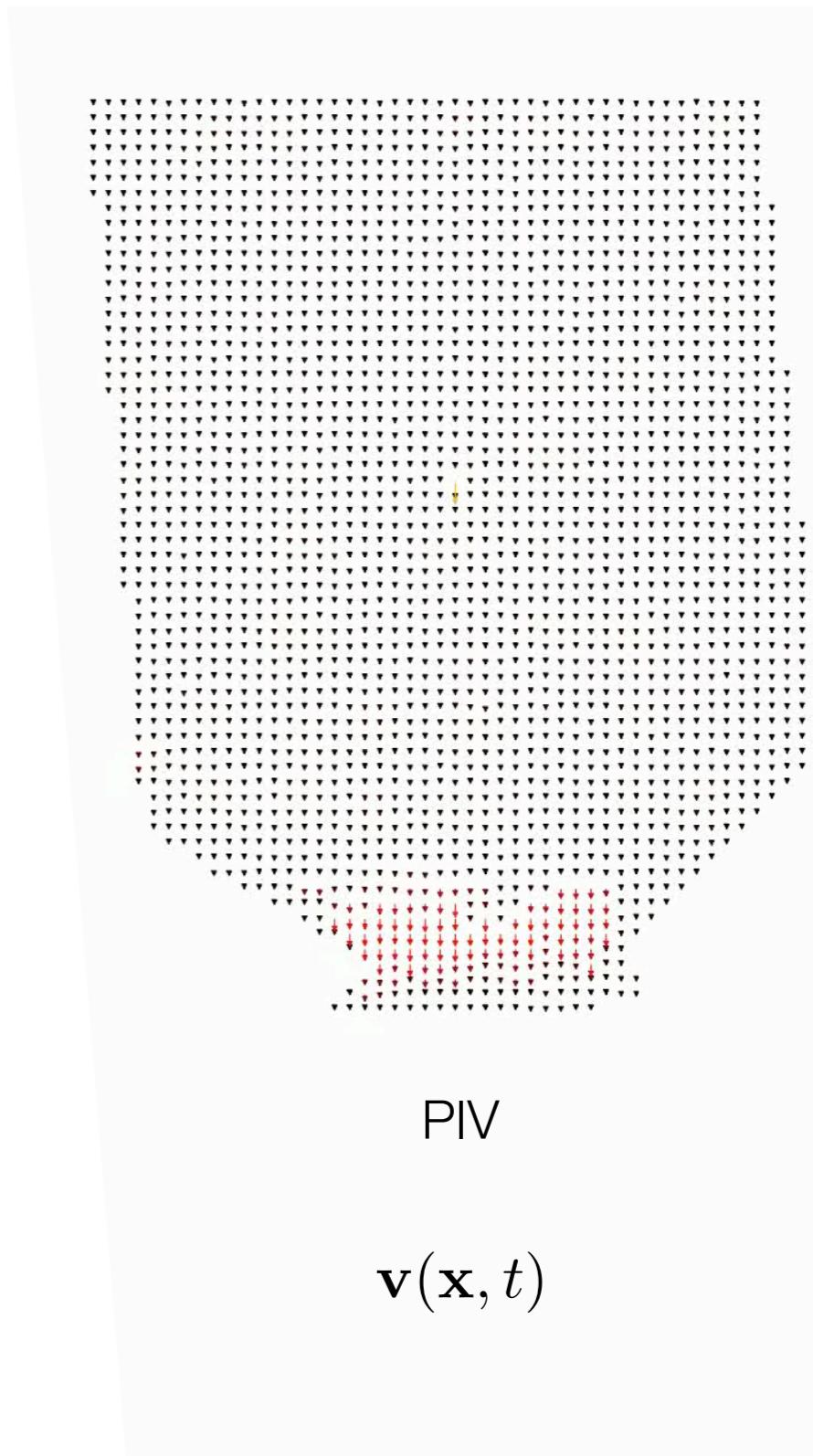
$$\rho_0 = 2.2 \pm .05 \text{ m}^{-2}$$

Crowd dynamics



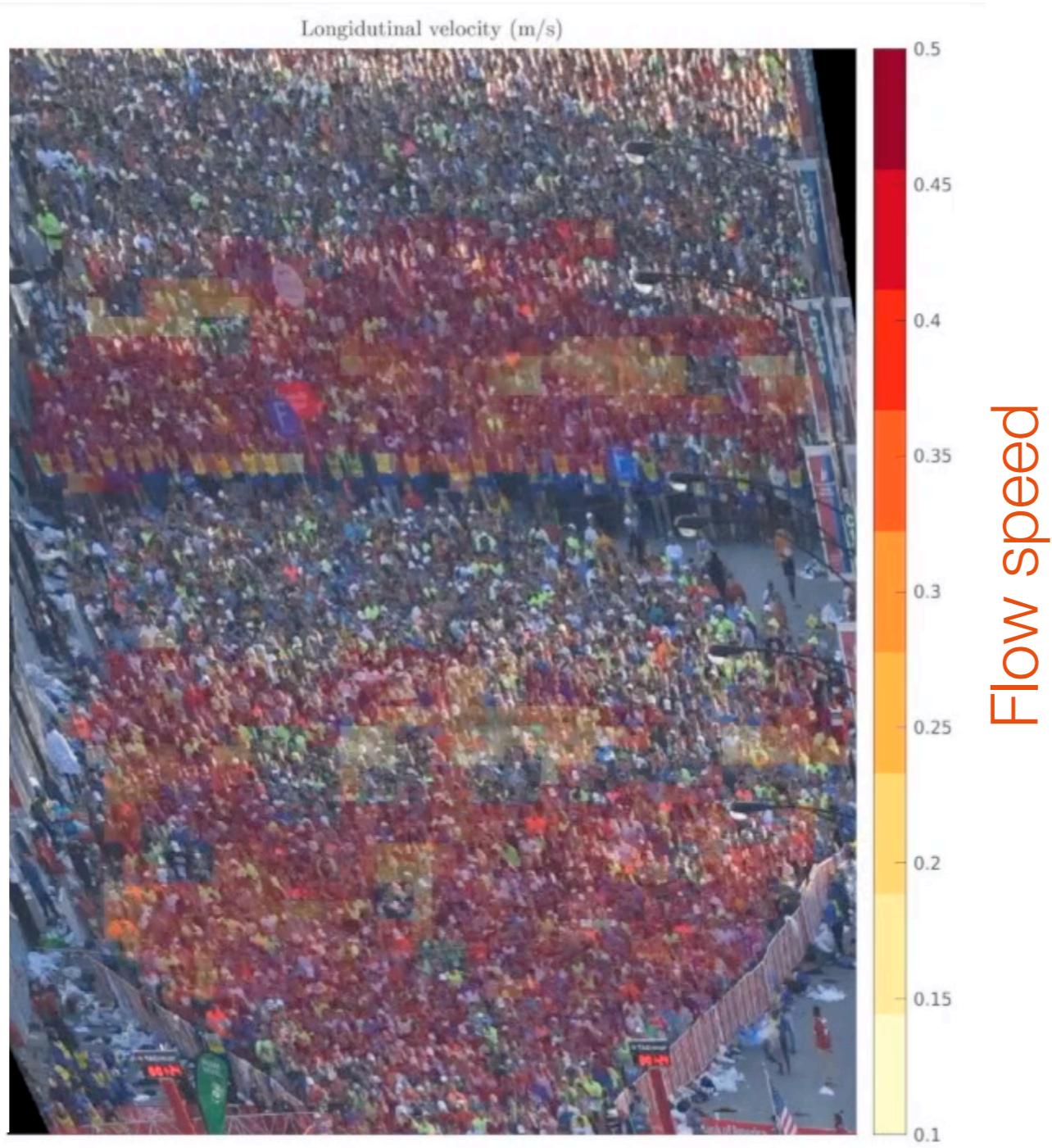
$10 \text{ m} \times 1 \text{ m}$

$\delta v \sim 10 \text{ cm/s}$

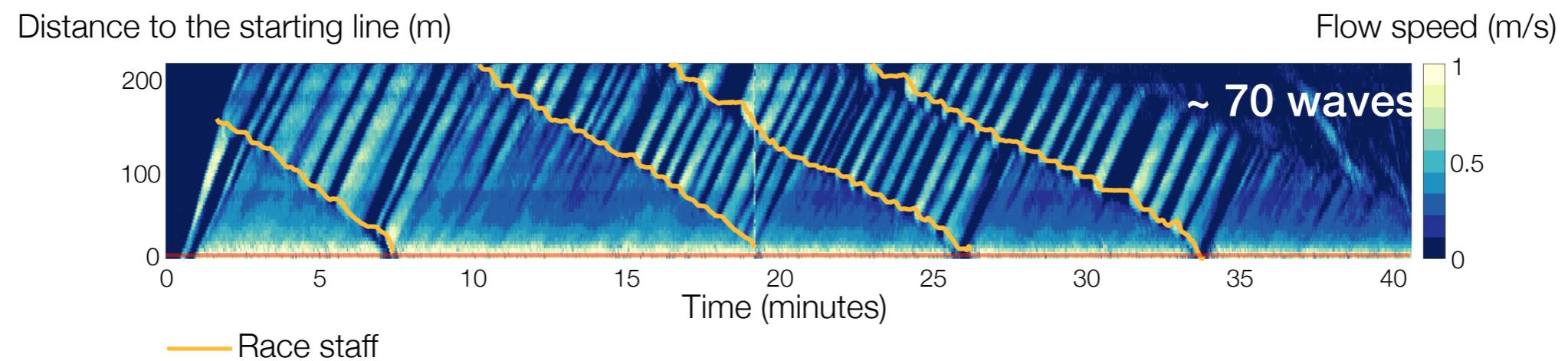


$\mathbf{v}(\mathbf{x}, t)$

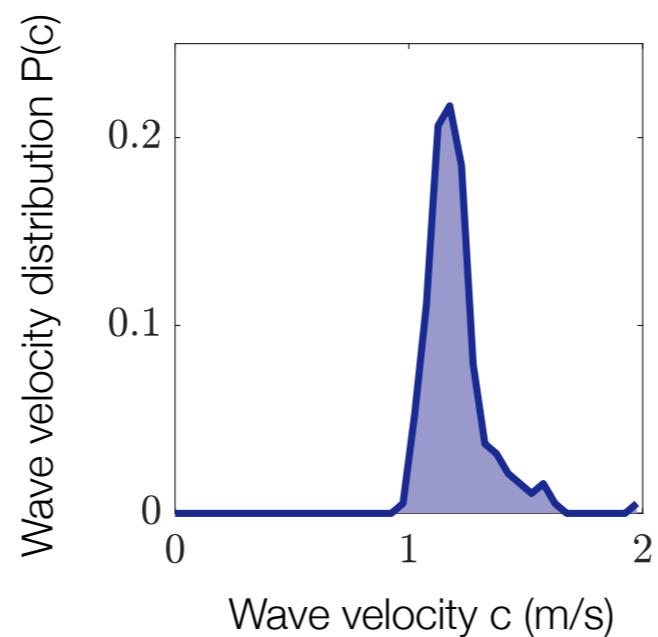
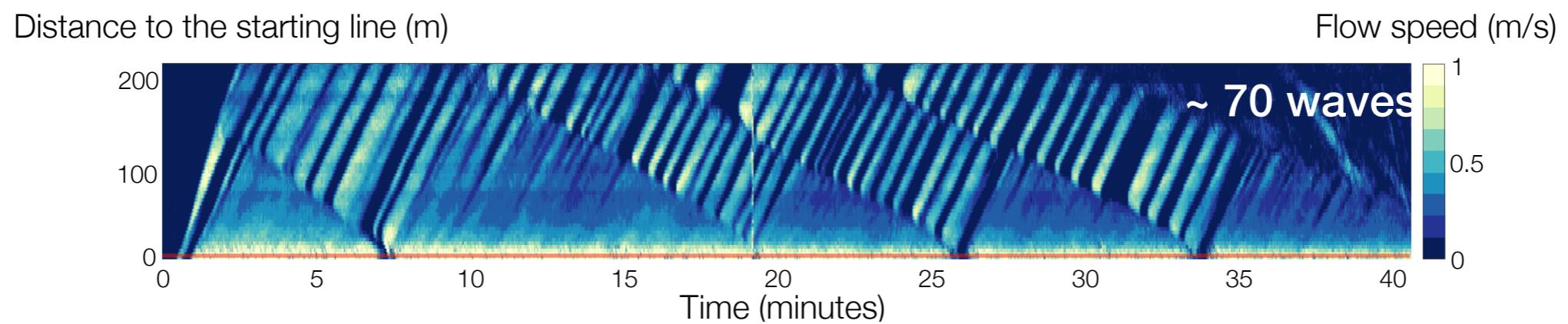
Density-speed waves



Dynamic response to boundary perturbations

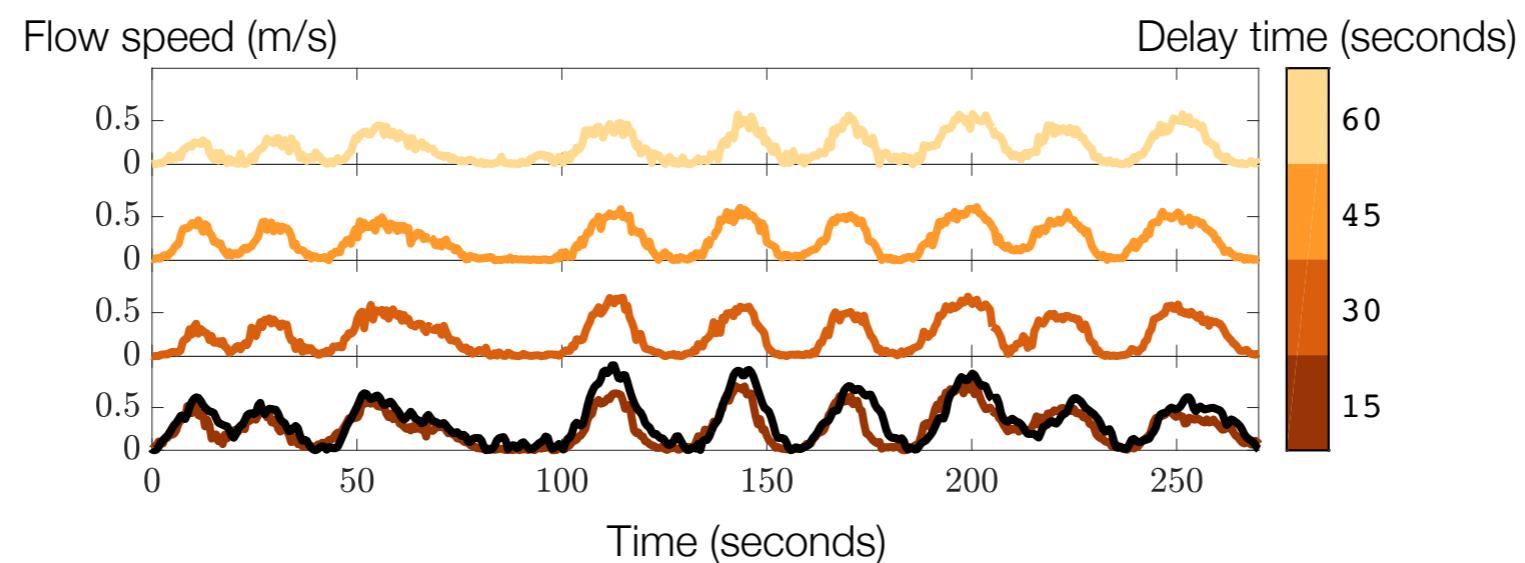
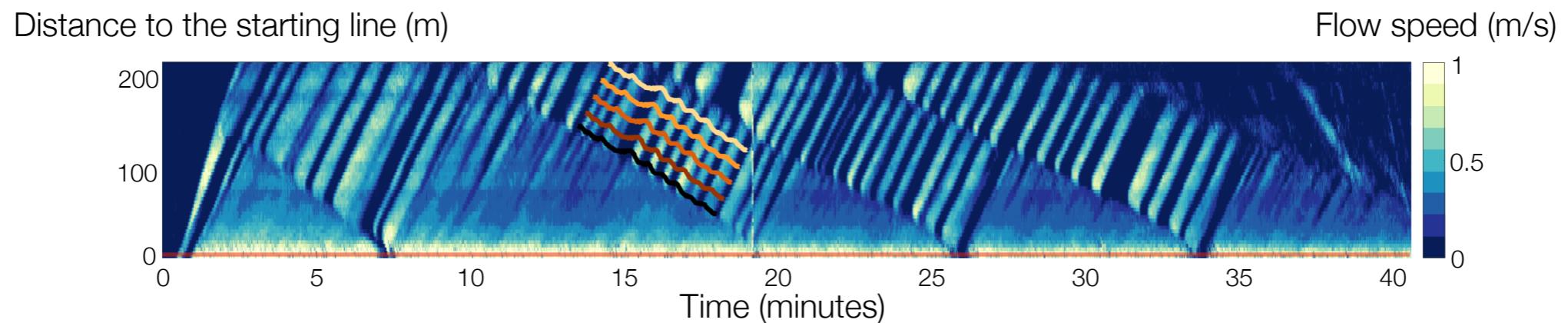


Constant wave speed



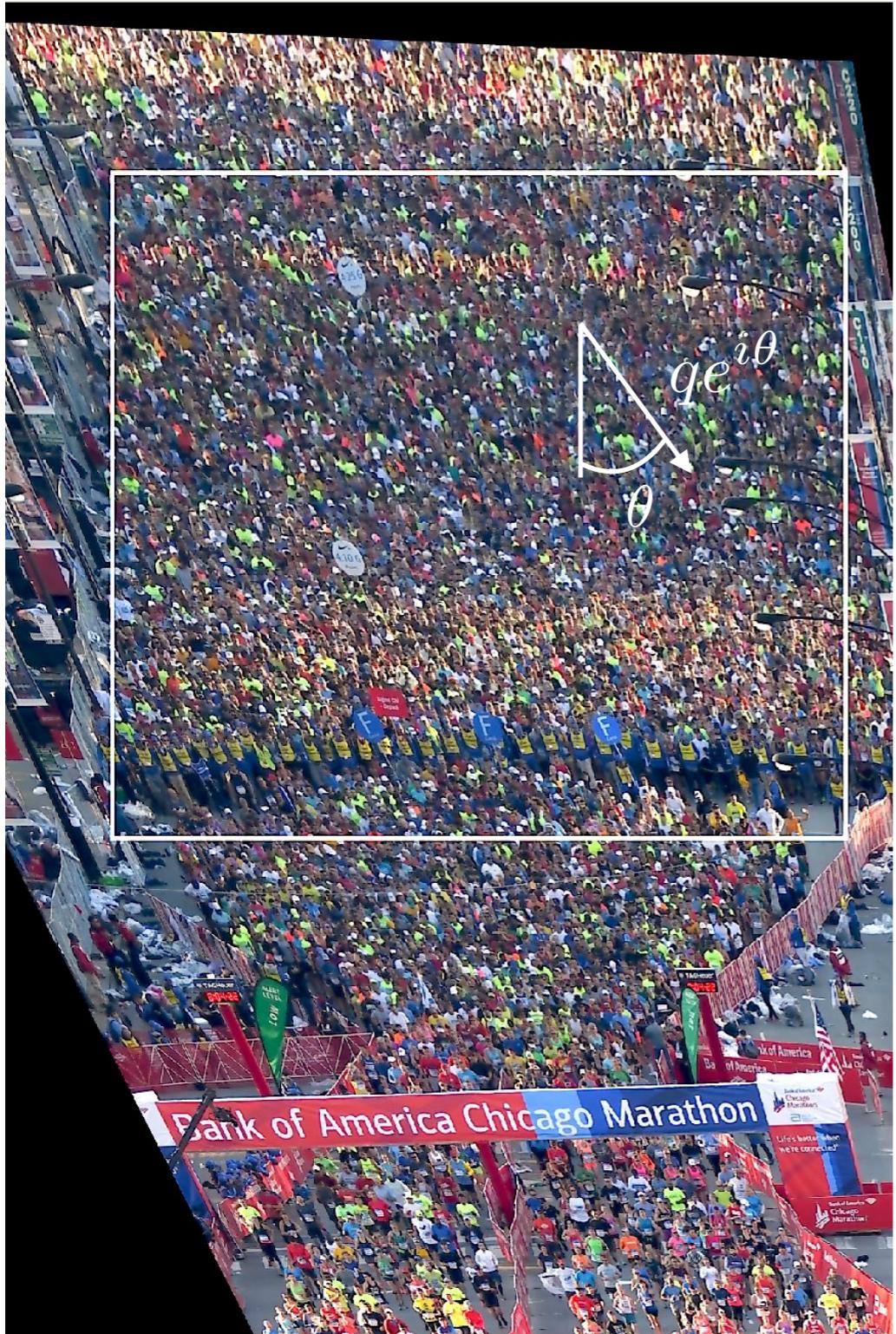
Wave velocity
 $\langle c \rangle = 1.2 \pm 0.3 \text{ m.s}^{-1}$

Linear response



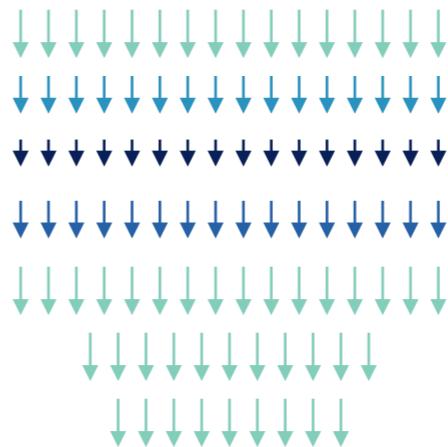
Linear, non-dispersive waves

Spectral analysis



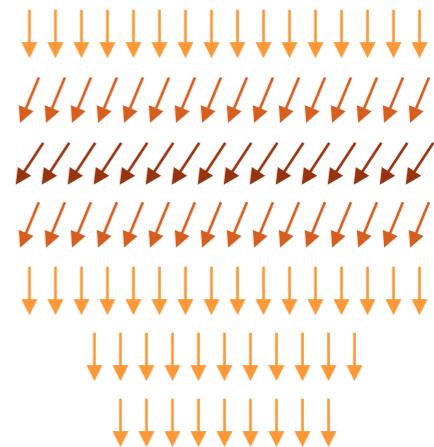
Flow speed

$$|\mathbf{v}|$$

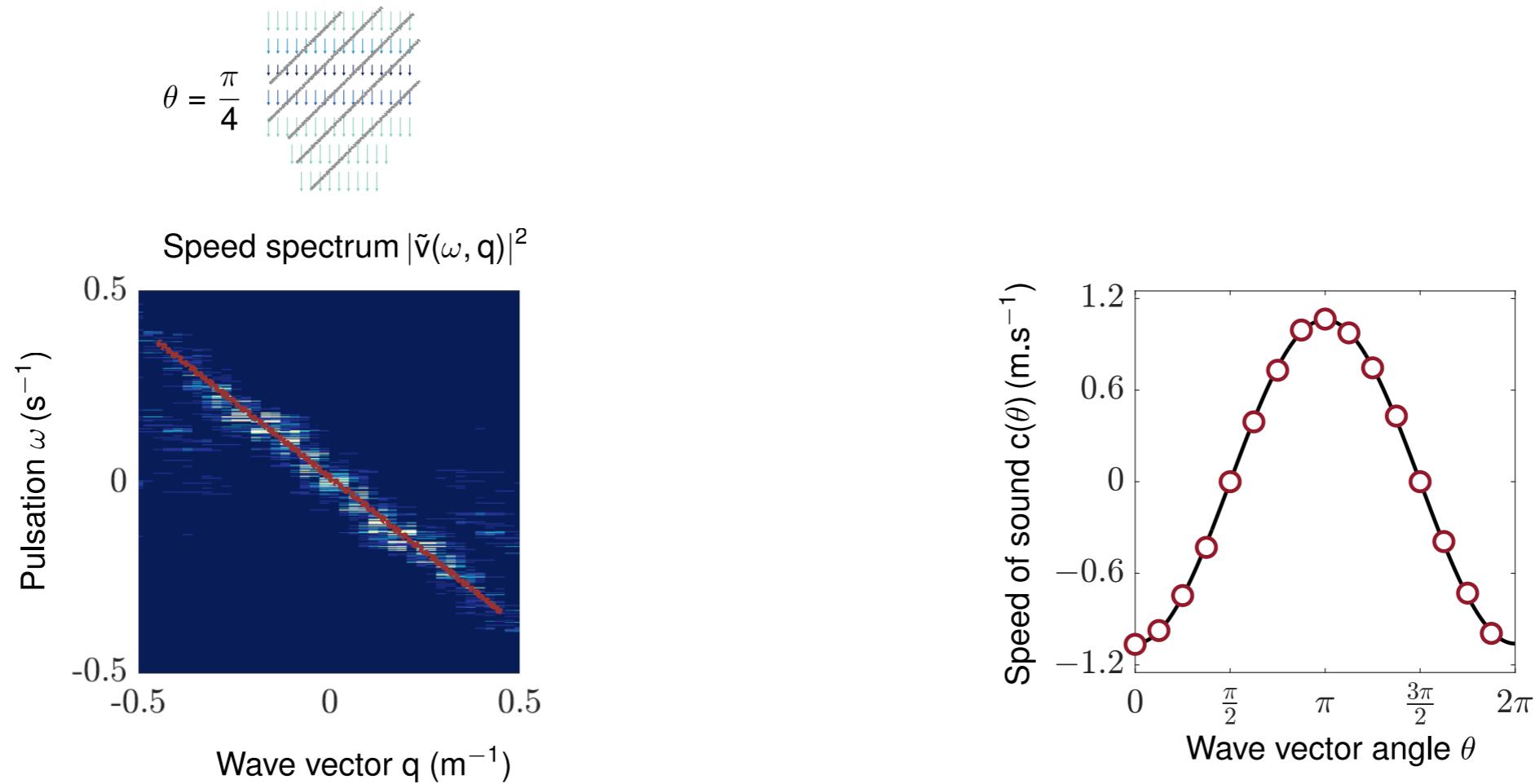


Flow orientation

$$\varphi = \arg(\mathbf{v})$$



Speed waves



Dispersion relation

$$\omega = c(\theta)q$$

Angular variations

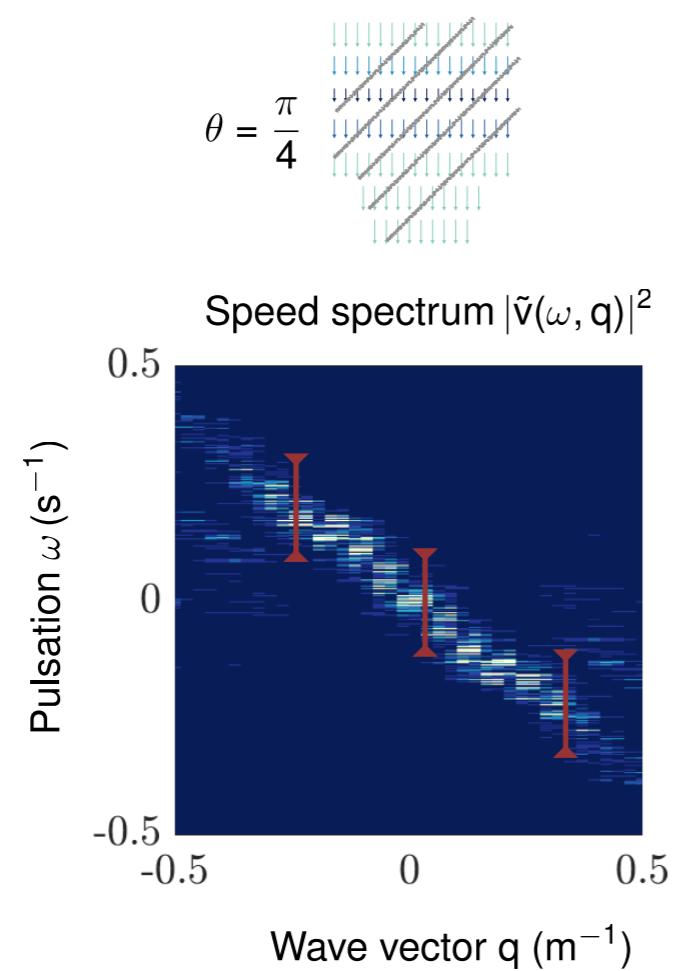
$$c(\theta) = -c_0 \cos \theta$$

Upstream transport only

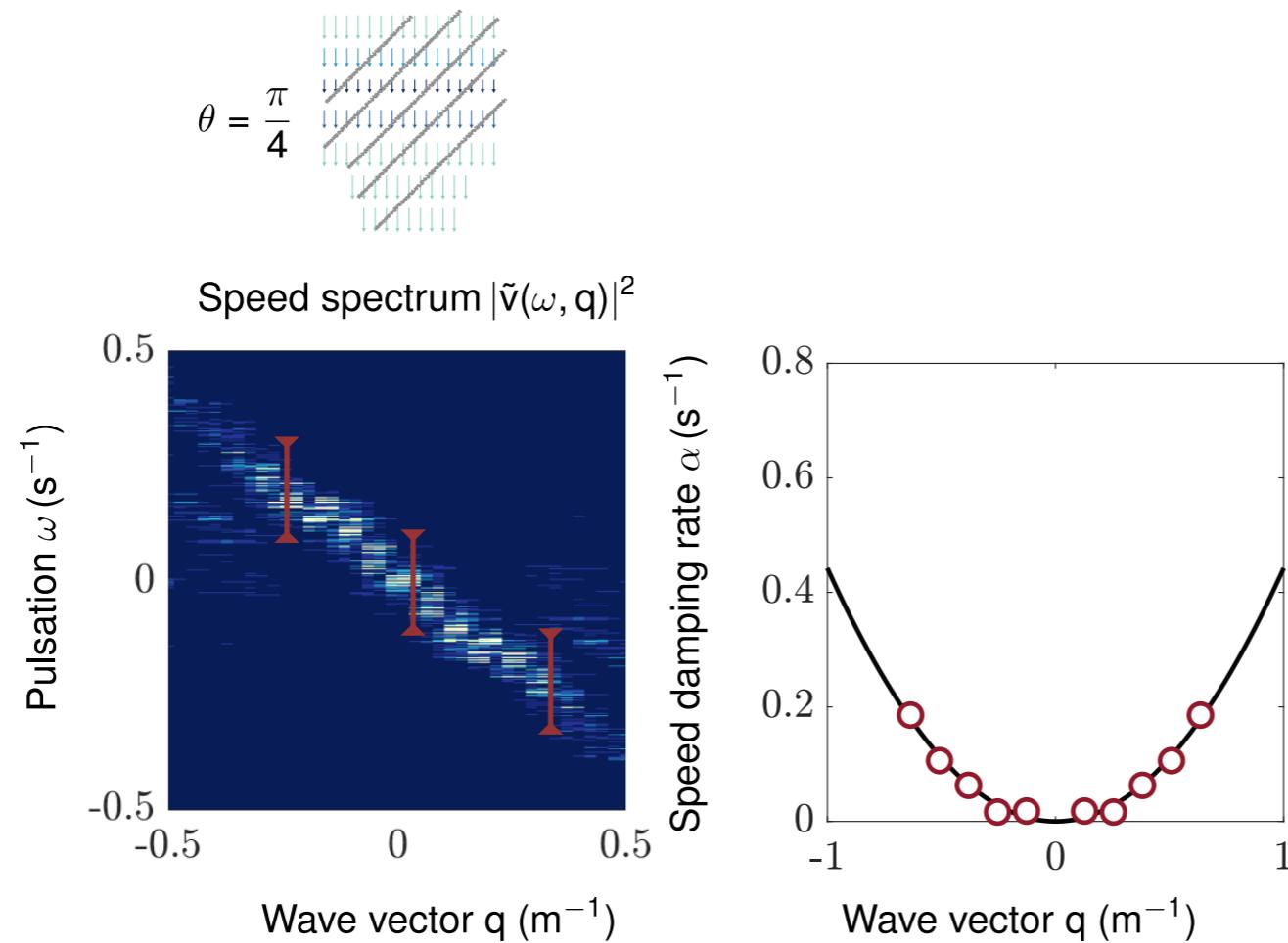
$$\omega = -c_0 q_x$$

$$c_0 = 1.2 \text{ m.s}^{-1}$$

Flow-speed damping

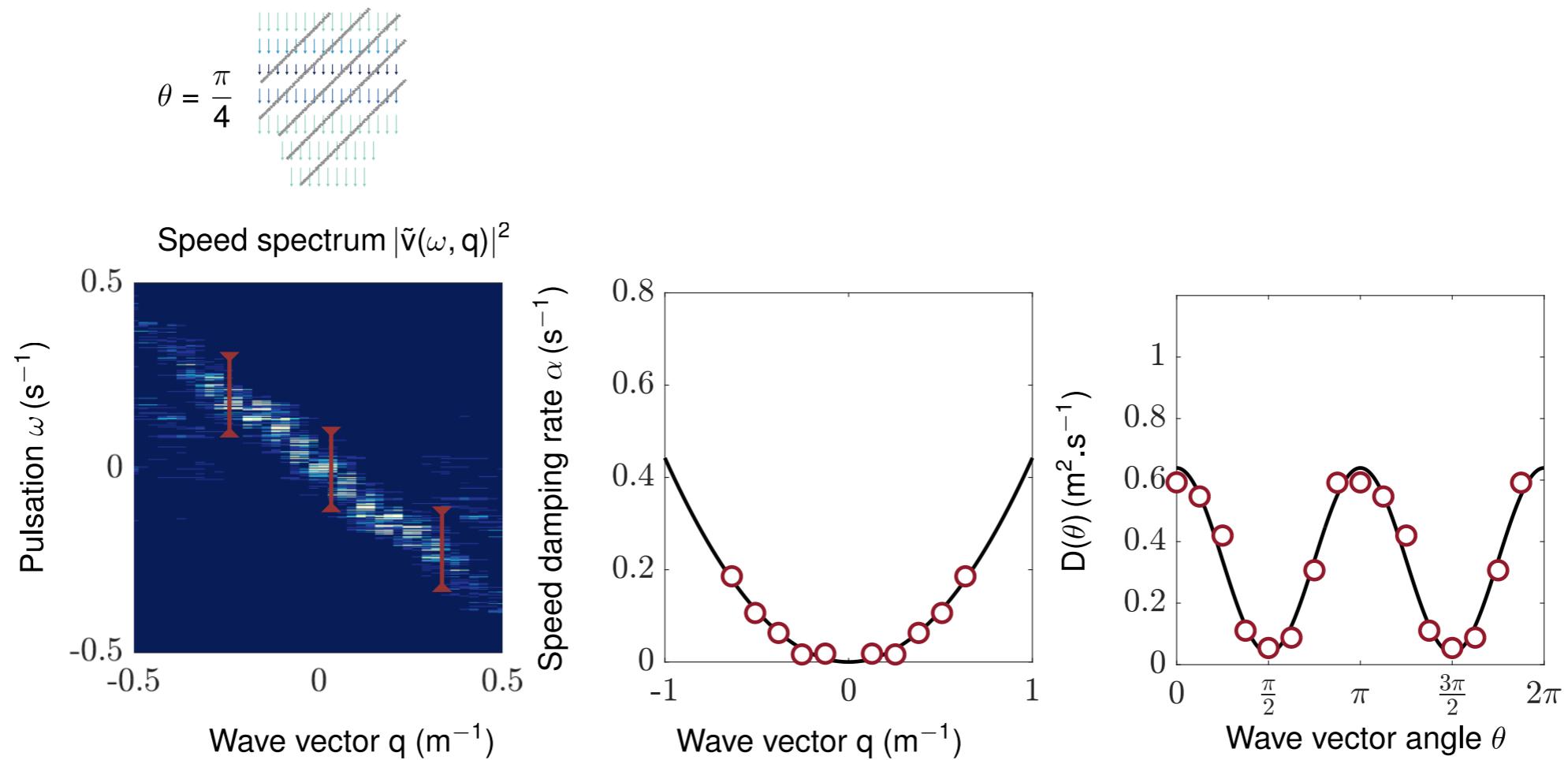


Flow-speed damping



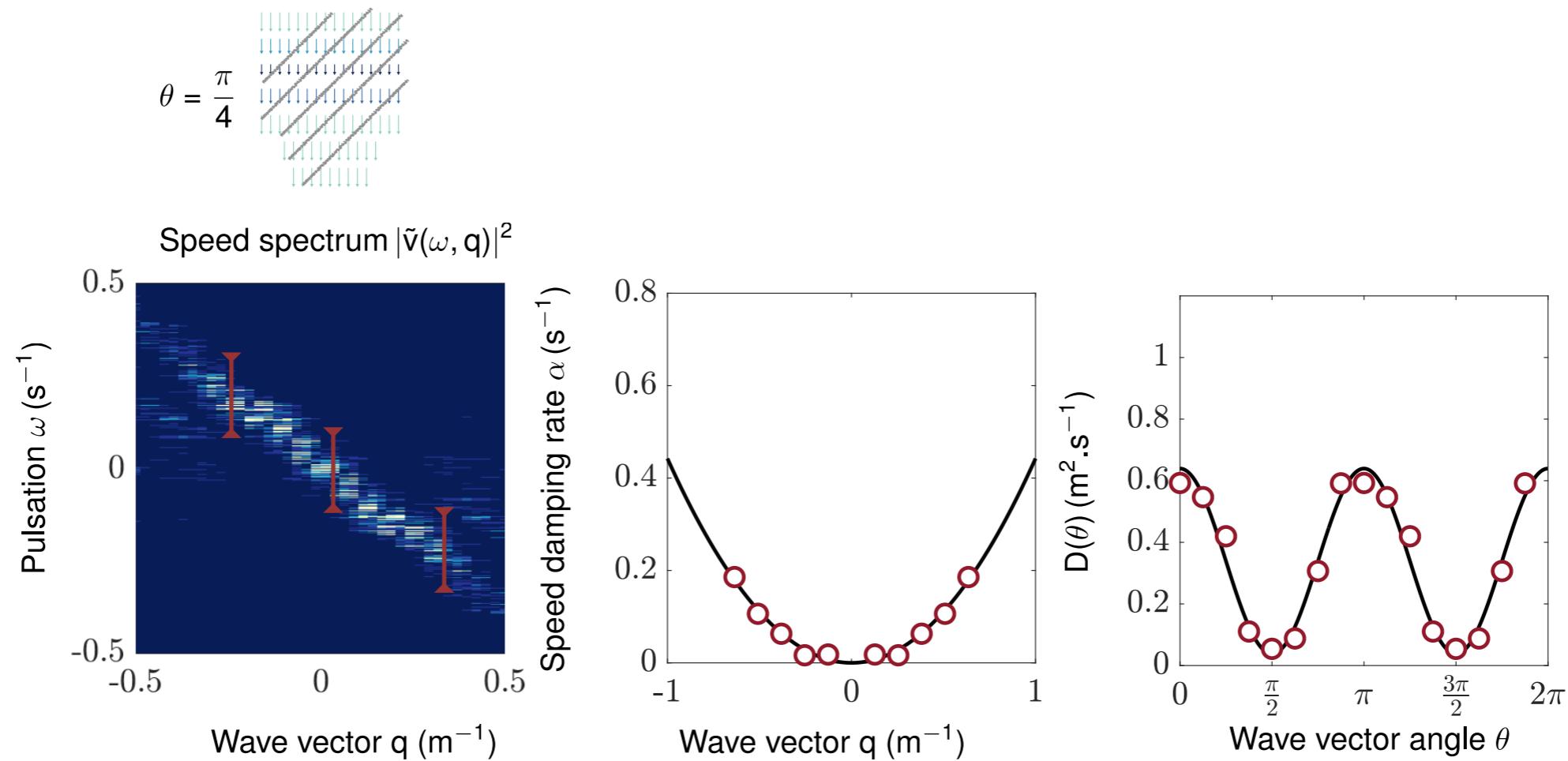
Diffusive damping
 $\alpha = D(\theta)q^2$

Flow-speed damping



$$D(\theta) = D_0 \cos^2 \theta$$
$$D_0 = 0.6 \text{ m}^2 \cdot \text{s}^{-1}$$

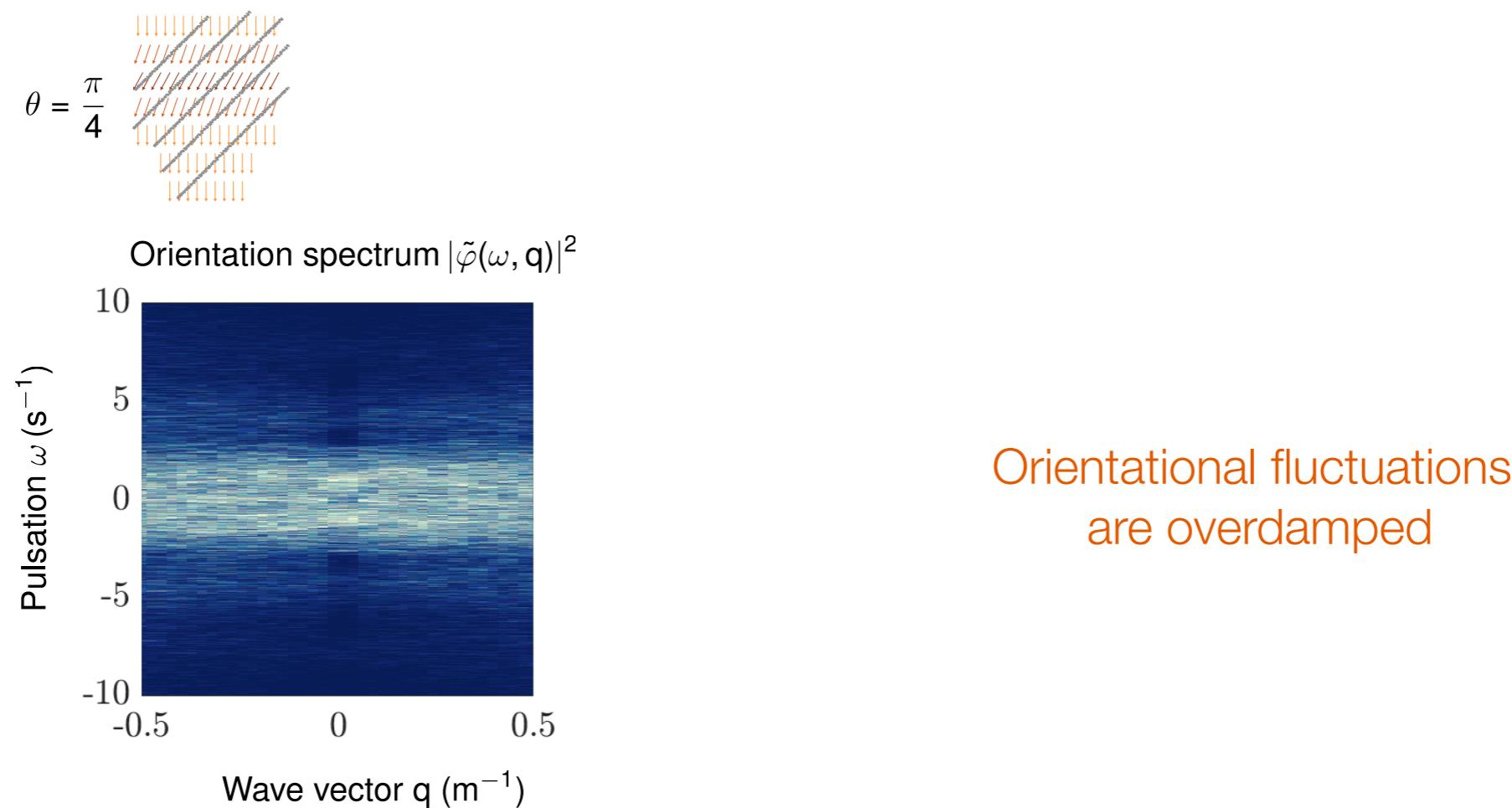
Flow-speed damping



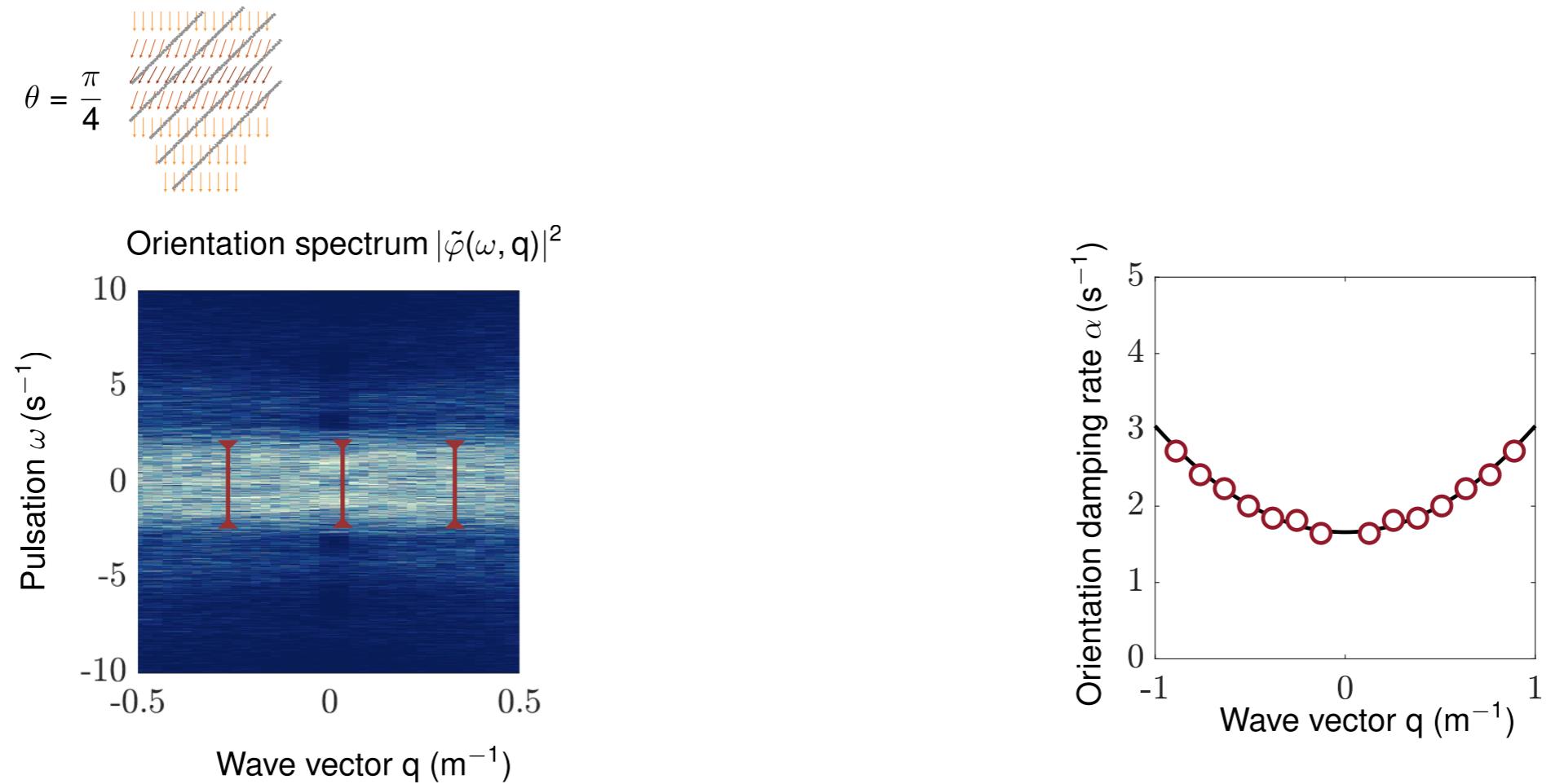
Slow 1D longitudinal dynamics

$$i\omega = -icq_x - D_0q_x^2$$

Orientational dynamics



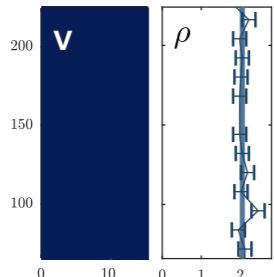
Orientational dynamics



Fast overdamped 2D dynamics

$$i\omega = -\alpha_0 - D_x q_x^2 - D_y q_y^2$$

Polarized crowds



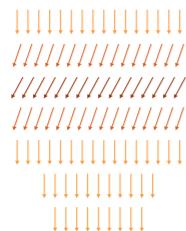
1) Static polarised crowds are homogeneous

$$\mathbf{v} = 0 \longrightarrow \rho = 2.2 \pm .05 \text{ m}^{-2}$$



3) Flow speed: slow 1D dynamics, no intrinsic relaxation scale

$$i\omega = -ic_0 q_x - D_0 q_x^2$$



4) Flow orientation: fast relation at all scales

$$i\omega = -\alpha_0 + \mathcal{O}(q^2)$$

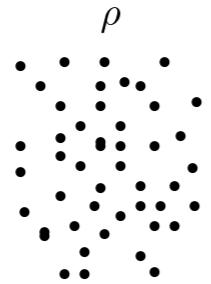
Crowd hydrodynamics

Conservation laws, symmetries & phenomenology

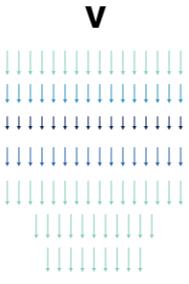
No behavioral assumption

Three fields

Density



Velocity



Polarization



Simplifying observation

- People do not walk sideways

$$\hat{\mathbf{v}} = \hat{\mathbf{p}}$$

Conservation laws

Mass conservation:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

Momentum conservation:

$$\rho D_t \mathbf{v} = \nabla \cdot \sigma + \mathbf{F}_f$$

Overdamped angular dynamic

$$\partial_t \mathbf{p} = \mathbf{T}$$

Stress field

Momentum conservation:

$$\rho D_t \mathbf{v} = \cancel{\nabla} \cdot \boldsymbol{\sigma} + \mathbf{F}_f$$

Pressure stress

$$\boldsymbol{\sigma} = -P(\rho) + \mathcal{O}(\nabla)$$

Linear response

$$\nabla \cdot \boldsymbol{\sigma} \sim -\beta \nabla \rho$$

Body force

Momentum conservation:

$$\rho D_t \mathbf{v} = -\beta \nabla \rho + \mathbf{F}_f$$

Friction Force



Body force

Momentum conservation:

$$\rho D_t \mathbf{v} = -\beta \nabla \rho + \mathbf{F}_f$$

Friction Force



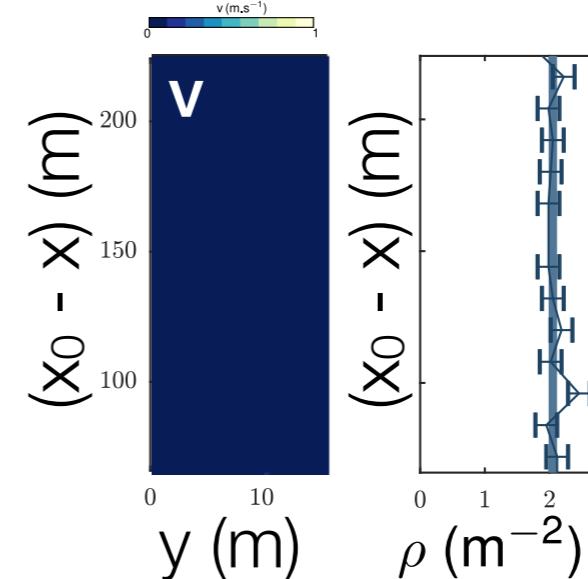
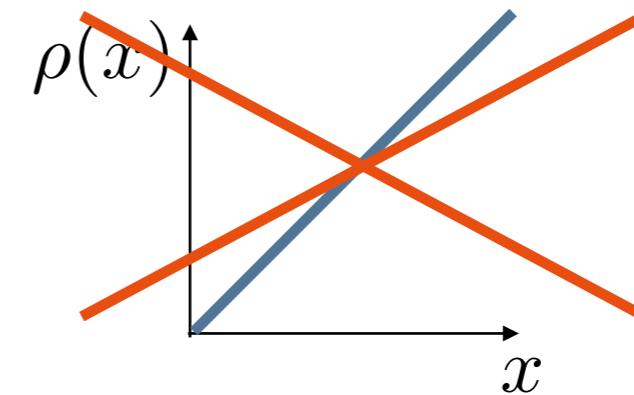
$$\mathbf{F}_f = -\Gamma \cdot (\mathbf{v} - \nu_0 \mathbf{p}) + \mathcal{O}(\nabla)$$

Crowd hydrostatics

Force Balance

$$0 = -\beta \nabla \rho + \nu_0 \mathbf{p}$$

$$\nu_0(\rho_0) = 0$$



Constant & uniform density

$$\rho_0 = 2.2 \pm .05 \text{ m}^{-2}$$

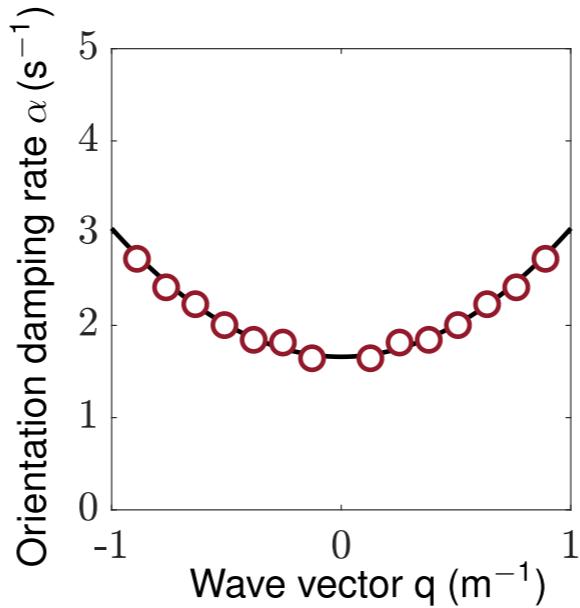
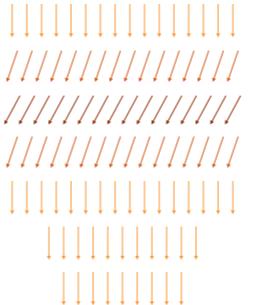
Body torque

Angular dynamics

$$\partial_t \mathbf{p} = \mathbf{T}$$

Friction Torque

$$\mathbf{T} = -\Gamma_r (\mathbf{p} - h\hat{\mathbf{x}})$$



Polarized crowd hydrodynamics

$$\partial_t v - c_0 \partial_x v - D_0 \partial_x^2 v = 0$$

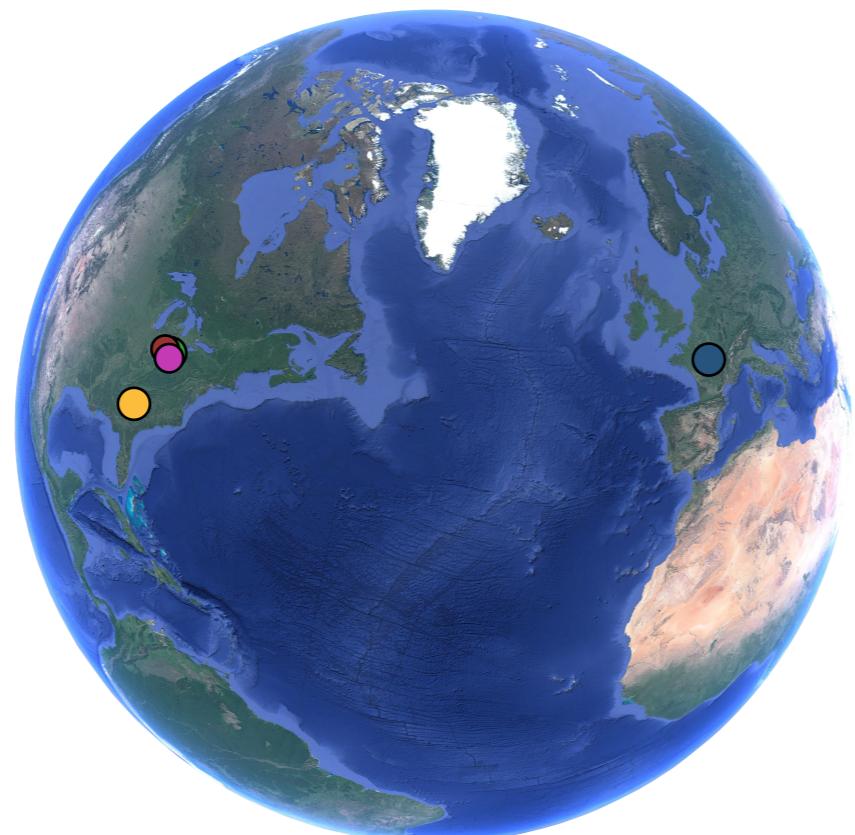
$$c_0 = -\rho_0 \nu'_0(\rho_0) \qquad \qquad D_0 = \frac{\rho_0 \beta}{\Gamma_x}$$

Active friction

Compressibility

Predictive theory?

$$\partial_t v - c_0 \partial_x v - D_0 \partial_x^2 v = 0$$



2016 Chicago Marathon

2017 Chicago Marathon

2017 Paris Marathon

2017 Peach Tree Road Race

2018 Chicago Marathon

Predictive theory

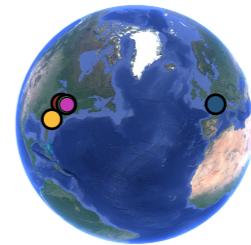
2016



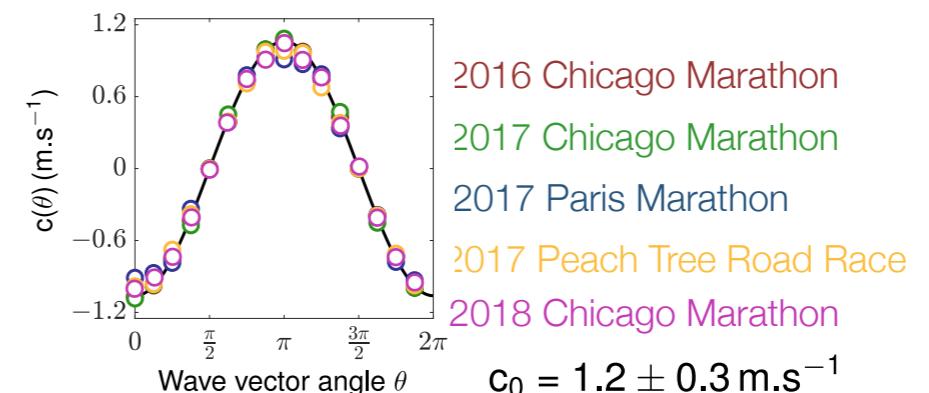
2017



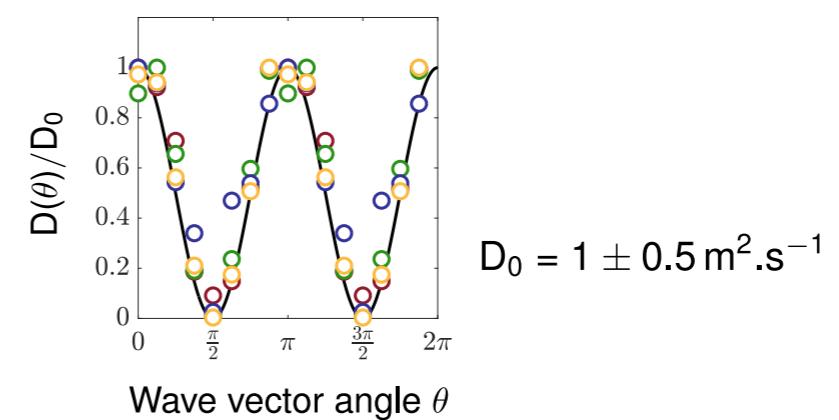
2018



Wave speed



Diffusive damping



Camille Jorge



Amélie Chardac



Alexis Poncet



Nicolas Bain



Alexandre Morin



Delphine Geyer





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